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Tarig Transform Using Fractional Delay Differential Equations

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Abstract

In this paper weuse the Tarigtransform andvariational iteration method (TVIM) to solve the fractional delay differential equations (FDDEs). Here, the fractional derivative was in Caputo form and also the convergences of these approximate solutions to the exact solution are very rapid. TVIM is defined for different type of problems and is beneficial method for estimating calculations. Here we are taking some illustrative examples and results for these problems compared numerically and implied method were shown by comparisons with methods such as Adomian Decomposition Method (ADM), Modified Adomian Decomposition Method (MADM), Laplace Adomian Decomposition Method (LADM) and also Homotopy Analysis Method (HAM).

Keywords: Variation Iteration Method; Tarig Transform; Fractional Order of Delay Differential Equation; Caputo derivative ; Approximate Solution.

1. Introduction

Fractional differential equations playing a role in modeling processes in fluid mechanics, mathematical biology, physics, population growth, and so on [1-3]. If we use differential equations with fractional derivatives its application used for describe some scientific problems. It is also define for pure mathematics and in applications such as water flow in pipe, blood flow, the analysis of pollution, and many other applications [4-6]. Delay differential equations (DDEs) is that type of functional differential equations which takes into account the past of a phenomenon [7]. In various types of dynamical models it is also defined [8]. Hence, Fractional delay differential equations have a large extent of uses and for solving scientific problem these are efficient models [9, 10]. For finding exact solutions for nonlinear we need some new

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methods. As to find out the exact solution for delay differential equation in fractional form is very complex. For solving out some types of non-linear problems we use some approximate and numerical methods. There are so many approximate methods which are modified such as Adomnian decomposition method, DGJ method, Iterative decomposition method, Laplace Adomian decomposition method [15], and Homotopy analysis method [16], TarigAdomnian decomposition method. The proposed method is combination of Tarig transform and variational iteration method. As we know, Tarig Transform has been earlier used with ADM to solve some FDDEs [11]. Also, we formulate the Tarig Transform with the variation iteration method for solving a class of non-linear FDDEs. Part 1 contains introductory and part 2 introduces some concepts of Tarig transform and fractional calculus. The TVIM is defined in part 3. Also part 4 introduces some numerical values of fractional order derivative I efficient method. Finally, in section 5, the conclusions are drawn. 2

2. Some important Definitions of Tarig Transform:

Here we are taking some definitions aboutTarig transform and related to fractional calculus.

Definition 2.1: Let us consider S be the set of functions having exponential order and is defined by

$$S=\left\{ f\left(t\right):\exists\ k_1,k_2>0\left|f\left(t\right)\right|<Me^{\frac{|t|}{k_j}},if\ \ t\in\left(-1\right)^j\times\left[0,\infty\right),\ j=1,2\right\} \qquad (1)$$

Where $M>0$ denotes a finite number while k_1,k_2 may be infinite, then for given function which satisfies the condition of S, the Tarig transformation of $f\left(t\right)$ is described as:

$$T\left[f\left(t\right)\right]=F\left(u\right)=\frac{1}{u}\int_0^\infty f\left(t\right)e^{\frac{-t}{u^2}}dt;t>0;\ u\neq 0 \qquad (2)$$

This table involves some elementary functions related to Tarig Transform

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$f(t)$	$T[f(t)]$
1	u
t	u^3
$t^n\ ;\ n \geq 1$	$n!u^{2n+1}$
e^{at}	$\frac{u}{1-au^2}$
$\sin at$	$\frac{au^3}{1+a^2u^4}$
$\cos at$	$\frac{u}{1+a^2u^4}$

Theorem 2: Let $F(u)$ definesTarig transform of Caputo fractional derivative of $f(t)$ of order α which is defined by:

$$T\left[D_0^\alpha f(t)\right] = \frac{1}{u^{2\alpha}} F(u) - \sum_{i=1}^{\alpha} u^{2(i-\alpha)-1} f^{(i-1)}(0) \quad ; m-1 < \alpha \leq m, \quad m \in N \qquad (3)$$

3. Tarig Variation Iteration Method (TVIM):

Here we define Fractional delay differential equation which is described as:

$$D_0^\alpha f(t) = g(t) + N\left(f(t), f(h(t))\right) \quad ; m-1 < \alpha \leq m\ ; t > 0 \qquad (4)$$

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$$f^{k}(0)=f_0^{(k)} \qquad ;k=0,1,2,...,m-1 \tag{5}$$

Where $D_0^{\alpha} f(t)$ represents the Caputo derivativeoforder α and Bounded operator which is non linear represented by and g(t) is continuous function, $f(t)$ denotes the unknown function, and $h(t)$ is the delay function. Some steps involved in this is process is:

1. Firstly apply Tarig transform on equation(4) we get ,

$$T\left[D_0^{\alpha} f(t)\right]=T\left[g(t)+N\left(f(t),f\left(h(t)\right)\right)\right]$$

Taking some initial conditions and applying above theorem and also condition of linearity onTarig transform are described as:

$$\frac{1}{u^{2\alpha}}F(u)-\sum_{i=1}^{\alpha}u^{2(i-\alpha)-1}f^{(i-1)}(0)=T\left[g(t)+N\left(f(t),f\left(h(t)\right)\right)\right] \tag{6}$$

where, $F(u)=\frac{1}{u}\int_0^{\infty} f(t)e^{\frac{-t}{u^2}}dt;t>0;\,u\neq 0$

2. Now by multiplying eq. (6) with Lagrange multipliers and using iteration formula, we get

$$\Rightarrow \frac{1}{u^{2\alpha}}F(u)-\frac{1}{u^{2\alpha-1}}f(0)-\frac{1}{u^{2\alpha-3}}f^{(1)}(0)-\frac{1}{u^{2\alpha-5}}f^{(2)}(0)....-\frac{1}{u^{2\alpha-(2m+1)}}=T\left[g(t)+N\left(f(t),f\left(h(t)\right)\right)\right] \quad ... \quad (6)$$

$$\Rightarrow F_{m+1}(u)=F_m(u)+\lambda(u)\left[\frac{1}{u^{2\alpha}}F(u)-\frac{1}{u^{2\alpha-1}}f(0)-\frac{1}{u^{2\alpha-3}}f^{(1)}(0)-\frac{1}{u^{2\alpha-5}}f^{(2)}(0)....-\frac{1}{u^{2\alpha-(2m+1)}}\right.$$

$$\left.-\,T\left[g(t)+N\left(f(t),f\left(h(t)\right)\right)\right]\right]... \tag{7}$$

Regarding the terms $T\left[N\left(\tilde{f}_m(t);\tilde{f}_m h(t)\right)\right]$ as confined variations, we make eq. (7) static with respect to F_m

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$$\delta F_{m+1}(u) = \delta F_m(u) + \lambda(u)\left[\frac{1}{u^{2\alpha}}\delta F_m(u)\right] \tag{8}$$

From (8), we determine the Lagrange multiplier as:

$$\lambda(u) = -u^{2\alpha} \tag{9}$$

The consecutive approximations are given by inverse Tarig transform T^{-1}, we have

$$\Rightarrow F_{m+1}(u) = F_m(u) - T^{-1}\left[u^{2\alpha}\left[\frac{1}{u^{2\alpha}}F_m(u) - \frac{1}{u^{2\alpha-1}}f(0) - \frac{1}{u^{2\alpha-3}}f^{(1)}(0) - \frac{1}{u^{2\alpha-5}}f^{(2)}(0) - \dots \frac{1}{u^{2\alpha-(2m+1)}} - T\left[g(t)\right] - T\left[N\left(f(t), f(h(t))\right)\right]\right]\right]$$

... (10)

Now

$$\Rightarrow F_{m+1}(u) = f(0) + u\,f^{(1)}(0) + u^3 f^{(2)}(0) + \dots + u^{2m-1} f^{(m-1)}(0) + T^{-1}\left[u^{\alpha} T\left[g(t)\right]\right] + T^{-1}\left[u^{\alpha} T\left[N\left(f_m(t), f_m(h(t))\right)\right]\right]$$

... (11)

Now, $f_0(t) = f(0) + u\,f^{(1)}(0) + u^3 f^{(2)}(0) + \dots + u^{2m-1} f^{(m-1)}(0)$

$$\Rightarrow f_{m+1}(t) = f(0) + u\,f^{(1)}(0) + u^3 f^{(2)}(0) + \dots + u^{2m-1} f^{(m-1)}(0) + T^{-1}\left[u^{\alpha} T\left[g(t)\right]\right] + T^{-1}\left[u^{\alpha} T\left[N\left(f_m(t), f_m(h(t))\right)\right]\right]$$

... (12)

$$f(t) = \lim_{m\to\infty} f_m(t)$$

4. Some applications and results:

Here some applications used in TVIM is defined as:

Example1: Let us take nonlinear Fractional Delay Differential Equation:

$$D_0^c f(t) = 1 - 2f^2\left(\frac{t}{2}\right) \quad ; 0 < \alpha \le 1 \quad ; \quad 0 \le t \le 1 \tag{13}$$

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$$f(0)=0 \tag{14}$$

Exact solution is

$$f(t)=\sin t \tag{15}$$

Solution: On Taking Tarig transform and using initial condition we get:

$$\frac{1}{u^{2\alpha}}T\left[f(t)\right]=T[1]-T\left[2f^2\left(\frac{t}{2}\right)\right]$$

