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On Proper Coloring Of Jelly Fish Graph ,Double Arrow Graph,Pyramid Graph And Shadow Graph

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ABSTRACT:

In this paper, we prove that the chromatic number of Jelly fish is 3, We also show that chromatic number of Double arrow graph is 3, Further, we prove that the proper coloring of Pyramid graph and Shadow graph is 2.

Keywords: Proper coloring**,** Jelly fish graph ,double arrow graph, pyramid graph and Shadow graph

INTRODUCTION:

Graph labeling have lately aroused considerable attention. They gave birth to families of graphs with attractive names such as magic, graceful, harmonious, felicitous, sequential and elegant [1]. They exhibited the delicacy of combinatorial constructions and promised interesting applications [2]. Labeled graphs serve as useful models for a broad range of applications such as, coding theory problems, missile guidance codes and convolution codes with optimal auto correlation properties

A labeling of graph G is an assignment of labels to either the vertices or the edges of G that induces for each edge uv in the former a label depending on the vertex labels f(u) and f(v) and in the latter for each vertex u a label depending on the labels of the edges incident with it.

Graph coloring is a special case of graph labeling. It is an assignment of labels traditionally called “Colors” to elements of a graph subject to certain constraints. In its simplest form, it is a way of coloring the vertices of a graph such that no two adjacent vertices share the same color.

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A proper vertex coloring of a graph G is an assignment of colors to the vertices of G such that no two adjacent vertices receive the same color.

The chromatic number of G is the minimum positive integer k such that there is a proper vertex coloring of G with k colors and it is denoted by $\chi(G)$.

Preliminaries:

Jelly fish graph:

The Jelly fish graph J(m,n) is obtained from a 4-cycle v1,v2,v3,v4 by joining v1 and v3 with an edge and appending m pendent edges to v2 and n pendent edges to v4. [3]

Double Arrow graph:

A Double arrow graph with width m and length n is obtained by joining two vertices v and m with superior vertices of Pm × Pn by m + m new edges from both ends [4]

Pyramid Graph:

A **graph** which procured by set out the vertices into a fixed number of rows with i vertices in i th line and every line the j th apex in that row is joined to the j th and the (j + 1) th vertex of the next line.[5]

Hanging Pyramid Graph:

Hanging pyramid graph obtained by attaching the apex of a pyramid graph to a new pendent edge and is denoted by HJn.[5]

Shadow graph:

The shadow graph D2(G) [ ] of a connected graph G is constructed by taking two copies of G, G′ and G″ and joining each vertex u ′ in G′ to the neighbors of the corresponding vertex u ″ in G″.[6]

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Main Results:

Theorem 1:

For x, y ≥ 1, there exists a Jelly fish graph J(x, y), which shows the existence of proper coloring.

Proof:

Let G(V, E) = J(x, y), then G has x + y + 4 vertices and x + y + 5 edges.

Let $V(G) = V_1 \cup V_2$ where $V_1 = \{s, t, u, v\}$, $V_2 = \{u_i, v_j \,;\, 1 \le i \le x;\ 1 \le j \le y\}$ and edge set $E = \{su, st, sv, ut, tv, uu_i, vv_j \,/\, 1 \le i \le x;\ 1 \le j \le y\}$.

Define $f : V \rightarrow \{1, 2, 3\}$ as follows,

Case i) when $x \neq y$

i) f(s) = 1

ii) f(t) = 2

iii) f(u) = 3

iv) f(v) = 3

v) $f(u_i) = 1$, for all i

vi) $f(v_j) = 1$, for all j

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Example: Jelly fish J(4, 2)

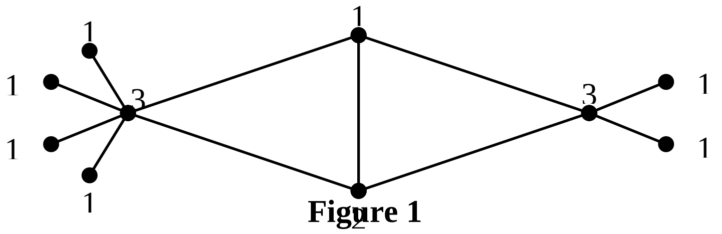

Figure 1

Case ii) when x = y

i) f(s) = 1

ii) f(t) = 2

iii) f(u) = 3

iv) f(v) = 3

v) $f(u_i) = 1$, for all i

vi) $f(v_j) = 1$, for all j

Example: Jelly fish J(4, 4)

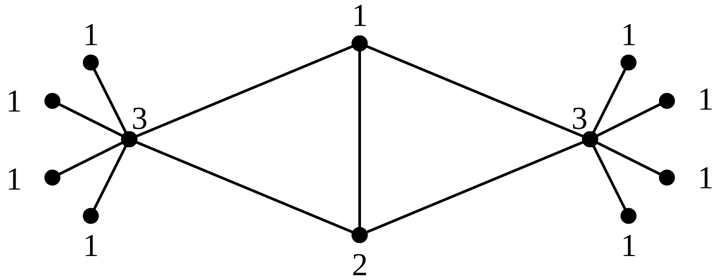

Figure 2

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Chromatic number of Jelly fish is always 3.

Theorem 2:

The double arrow graph DA_n^2 for n > 2, which shows the existence of proper coloring, whose chromatic number is 3.

Proof:

Consider DA_n^2, be a double arrow graph, obtained by connecting two vertices x and y with $P_2 \times P_n$ by adding 2 edges to both sides [4].

Let the vertex set $V = \{x, y, x_i, y_i \,;\, 1 \le i \le n\}$ and edge set $E = \{xx, x_i x_{i+1}, x_n y, xy_1, y_i y_{i+1}, y_n y, x_i y_i \,;\, 1 \le i \le n\}$.

