

On Proper Coloring Of Some Special Graphs

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Abstract:

In this paper we prove that the chromatic number for Moser-Spindle graph is 4.And also showed the existence of chromatic number of lollipop graph is m. where m represents number of vertices in the complete graph. An open star of finite r copies of fan graph which admits proper coloring and whose chromatic number is 3. In a triple triangular snake, and Alternate triple triangular snake having its chromatic number 3.

Keywords: Triple Triangular snake, Alternate, Triple Triangular snake Lollipop graph, , Chromatic number

Introduction:

Graph theory is an old subject with many modern applications. Its basic ideas were introduced in the eighteenth century by the great Swiss mathematician Leonhard Euler. In the last three decades graph theory has established itself a worthwhile mathematical discipline and there are many applications of graph theory to a wide variety of subjects such as operation research, physics etc. [5]. A graph G consists of pair (V (G), E (G)) where V (G) is a non empty finite set whose elements are called **vertices** and E (G) is set of unordered pairs of distinct elements of V (G) called **edges** of graph .

Preliminaries:

Definition 1

Graph coloring is a special case of graph labeling. It is an assignment of labels traditionally called "colors" to elements of a graph subject to certain constraints. In its simplest form, it is a way of coloring the vertices of graph such that two adjacent vertices share the same color.

Definition 2

The Moser graph which is also called Moser spindle is an undirected graph with 7 vertices and 11 edges [3].

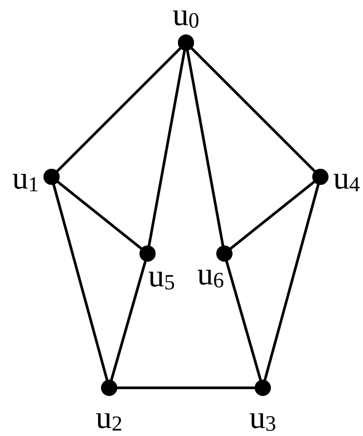

Figure 1:MoserSpindle graph

Definition 3

The (m, n)-Lolligraph is a special type of graph consisting of a complete graph on m vertices and a path graph on n vertices connected with a bridge[2].

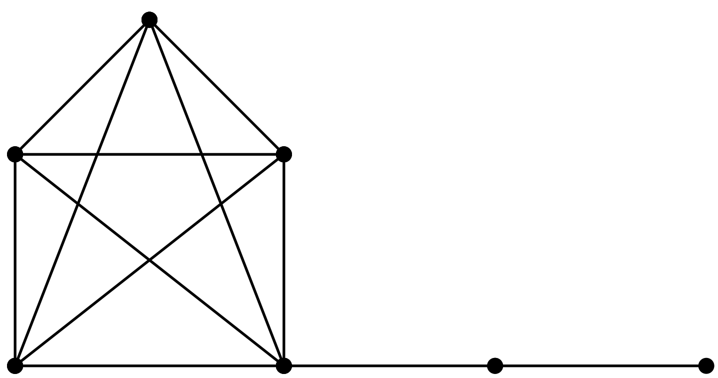

Figure 2: Lollipop L(5, 2) graph

Definition 4

An alternate triangular snake $A(T_n)$ is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} to new vertex v_i. That is every alternate edge of a path is replaced by C_3.

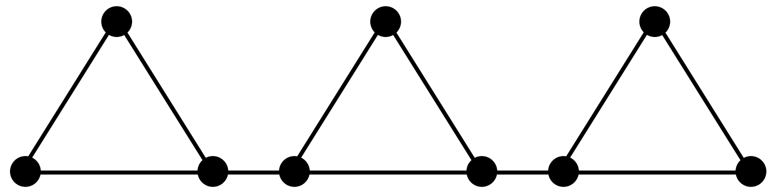

Figure 3 Alternate triangular snake AT_3

Definition 5

A double alternate triangular snake $DA(T_n)$ consists of two alternate triangular snakes that have a common path. That is a double alternate triangular snake is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} to two new vertices v_i and w_i [1].