By using Eq. (7), we obtain:

$$\Rightarrow F_{m+1}(u)=F_m(u)+\lambda(u)\left(\frac{1}{u^{2\alpha}}F_m(u)-T[1]+T\left[2f^2\left(\frac{t}{2}\right)\right]\right)$$

$$\Rightarrow F_{m+1}(u)=u^{2\alpha+1}-u^{2\alpha}T\left[2f^2\left(\frac{t}{2}\right)\right]$$

Taking inverse of Tarig transform yields:

$$\Rightarrow f_{m+1}(u)=T^{-1}\left[u^{2\alpha+1}\right]-T^{-1}\left[u^{2\alpha}T\left[2f^2\left(\frac{t}{2}\right)\right]\right]$$

By using Eq. (12), we obtain:

$$f_0(t)=T^{-1}\left[u^{2\alpha+1}\right]=\frac{t^{\alpha}}{\Gamma(\alpha+1)}$$

$$f_{m+1}(t)=\frac{t^{\alpha}}{\Gamma(\alpha+1)}-T^{-1}\left[u^{2\alpha}T\left[2f^2\left(\frac{t}{2}\right)\right]\right]\ ;\ n\geq 0$$

We will calculate the components f_1 , f_2 as:

$$f_1(t)=\frac{t^{\alpha}}{\Gamma(\alpha+1)}-T^{-1}\left[u^{2\alpha}T\left[2f_0^{\,2}\left(\frac{t}{2}\right)\right]\right]$$

$$= \frac{t^{\alpha}}{\Gamma(\alpha+1)} - T^{-1}\left[u^{2\alpha}T\left[2\left(\frac{\left(\frac{t}{2}\right)^{\alpha}}{\Gamma(\alpha+1)}\right)^{2}\right]\right]$$

$$= \frac{t^{\alpha}}{\Gamma(\alpha+1)} - T^{-1}\left[u^{2\alpha}.2\frac{\Gamma(2\alpha+1)u^{4\alpha+1}}{2^{2\alpha}\Gamma^{2}(\alpha+1)}\right]$$

$$= \frac{t^{\alpha}}{\Gamma(\alpha+1)} - \frac{2\Gamma(2\alpha+1)}{2^{2\alpha}\Gamma^{2}(\alpha+1)}\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}$$

$$f_2(t) = \frac{t^{\alpha}}{\Gamma(\alpha+1)} - T^{-1}\left[u^{2\alpha}T\left[2f_1^{2}\left(\frac{t}{2}\right)\right]\right]$$

$$= \frac{t^{\alpha}}{\Gamma(\alpha+1)} - T^{-1}\left[u^{2\alpha}T\left[2\left(\frac{\left(\frac{t}{2}\right)^{\alpha}}{\Gamma(\alpha+1)} - \frac{2.\Gamma(2\alpha+1)\left(\frac{t}{2}\right)^{3\alpha}}{2^{2\alpha}\Gamma^{2}(\alpha+1)\Gamma(3\alpha+1)}\right)^{2}\right]\right]$$

$$= \frac{t^{\alpha}}{\Gamma(\alpha+1)} - T^{-1}\left[u^{2\alpha}T\left[2\left(\frac{\left(\frac{t}{2}\right)^{\alpha}}{\Gamma(\alpha+1)}\right)^{2}\right]\right] + T^{-1}\left[u^{2\alpha}T\left[\frac{8\Gamma(2\alpha+1)\left(\frac{t}{2}\right)^{4\alpha}}{2^{2\alpha}\Gamma^{3}(\alpha+1)\Gamma(3\alpha+1)}\right]\right]$$

$$-T^{-1}\left[u^{2\alpha}T\left[2\left(\frac{-2\Gamma(2\alpha+1)\left(\frac{t}{2}\right)^{3\alpha}}{2^{2\alpha}\Gamma^{2}(\alpha+1)\Gamma(3\alpha+1)}\right)^{2}\right]\right]$$

$$= \frac{t^{\alpha}}{\Gamma(\alpha+1)} - T^{-1}\left[u^{2\alpha}\frac{2\Gamma(2\alpha+1)}{2^{2\alpha}\Gamma^{2}(\alpha+1)}u^{4\alpha+1}\right] + T^{-1}\left[u^{2\alpha}\frac{8\Gamma(4\alpha+1)}{2^{6\alpha}\Gamma^{3}(\alpha+1)\Gamma(3\alpha+1)}u^{8\alpha+1}\right]$$

