Fractional Calculus for Advanced Materials Modeling: Obstacles and Opportunities

Miss. Anuja Arun Abhale
SMBST Arts, Science and Commerce College, Sangamaner
Miss. Gita Balasaheb Nawale
SMBST Arts, Science and Commerce College, Sangamaner

Abstract- This extensive review explores the numerous functions of fractional calculus in modelling of advanced materials, with a focus on nonlocal motion, fracture cases, diffusion in porous media, heat transfer, and viscosity in particular. This helps explain phenomena such as reactive transport in porous media, heterogeneous sub diffusion, and diffusion processes in irregularly shaped blocks. For fracture media and porous materials to be continuum mechanics, as well as explain linear and nonlinear viscoelastic behaviour, research finds several examples of using different derivatives of fractional calculation to predict her Covers usage, dynamic processes in fractured materials, and thermodynamic concepts. Although the work provides a comprehensive survey of the application of fractional computation to many material situations, further research on computational techniques, theoretical constraints, and empirical validation may increase the breadth and utility of these models.

Keywords: *Fractional Calculus, Advanced Materials, Nonlocal Motion, Porous Media, Viscoelasticity & Fracture Cases.*

Introduction: Overview

Fractional operations can be thought of as convolution-type integro-differential operators with supersingular power-law kernels. To date, many prominent mathematicians have discussed the notion of derivatives of order ½ [1, 2]. But fractional calculus was long thought to be an elegant but completely speculative area of mathematics with little application in real-world situations. In applied research, more recently, fractional calculus has been of great importance. It has been shown to be an example model of the behaviour of nonlocal objects, important for many phenomena but poorly suited for standard mathematical methods such as classical differential calculus. Unexpected possibilities arose with the creation of new objects in order to provide a coherent framework for modelling non-local and other unconventional media.

Perhaps the most interesting and important applications of fractional computing are to understand complex material behaviour and to simulate unconventional media. Potential impacts in materials science and engineering domain require further research. This thematic journal, including more articles on viscos – elasticity behaviour, thermal transfer, then Porous media diffusion and distant continuum, seeks to provide a comprehensive view of the Essence of the craft and of latest advancements. The cost of which provides a comprehensive explanation of every facet of fractional computational functions in material simulation, despite its often rohibitive nature. Applications use many fractional operators in order to live with variables and constants. Variable-order fractional operators are gaining increasing popularity in recent literature as a useful tool for modelling evolutionary processes without changing the governing equations.

Material inheritance: viscoelastic properties

The time nonlocality of fractional operators is well suited for simulating material properties, especially viscoelastic behavior. Boltzmann and Volterra, who initially proposed the concept of constitutive equations provided by convolutional integral modeling for the memory of prior stress or strain history, pioneered work in this area. Gemant [3] and Bosworth extended this by introducing fractional viscosity. Boltzmann's special derived prototype for the structural viscoelastic medium's behaviour was previously proposed. Nutting, in his introduction to the Volterra convolution integral, found that stress-strain data sets for many solids do exhibit power-law curves. Later, Scott-Blair & Caffin and Caputo went on to investigate the fitting of data from experiments using partial derivatives. However, in the early 1980s, Bagley & Torwick formulated their model in the framework of molecular concept and sought to lay a theoretical foundation for various derivative models of viscosity. They also showed that viscoelasticity differs from traditional models of rheology, like the Kelvin-Voigt system. Fractional derivatives are well suited to investigate the frequency-dependent properties of substances**.** Another advantage is the avoidance of non-causal responses when using fractional derivatives over complex stiffness models of damping. Many studies in the last 20 years have successfully used fractional viscoelastic models, which means that the behaviour of solid materials can be described at the macroscopic level as equations with multiple constraints. Additionally, they have consequently become well-estimated methods for diverse kinds of viscoelastic media. [4].