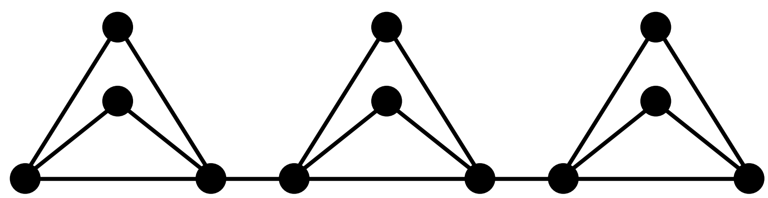

Figure 4 Double alternate triangular snake $AD(T_3)$

Definition 6

Double triangular snake $D(T_n)$ consists of two triangular snakes that have a common path[1]

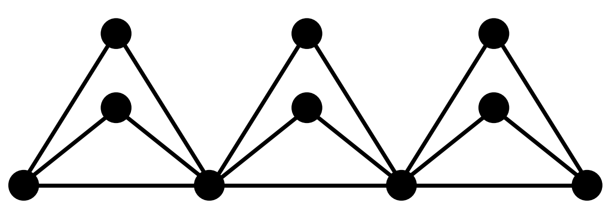

Figure 5 Double triangular snake DT_3

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Main Results:

Theorem 1:

The Moser-Spindle graph acknowledges proper coloring and whose chromatic number is 4.

Proof:

Consider the Moser Spindle graph with 7 vertices and 11 edges. Let uj be the vertex set for $0 \leq j \leq 6$.

Then the vertex function $f : V(G) \rightarrow \{1, 2, 3, 4\}$ as defined as follows:

(i) $f(u_0) = 1$
(ii) $f(u_1) = 1$
(iii) $f(u_2) = 3$
(iv) $f(u_3) = 2$
(v) $f(u_4) = 4$
(vi) $f(u_5) = 2$
(vii) $f(u_6) = 3$

Illustration:

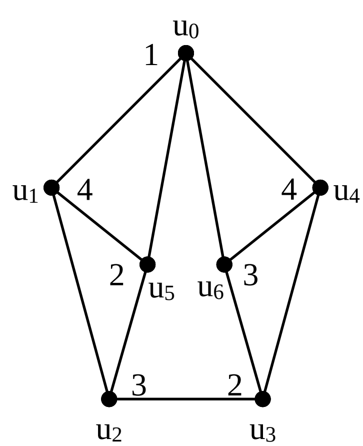

Figure 6: Moser-Spindle graph

Theorem 2:

The lollipop graph which acknowledges proper coloring, whose chromatic number m where m represents number of vertices in the complete graph.

Proof:

Let x_1, x_2 … x_n be the vertices of the complete graph K_m. x_{n+1}, x_{n+2} … x_{n+m} be the vertices of the path.

(i) $f(x_i) = i$, for $i = 1, 2, 3 \ldots n$

(ii) $f(x_{n+1}) = 1$

(iii) $f(x_{n+2}) = 2$

For path vertices coloring has to be given 1, 2 alternatively

Illustration 1:

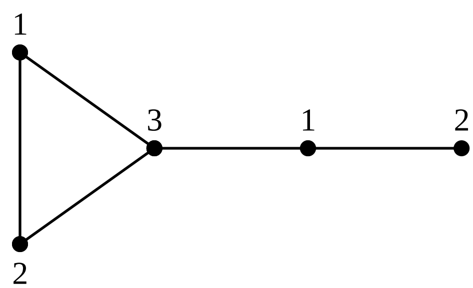

Figure 7: Lollipop graph $L_{3,2}$

Illustration 2:

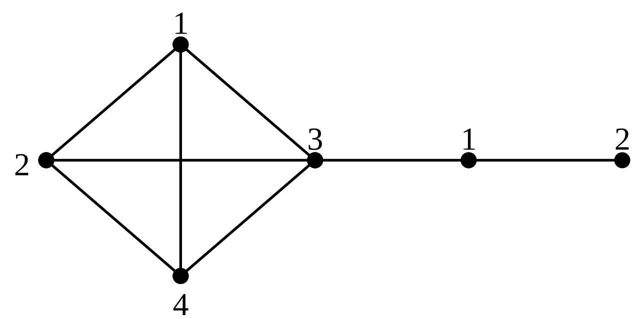

Figure 8: Lollipop graph $L_{4,2}$

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Theorem 3:

An open star of finite r copies of fan graph which admits proper coloring and whose chromatic number is 3.

Proof:

Consider a fan graph $F_n = P_n + K_1$. Let $G = S(r, F_n)$.

Let u_i $(1 \le i \le r)$ be the apex vertex of q^{th} copy of F_n. Let u_{ij} $(1 \le i \le r, 1 \le j \le n)$ be the j^{th} vertex in P_n of i^{th} copy of F_n. Let u be the apex vertex of the star graph $K_{1,n}$. We note that $|V(G)| = r(n+1) + 1$ and $|E(G)| = 2nr$.

Let we define the coloring $f : V(G) \rightarrow \{1, 2, 3\}$ as follows.

(i) $f(u) = 1$

(ii) $f(u_i) = 2$, when $i = 1, 3 \dots n-1$

(iii) $f(u_i) = 3$, when $i = 2, 4 \dots n$

(iv) $f(u_{1i}) = 1$, when $i = 1, 3$

(v) $f(u_{1i}) = 3$, when $i = 2, 4$

(vi) $f(u_{2i}) = 2$, when $i = 1, 3$

(vii) $f(u_{2i}) = 1$, when $i = 2, 4$

(viii) $f(u_{3i}) = 3$, when $i = 1, 3$

(ix) $f(u_{3i}) = 3$, when $i = 2, 4$

(x) $f(u_{4i}) = 1$, when $i = 1, 3$

(xi) $f(u_{4i}) = 2$, when $i = 2, 4$

Illustration 1:

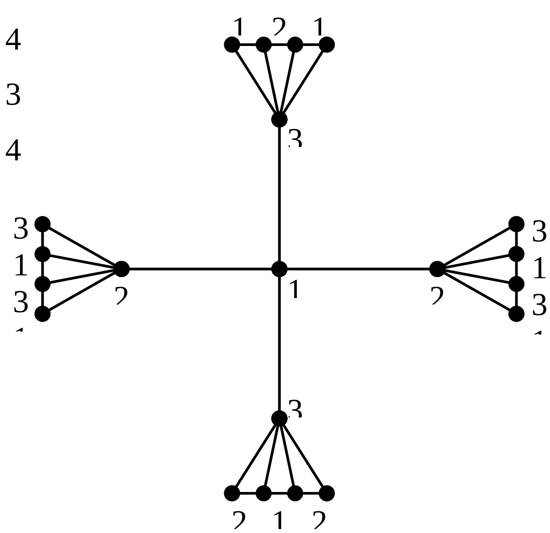

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Figure 9 Finite 4 copies of fan graph

Illustration 2:

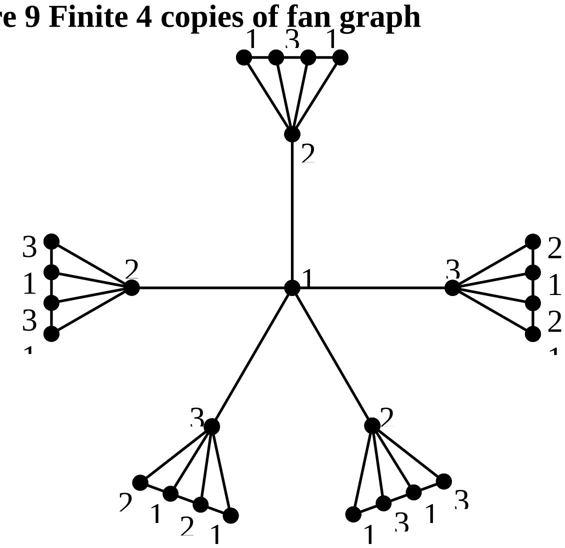

Figure 10: Finite 5 copies of fan graph

Theorem 4:

In a triple triangular snake, the existence of chromatic number is 3.