Let $f : V(G) \to \{1, 2, 3\}$ the vertex labeling is given by

i) f(x) = 1

ii) f(y) = 1

iii) $f(x_i)$ = 2, when 'i' is an odd

= 3, when 'i' is an even

iv) $f(y_i)$ = 2, when 'i' is an even

= 3, when 'i' is an odd

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Example: Double arrow graph DA_7^2

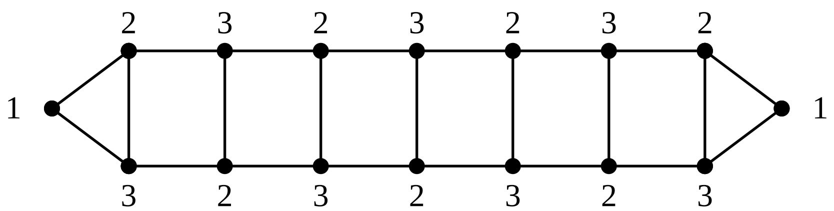

Figure 3

Example: Double arrow graph DA_9^2

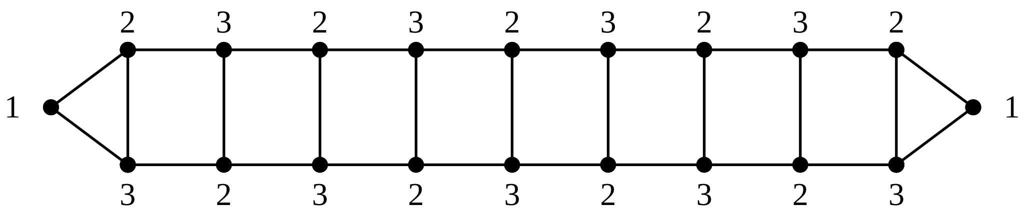

Figure 4

Chromatic number of double arrow graph is 3.

Theorem 3:

For $n \geq 3$, pyramid graph J_n whose chromatic number is 2.

Proof:

Let G be the pyramid graph with the vertex set $V = \{x, u_i, v_i, y_i\ ;\ 1 \leq i \leq n-1\}$ and $V(G) = \frac{n^2+n}{2}$; and $E(G) = n^2 - n$.

Let the function $f : V(G) \rightarrow \{1, 2\}$, the vertex labeling is given by

i) $f(x) = 1$

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ii) $f(u_i) = 2$, when 'i' is an odd

$= 1$, when 'i' is an even

iii) $f(v_i) = 2$, when 'i' is an odd

$= 1$, when 'i' is an even

iv) $f(y_i) = 1$, when $i = 1, 4, 5, 6, \ldots$

$= 2$, when $i = 2, 3, 7, 8, 9, 10, \ldots$

Example: PY_3 pyramid graph

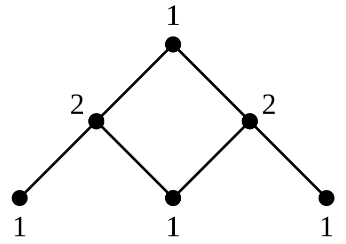

Figure 5

Example: PY_5 pyramid graph

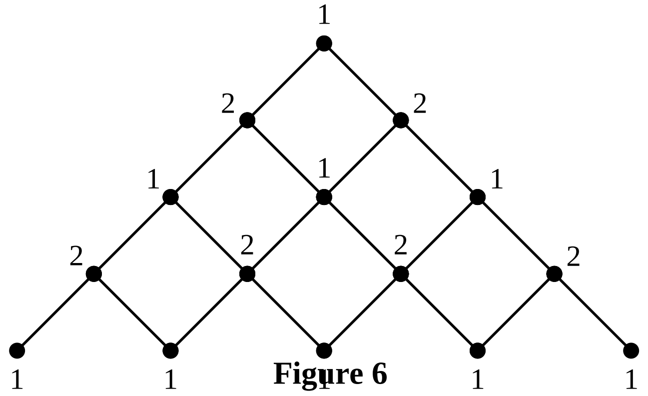

Figure 6

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Chromatic number of the above two graph is 2.

Theorem 4:

A shadow graph of a path graph admits a proper coloring and its chromatic number is 2.

Proof:

Let $u_1\ u_2\ \ldots\ u_n$ be the vertices of the first copy of P_n and $v_1\ v_2\ \ldots\ v_n$ be the vertices of the second copy of P_n.

Let $f : V(G) \rightarrow \{1, 2\}$, then the vertex labeling is given by

i) $f(u_i) = 1$, when 'i' is odd

ii) $f(u_i) = 2$, when 'i' is an even

iii) $f(v_i) = 1$, when 'i' is an odd

iv) $f(v_i) = 2$, when 'i' is an even

Example: Shadow graph $D_2(P_5)$

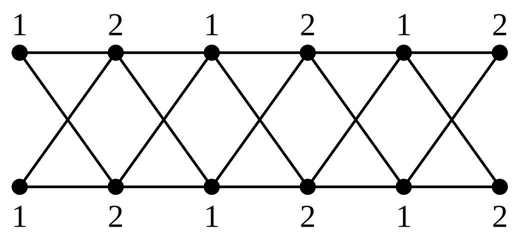

Figure 7

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CONCLUSION:

In this paper the proper coloring of Jelly fish graph, Double arrow graph, Pyramid graphs, and Shadow graph were discussed .Hence it is of interest to apply Prime cordial labeling, square difference labeling, Cube difference labeling of these type of graphs.

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