Linear viscoelasticity is the subject of study, and Mainardi [6] provides a detailed Historic viewpoint on fractional computational functions within this area. Several research articles have reported linear fractional viscous models with different fractional operations. Caputo fractional derivatives for polymers, epoxy resin [7], and fruits and vegetables Di Paola et al. have presented viscoelastic models using data acquired by Mahiuddin and colleagues in their experiments. [8] demonstrated the close relationship between the relaxation and the fractional order parameter of the material and ultimately based on the degree of distortion constant of the feed tissues. In recent years, linear chipchipa-lock-copik is an important improvement in the understanding of the theoretical basis, included in the best test-signal-filting-oriented, for example, D Paola and zingles in which the problems arise. A coherent mechanistic account of the power law recovery was revealed and presented.

Deseri et.al. described a thermodynamic diagram including the condition of a substance having power-law memories, showing that the derivatives from the pressure or tension history to $t = 0$ have different order integrals describing the state. [9] also presented a Power-law feed residuals defined thermally in terms of the Clausius-Duhem inequality, which implies that Staverman-Schwarzl dissipation is equivalent to mechanical energy dissipation. In a recent publication [10], more insights into thermodynamics in fractional-order viscosity were explored. Recently, a fractional-order multiaxial model inheritance for homogeneous materials was presented [11], after placing thermodynamic constraints on the power law scheme involved in numerical models of relaxation functions and numerical axial arrays.

Another problem in viscosity theory is the modelling of the mechanical properties of materials, which can change due to various factors. Several authors have proposed operators with fractions

ISSN PRINT 2319 1775 Online 2320 7876
 UGC CARE Listed (Group -I) Journal Volume 11, Iss 09, 2022

of varying order—that is, operators with a changing order over time or in reaction to specific variable states—for this purpose. Interest in variable-order operators stems from the early work of Samko & Ross, and at the beginning of the last decade, some new contributions to this field were made. Operators of fractions with different orders have been proposed in [12], apparently relying on a thermal region characterized by random fluctuations. Furthermore, Beltempo et al. have investigated whether Model B3 can be substituted with variable-order fractional calculus in dealing with the ageing of reinforced concrete. In order to provide constant theoretical relaxation functions that can be programmed into computer programmes tailored for finite element simulations, variable-order fractional computation has been employed to depict the deterioration of concrete in real-case structures [13].

There have been several interesting proposals on applying variable-order fractional differential calculus for solving nonlinear problems. In this case, the system's friction force is constantly varyingIn contrast to the proposed solution of the nonlinear fixed-order differential equation of Ramirez and Coimbra, the study reveals compressive viscoelastic material composites exhibiting constant stresses with variable-order fractional derivatives, according to Coimbra, which shows the expression. Additive assembly was developed. The authors used a statistical mechanical approach to fit Epoxy resin and carbon/epoxy investigational facts composites and presented that the quick alteration of molecules' order over long periods at the mesoscale within the content is related to the change order provided by the operator. A recent study by Meng et al. [14] demonstrated a new and effective application of Coimbra's suggested variable-order operation for managing the stress response of polymers at different glass transitions and temperatures. The study examines the relationship between actual stresses and transition processes at different temperatures.

Ingman and Suzdalnitsky used a time-based fractional operator developed as a generalisation of the operator for Riemann-Liouville to predict the behaviour of polymeric materials. In this case, the order changes independently of the problem variable. The nonlinear behaviour of steel and asphalt mixtures has been obtained through fractional operators of variable order [15]. Patnaik and others [16] highlights the benefits of solving nonlinear problems with variable-order operators, emphasising many areas of application, such as nonlinear viscoelasticity.

Finally, another interesting topic application for the fractional viscoelasticity model of biological and biomedical systems is presented to fit experimental data of interfaces of pulmonary tissue [17], venous blood flowing, bovine serum albumin, and acacia oral solution. Indeed, as previously shown for aortic valves, many physics studies show a power-law-dependent stress relaxation, sometimes clearly nonlinear depending on the applied pressure.