Proof:

Let g be the graph obtained from a path x_1 x_2 … x_n by joining x_i x_{i+1}, to three new vertices y_i, w_i and a_i, for $1 \leq i \leq n-1$.

Define the function $f : V(G) \rightarrow \{1, 2, 3\}$, by

i) $f(x_i) = 1$, when 'i' is odd

ii) $f(x_i) = 2$, when 'i' is even

iii) $f(y_i) = 3$, for all i

iv) $f(w_i) = 3$, for all i

v) $f(a_i) = 3$, for all i

Example: Proper coloring of triple triangular snake $T(T_4)$

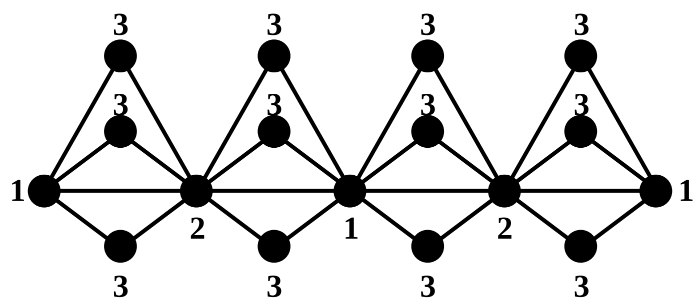

Figure 11

Example: Proper coloring of triple triangular snake $T(T_6)$

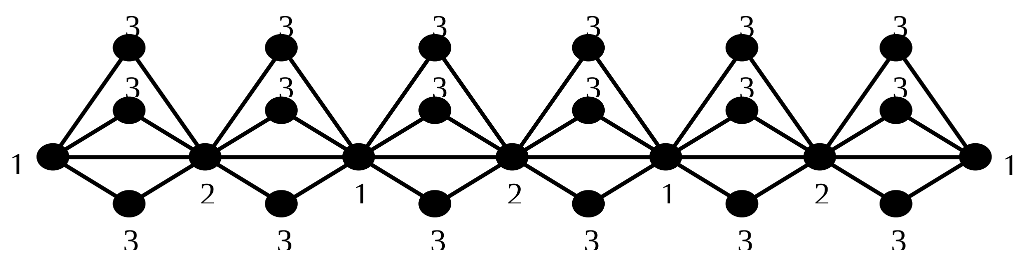

Figure 12

In the above two graphs having its chromatic number 3.

Theorem 5:

Alternate triple triangular snake having its chromatic number 3.

Proof:

Let G be the alternate triple triangular snake graph,[6] and let $a_1, a_2, \ldots, a_n$ be the path vertices by joining a_i, a_{i+1} to three new vertices b_i, c_i and d_i, $1 \le i \le n-1$.

Define the function $f : V(G) \to \{1, 2, 3\}$ by

i) $f(a_i) = 1$, for 'i' is an odd

ii) $f(a_i) = 2$, for 'i' is an even

iii) $f(b_i) = 3$, for all 'i'

iv) $f(c_i) = 3$, for all 'i'

v) $f(d_i) = 3$, for all 'i'

Example: Proper coloring of alternate triple triangular snake $T(T_3)$

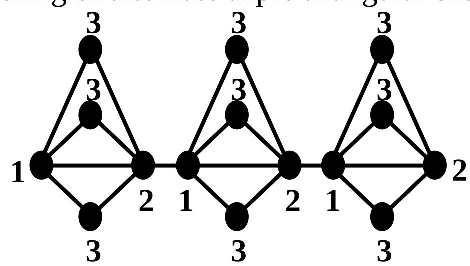

Figure 13

From the above graphs the proper coloring of $T(T_n)$ is 3.

Theorem 6:

The triangular belt TB(n) ($\downarrow^n$) admits a proper coloring and its chromatic number 4.

Proof:

Let G = TB(n) be the triangular belt. Let $L_n = P_n \times P_2$ ($n \geq 2$) be the ladder graph with vertex x_i and y_i, i = 1, 2, …, n.[4] The triangular belt is obtained from the ladder by adding the edges $x_i\, y_{i+1}$ for all $1 \leq i \leq n-1$.

Let the function $f : V(G) \rightarrow \{1, 2, 3, 4\}$ is defined by,

i) $f(x_i) = 1$, when 'i' is an odd

ii) $f(x_i) = 2$, when 'i' is an even

iii) $f(y_i) = 3$, when 'i' is an odd

iv) $f(y_i) = 4$, when 'i' is an even

Example: Triangular belt TB(4)

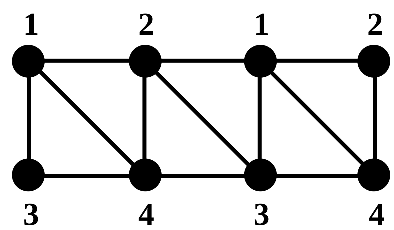

Figure 14

Theorem 6:

An alternate triangular belt ATB($\downarrow^n$) which admits acyclic coloring whose chromatic number is 4.

Proof:

The triangular belt is obtained from the ladder graph by adding the edges x_i, y_{i+1}, for i = 1, 3, 5, …, n–1 and adding edges x_{i+2} y_{i+1} for i = 1, 3, 5, …, n–1.

Let the function f : V(G) → {1, 2, 3, 4} is defined by,

i) $f(x_i) = 1$, when ‘i’ is an odd

ii) $f(x_i) = 2$, when ‘i’ is an even

iii) $f(y_i) = 3$, when ‘i’ is an odd

iv) $f(y_i) = 4$, when ‘i’ is an even

Example: Alternate triangular belt ATB(5)

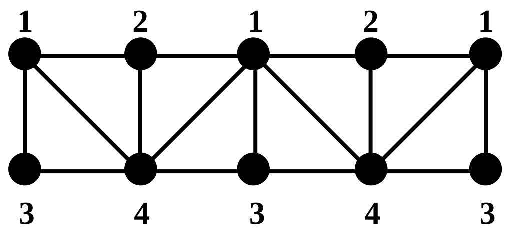

Figure 15

Example: Alternate triangular belt ATB(5)

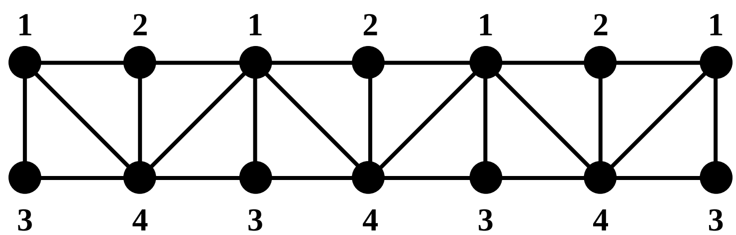

Figure 16

From the above graph chromatic number of AT(T_n) is 4.

Conclusion:

In this paper ,Moser-Spindle graph acknowledges proper coloring and whose chromatic number is 4. The lollipop graph which acknowledges proper coloring, whose chromatic number m where m represents number of vertices in the complete graph .

An open star of finite r copies of fan graph which admits proper coloring and whose chromatic number is 3. Alternate triple triangular snake having its chromatic number 3. An alternate triangular belt ATB($\downarrow^n$) which admits coloring whose chromatic number is 4. Hence it is of interest to compute the certain classes of graphs, like Neighbourhood graph, Cartiesian product graph,Tensor product graph, Corona product graph, Duplication of graphs. etc.,

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