Several cases of linear and nonlinear fractional viscosities as related to materials have added to the topic [18, 19]. Thermodynamic constraints on structural equations for viscoelastic fluids arising from weak-type entropy inequalities under isothermal conditions have been reviewed by Atnaković et al. The constraints are obtained for a wide range of linear constitutive equations with various derivatives and fractional Burger models. The authors point out that the proposed constraints have been found to be much less stringent than those commonly used at present, providing potentially better methods for recording creep activity in a viscoelastic medium. The subject of Ionescu et al. is a non-Newtonian fluid. The researchers propose the concept of fractional-order impedance, which incorporates five basic parameter estimates to represent various viscosity quantities in non-Newtonian fluids. Genetic particle ensemble algorithms A method for determining frequencies using nonlinear least squares Soap detergent sugar exhibits exceptionally strong fits to experimental data over a wide frequency range for oil-like liquids. Furthermore, relationships between several fluid properties and several fractional-order impedance model parameters have been proposed.

A critical analysis of the model's shortcomings and potential future applications rounds out the article. A three-branched, fractionally derived viscoelastic model for solid propellants was developed by Fang et al. [20]. Compared to standard models featuring derivatives of integer order, the model uses fewer parameters and gives better conformity with experimental data regarding the parameters for stress relaxation and storage. Supporting the study Easy-to-use direct data-fitting methods are employed through discovery approaches. Contrasting the use of fractional counter and electrical impedance spectroscopy for the mechanical characterization of epoxy resins by Tenrero Machado et al. and fractional integer models, the authors show that the latter performs better at lower frequencies than before and provides a better fit of experimental data; usually, fewer parameters are necessary. Finally, a new technique for variable-order, time-dependent differential viscosity is addressed by using a hierarchical clustering method to compare data from other adhesives and sealants with data from electrical impedance spectroscopy collected from epoxy samples to characterise the relationships between different factors (Di Paola et al [21]). The authors present a new solution that always uses the Boltzmann principle in a homogeneous system to obtain the response of the system constructed at any instantaneous time based on the response and distribution pattern of all previous ones. This approach stems from the finding that Boltzmann's linear superposition theory is not employed in this instance as a criterion, primarily because the fractional order changes over time.

Heat conduction

Time-nonspatial fractional operators have been employed to describe memory consequences in a variety of processes, including heat transfer. Numerous equations for time-fractional thermal transfer were created, with differences in the fractional operators involved between them [22]. Jingless recently proposed a thermodynamic model with this structure and potential with proportional temperature and thermal conductivity. Consistent with growth laws is the relationship between partial heat transfer time plus physical processes. In this regard, concepts such as fractional thermoelasticity, differential thermo-viscosity, and differential electro thermo elasticity were born in this context. For example, the Caputo fractional operator has been introduced to generalise Fourier scattering in a model of thermomechanical processes with a different-order thermal formula [23]. In addition to possible thermal stresses from thermal shocks around the fracture region, they are of particular importance.

Povstenko & Kyrylych addressed this issue in their thesis. Their use of the Caputo derivative simplified the solution of the time-fraction heat transfer equation for an infinite axisymmetric cube, incorporated penny-shaped fractures, and considered a specific thermal flow. An analytical process is used to produce a heat field, a heat stress field, as well as stress level factor—that is, the generalised two-parameter Mittag. By using the Laplace, Hankel, and co-Fourier integral transforms as integrals using Leffler functions, parametric studies can be conveniently performed regarding various systems of the Caputo derivative from the obtained equations. Li & Cao [24] provide further support for the topic of heat transfer. It is shown that, especially at low order, the residual impacts of phonon heat transport are described by the fractional order Phonon-Boltzmann transport equations, can be used for known representative temperatures because of their anomalous dissipation. The analysis reveals the importance of the first effect on instantaneous entropy as well as the non-trivial relationship that exists between heat flux and entropy flux, characterised by fractional-order dynamics.

Diffusion in media with pores

Diffusion processes in complex systems can also be solved by the time and/or spatial nonlocality of fractional operators. Metzler and Klafter carried out an original study that investigated the relationship between their fractional computation strategy and the random walk theory of continuous time. They emphasised the benefits of using fractional operators, particularly in the context of boundary-valued diffusion phenomena. Exterior speed fields, or fields of force that are similarly characteristic of complex systems and anomalous subdiffusion, are best described by time-fraction operators. Since solutes can interact with static pore media in highly nonlinear ways, it has been possible to model reactive transport using time-fractional derivatives, e.g., dry for random times or settle into asymmetrical blocks in comparatively calm water. There is evidence to suggest such behaviour occurs [25]. It has been shown that the fractional-order extension of the widely-known Darcy equation for mass transport takes anomalous diffusion into account. The extended pore-elastic model is able to capture the rapid swelling and relaxation of pore media under externally applied pressure, as the fractional-order Darcy equation is further applied to the flow time over an elastic medium.

Fractional space derivatives expressed in spatial coordinates are used in conjunction with time-fractional models to model diffusion phenomena in porous media. These non-local fractional operators are used to describe the sizes of the layers that make liquid highly permeable or break through as liquid flows in porous media.

For experimental evidence for the movement of methanol along porous media, such as pelletized zeolite-based catalysts, Caputo fractional derivatives are considered ideal [26]. Space-time fractional differential equations for stochastic convection and diffusion in fractured media with correlated spatial fluctuations have been studied, in particular the Riemann diffusion, which has been modelled by space-time fractional equations. There, a Riesz-Feller space derivative and a Liouville temporal derivative were used.

Non-local continuities

Nonlocal continuous, i.e., continuous containing appropriate nonlocal terms in their governing equations, is formulated using space fractional operators. The effects of microstructure on elastic wave propagation in materials with different compositions cannot be handled by classical local continuity, guaranteeing the stability of the equations. In order to define the concept of system-alpha motion and obtain appropriate stress and strain tensors, Drapaca & Sivaloganatha presented the application of space-fractional continuity to Plasticity that is independent of rate and linear elasticity, and Sumelka et al. [27] present a space-fractional continuum formulation of power-law length interactions in n-dimensional lattice structures between particles, employing the Riesz-Caputo fractional derivative. This formulation exhibits nonlocal deformation gradients at small or finite strains. They are space fractional and have been the model of the operator. Interestingly, a bridge has been established after the introduction of fractional derivatives and integrals on a lattice of general form by corresponding lattice structures and nonlocal continuums, leading back to Riesz fractional derivatives and integrals of continuity at the continuum boundary.

Carpinteri and so on provide a fractional generalisation of the conventional non-locality Eringen integral structure, representing linearly elastic stress-strain integral combinations with four groups of springs—three nonlocal and one—the authors' interpretation of point-spring mechanics for aspect one dimension (1D) gave a condition, which can be understood as describing a volumetric surface, a detailed volume-volume interaction in a cube. Di Paola et al. proposed a non-local fractional 1D continuum with a mechanical foundation and linear elastic properties, predicated on the idea that non-adjacent volumes apply pressures according to relative displacements through the use of distance-decay power-law functions. Paola and colleagues did so by providing equilibrium equations involving Marchaud fractional derivatives and revealing thermodynamic consistency. Moreover, some shortcomings of the fractional Ehringen model for bounded domains were revealed; previously, nonlocal fractional 1D continuity was determined by heat loss calculations; and finally, its related numerical solution methods proposed the use of equations [28].

Two-dimensional (2D) base subgrade models have been used for additional purposes, where non-local terms are introduced through interactions between the sublayer and the base body. Alotta et al. further developed the concept of long-range power-law interaction in fractional nonlocal Timoshenko beam theory, where non-adjacent beam elements change as they decay with transverse power and power-law functions with beam axes that are far apart. - Provided Tarasov & Aifantis proposed a more detailed theory of fractional linear gradient elasticity based on the three-dimensional Riez fractional Laplacian modelling power law and non-local constitutive behaviour. The same authors as before, based on the related fractional variation theory, presented a new formulation of the fractional linear slope elasticity theory, suggesting that chemical synthesis can be used to manipulate the unique properties of nanomaterial. The concept of intermediate fractional order interval operators, which is a generalisation of the second-order centred interval operator found in the context of the classical lattice model, was introduced by Michelitsch et al. for n-dimensional periodic infinite lattices. As part of the fractional grid method. Relationships between fractional-order difference operators and the classical Riesz fractional Laplacian derivative with a continuous limit have been found. Nonlocal spatial fractions have also been used to simulate long-range viscoelastic interactions and blood flow in capillaries.

Other applications have been found in two-dimensional (2D) base subgrade models, where non-local terms are introduced through interactions between the sublayer and the base body. Alotta et al. further developed the concept of long-range power-law interaction in fractional nonlocal Timoshenko beam theory, where non-adjacent beam elements change as they decay with transverse power and power-law functions with beam axes that are far apart. - Provided Tarasov & Aifantis proposed a more detailed theory of fractional linear gradient elasticity based on the three-dimensional Riez fractional Laplacian modelling power law and non-local constitutive behaviour. The same authors as before, based on the related fractional variation theory, presented a new formulation of the fractional linear slope elasticity theory, suggesting that chemical synthesis can be used to manipulate the unique properties of nanomaterial. The concept of intermediate fractional order interval operators, which is a generalisation of the second-order centred interval operator found in the context of the classical lattice model, was introduced by Michelitsch et al. for n-dimensional periodic infinite lattices. As part of the fractional grid method. Relationships between fractional-order difference operators and the classical Riesz fractional Laplacian derivative with a continuous limit have been found. Nonlocal spatial fractions have also been used to simulate long-range viscoelastic interactions and blood flow in capillaries [29], these applications showcase the versatility of such operators in modeling complex phenomena.

Fractal media

It is undoubtedly an intriguing and difficult subject to explain the physics of fractal media using concepts from fractional calculus. Using the concept of local fractional derivatives, Carpinteri et al. and Carpinteri & Mainardi first presented a space-fractional derivative method for fractal media. Later, Tarasov suggested a new fractional integral method that uses non-integer-order fractional integration to model fractal media continuously. Tarasov also presented a series of continuous models with fractional integrals using describe dynamical processes in a fractured material and related equations, such as mass density balance, momentum density, internal energy, Navier Stokes, and Euler's equation, applied to porous media.

Tarasov & Aifantis presented a gradient elasticity theory for brittle materials based on a fractional-integral formulation. The study provides a brief but instructive discussion of continuous models of fractal media based on concepts from fractional calculus. Balankin presented a fractional space approach to fractal physics based on Stillinger's concept of fractional scale space. Mashayekhi et al. used the fractal medium description in to develop linear and nonlinear fractional viscosity models for fractal media from thermodynamic principles. Using Bayesian data for calibration, the authors proposed fractional order models are more accurate than integer order models by several orders of magnitude. Having wide deformation rates Experimental validation was provided on dielectric elastomers, characterized by significant rate-dependent deformation when measuring uniaxial strain.

In a linear fractional model of viscosity, the order of the fractional derivative was found to be a rate-dependent material property closely related to the fracture scale and spectral scale, indicating diffusion in fracture media (Mashayekhi et al.). In another recent work, investigated the physical relation between the time-fractional derivative of the fractal medium and the fractal geometry. Power-law functions have been shown to be important to describe phenomena in fractured media for flow-time relationships between fractures and for the biomechanics of skeletal muscles. Li & Ostoja-Starzewski [30] discuss the physics of fractured media in the subject matter, focusing on porous media. A continuum model is established for fractal-type anisotropic filter media, this model expresses simulations of global equilibrium laws in traditional (Euclidean) space as fractal integrals based on suitable product measures. Local equilibrium laws for fractal media are then derived. Then, by appropriate second laws of

ISSN PRINT 2319 1775 Online 2320 7876

thermodynamics, the conservation of mass, energy, linear and angular momentum, and microinertia are included through a process of localization. Fractal derivatives, commonly called gradients, are a concept associated with local equilibrium law.

Throughout the study, the relationship between the proposed porous material and other fractional continuum models is investigated. The authors present a specific application for the thermal expansion of liquids and gases in fractured porous elastic media. The suggested model has the capability to extend the utility of continuum mechanics and physics to a variety of particularly complicated and fractured media. One of the hardest problems in complicated media modelling is determining the strong response of media with degenerate Hurst additives to random mass density. Zhang & Ostoja-Starzewski address this issue in the title focusing on sheep-like cases. This method has the advantage of cell-to-cell heterogeneous material properties and continues in the limit for an infinitely few cells. The equivalence of elasto-kinematic equations is checked. The authors also delve into the theoretical constraints of developing a coherent space-fraction derivative model of fractal media and finding appropriate solutions. The study of wave propagation on an elastic half-plane belongs to two mesh-type problems: the central location moment and tangent-point weight.

Conclusion

The thorough investigation of the concern shows the usefulness of fractional calculus in quite a few fabric conditions and affords new understandings of difficult behaviours including diffusion, viscoelasticity, and nonlocal movement. They have a look at illustrates the huge applicability of fractional fashions; however, it also emphasises the need for further investigation into computational methods, empirical validation, and resolving theoretical obstacles so as to enhance the models' usefulness and complexity in actual global settings.

References:

1. Caputo M, Fabrizio M. 2015 A new definition of fractional derivative without singular kernel. Prog. Fract. Differ. Appl. 1, 73–85. (doi:10.12785/pfda/010201)
2. Kilbas AA, Srivastava HM, Trujillo JJ. 2006 Theory and applications of fractional differential equations. Amsterdam, The Netherlands: Elsevier
3. Gemant A. 1938 On fractional differentials. Lond. Edinb. Dublin Phil. Mag. J. Sci. 25, 540–549. (doi:10.1080/14786443808562036)
4. Adolfsson K, Enelund M, Olsson P. 2005 On the fractional order model of viscoelasticity. Mech. Time-Depend. Mater. 9, 15–34. (doi:10.1007/s11043-005-3442-1)
5. Zafar AA, Riaz MB, Shah NA, Imran MA. 2018 Influence of non-integer-order derivatives on unsteady unidirectional motions of an Oldroyd-B fluid with generalized boundary conditions. Eur. Phys. J. Plus 133, 127. (doi:10.1140/epjp/i2018-11981-4)
6. Mainardi F. 2012 An historical perspective on fractional calculus in linear viscoelasticity. Fract. Calc. Appl. Anal. 15, 712–717. (doi:10.2478/s13540-012-0048-6)

7. Di Paola M, Fiore V, Pinnola FP, Valenza A. 2014 On the influence of the initial ramp for a correct definition of the parameters of fractional viscoelastic materials. Mech. Mater. 69, 63–70. (doi:10.1016/j.mechmat.2013.09.017)
8. Mahiuddin M, Godhani D, Feng L, Liu F, Langrish T, Karim MA. 2020 Application of Caputo fractional rheological model to determine the viscoelastic and mechanical properties of fruit and vegetables. Postharvest Biol. Technol. 163, 111147. (doi:10.1016/j.postharvbio.2020. 111147)
9. Deseri L, Di Paola M, Zingales M. 2014 Free energy and states of fractional1order hereditariness. Int. J. Solids Struct. 51, 3156–3167. (doi:10.1016/j.ijsolstr.2014. 05.008)
10. Fabrizio M, Lazzari B, Nibbi R. 2017 Existence and stability for a visco-plastic material with a fractional constitutive equation. Math. Methods Appl. Sci. 40, 6306–6315. (doi:10.1002/mma.4458)
11. Bologna E, Deseri L, Graziano F, Zingales M. 2019 Power-laws hereditariness of biomimetic ceramics for cranioplasty neurosurgery. Int. J. Non-Linear Mech. 115, 61–67. (doi:10.1016/j.ijnonlinmec.2019.01.008)
12. Colinas-Armijo N, Di Paola M, Di Matteo A. 2018 Fractional viscoelastic behaviour under stochastic temperature process. Probab. Eng. Mech. 54, 37–43. (doi:10.1016/j. probengmech.2017.06.005)
13. Beltempo A, Bonelli A, Bursi OS, Zingales M. 2020 A numerical integration approach for fractional-order viscoelastic analysis of hereditary-aging structures. Int. J. Numer. Methods Eng. 121, 1120–1146. (doi:10.1002/nme.6259)
14. Meng R, Yin D, Lu S, Xiang G. 2019 Variable-order fractional constitutive model for the time1dependent mechanical behavior of polymers across the glass transition. Eur. Phys. J. Plus 134, 376. (doi:10.1140/epjp/i2019-12767-x)
15. Pinnola FP, Zavarise G, Del Prete A, Franchi R. 2018 On the appearance of fractional operators in non-linear stress-strain relation of metals. Int. J. Non-Linear Mech. 105, 1–8. (doi:10.1016/j.ijnonlinmec.2018.08.001)
16. Patnaik S, Hollkamp JP, Semperlotti F. 2020 Applications of variable-order fractional operators: a review. Phil. Trans. R. Soc. A 476, 20190498. (doi:10.1098/rspa.2019.0498)
17. Dai Z, Peng Y, Mansy HA, Sandler RH, Royston TJ. 2015 A model of lung parenchyma stress relaxation using fractional viscoelasticity. Med. Eng. Phys. 37, 752–758. (doi:10.1016/ j.medengphy.2015.05.003)
18. Atanackovi´c TM, Janev M, Pilipovi´c S. 2020 On the thermodynamical restrictions in isothermal deformations of fractional Burgers model. Phil. Trans. R. Soc. A 378, 20190278. (doi:10.1098/rsta.2019.0278)
19. Machado JAT, Lopes AM, de Camposinhos R. 2020 Fractional-order modelling of epoxy resin. Phil. Trans. R. Soc. A 378, 20190292. (doi:10.1098/rsta.2019.0292)
20. Fang C, Shen X, He K, Yin C, Li S, Chen X, Sun H. 2020 Application of fractional calculus methods to viscoelastic behaviours of solid propellants. Phil. Trans. R. Soc. A 378, 20190291. (doi:10.1098/rsta.2019.0291)

21. Di Paola M, Alotta G, Burlon A, Failla G. 2020 A novel approach to nonlinear variable-order fractional viscoelasticity. Phil. Trans. R. Soc. A 378, 20190296. (doi:10.1098/rsta.2019.0296)
22. Zingales M, Failla G. 2015 The finite element method for fractional non-local thermal energy transfer in non-homogeneous rigid conductors. Commun. Nonlinear Sci. Numer. Simul. 29, 116–127. (doi:10.1016/j.cnsns.2015.04.023)
23. Piccolo V, Alaimo G, Chiappini A, Ferrari M, Zonta D, Zingales M, Deseri L. 2018 Fractional1order theory of thermoelasticity. II: Quasi-static behavior of bars. J. Eng. Mech. 144, 04017165. (doi:10.1061/(ASCE)EM.1943-7889.0001395)
24. Li S-N, Cao B-Y. 2020 Fractional-order heat conduction models from generalized Boltzmann transport equation. Phil. Trans. R. Soc. A 378, 20190280. (doi:10.1098/rsta.2019.0280)
25. Meerschaert MM, Benson DA, Scheffler HP, Baeumer B. 2002 Stochastic solution of space-time fractional diffusion equations. Phys. Rev. E 65, 041103. (doi:10.1103/ PhysRevE.65.041103)
26. Zhokh A, Strizhak P. 2018 Non-Fickian transport in porous media: always temporally anomalous? Transp. Porous Med. 124, 309–323. (doi:10.1007/s11242-018-1066-6)
27. Sumelka W, Zaera R, Fernández-Sáez J. 2015 A theoretical analysis of the free axial vibration of non-local rods with fractional continuum mechanics. Meccanica 50, 2309–2323. (doi:10.1007/s11012-015-0157-5)
28. Failla G, Santini A, Zingales M. 2010 Solution strategies for 1D elastic continuum with long-range interactions: smooth and fractional decay. Mech. Res. Commun. 37, 13–21. (doi:10.1016/j.mechrescom.2009.09.006)
29. Alotta G, Di Paola M, Pinnola FP, Zingales M. 2020 A fractional nonlocal approach to nonlinear blood flow in small-lumen arterial vessels. Meccanica 55, 891–906. (doi:10.1007/ s11012-020-01144-y)
30. Li J, Ostoja-Starzewski M. 2020 Thermo-poromechanics of fractal media. Phil. Trans. R. Soc. A 378, 20190288. (doi:10.1098/rsta.2019.0288)