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$$-T^{-1}\left[u^{2\alpha}\cdot\frac{8\Gamma^2(2\alpha+1)\Gamma(6\alpha+1)}{2^{10\alpha}\Gamma^4(\alpha+1)\Gamma^2(3\alpha+1)}u^{12\alpha+1}\right]$$

$$=\frac{t^{\alpha}}{\Gamma(\alpha+1)}-\frac{2\Gamma(2\alpha+1)t^{3\alpha}}{2^{2\alpha}\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}+\frac{8\Gamma(2\alpha+1)\Gamma(4\alpha+1)t^{5\alpha}}{2^{6\alpha}\Gamma^3(\alpha+1)\Gamma(3\alpha+1)\Gamma(5\alpha+1)}$$

$$-\frac{8\Gamma^2(2\alpha+1)\Gamma(6\alpha+1)t^{7\alpha}}{2^{10\alpha}\Gamma^4(\alpha+1)\Gamma^2(3\alpha+1)\Gamma(7\alpha+1)}$$

$$\therefore f(t)=\lim_{r\to\infty}f_r(t)$$

$$=\frac{t^{\alpha}}{\Gamma(\alpha+1)}-\frac{2\Gamma(2\alpha+1)t^{3\alpha}}{2^{2\alpha}\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}+\frac{8\Gamma(2\alpha+1)\Gamma(4\alpha+1)t^{5\alpha}}{2^{6\alpha}\Gamma^3(\alpha+1)\Gamma(3\alpha+1)\Gamma(5\alpha+1)}$$

$$-\frac{8\Gamma^2(2\alpha+1)\Gamma(6\alpha+1)t^{7\alpha}}{2^{10\alpha}\Gamma^4(\alpha+1)\Gamma^2(3\alpha+1)\Gamma(7\alpha+1)}$$

For $\alpha=1$ the approximate solution in the series form of (13)-(14) is expressed as :

$\therefore f(t)=\lim_{r\to\infty}f_r(t)=\sin t$; is the exact solution of (13) - (14).

Table 1: Table 1 represents the approximate solution of Eqn. (13-14) using Tarig transform for different values of α and the exact solution when $\alpha = 1$.

T	$\alpha=0.5$	$\alpha=0.75$	$\alpha=0.95$	$\alpha=1$			
	TVIM	TVIM	TVIM	TVIM	MVIM	Exact	Error
0.1	0.3285933479	0.1910488793	0.1142169112	0.0998334166	0.099833	0.0998334166	0.000000
0.2	0.4305932058	0.3139358691	0.2191334491	0.1986693333	0.198669	0.1986693307	0.000000
0.3	0.4927180817	0.4129220909	0.3185844185	0.2955202066	0.29552	0.2955202066	0.000000
0.4	0.5371599438	0.4944874073	0.4125041145	0.3894183423	0.389418	0.3894183423	0.000000

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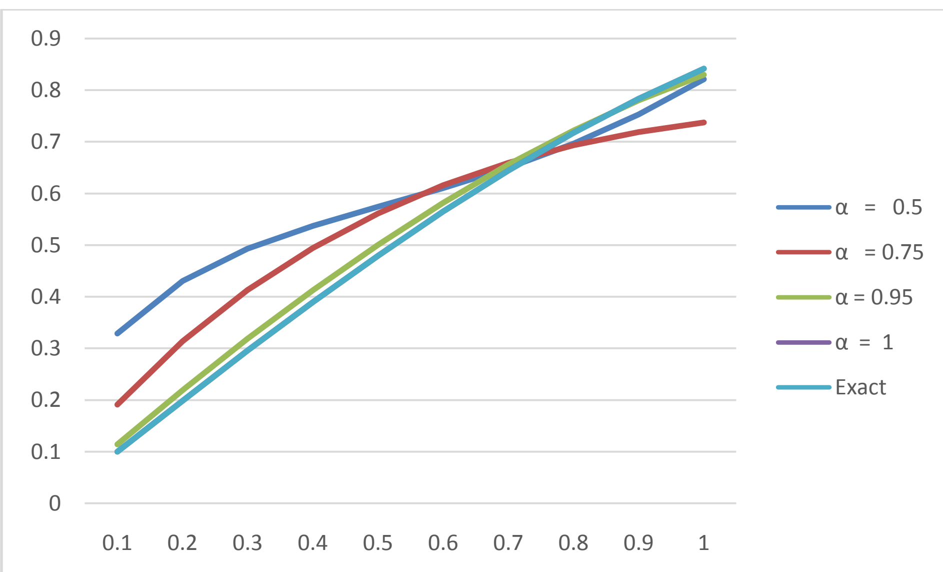
0.5	0.5742279366	0.5615830088	0.5003934631	0.4794255386	0.479426	0.4794255386	0.000000
0.6	0.6102624987	0.6161078768	0.5816559326	0.5646424733	0.564645	0.5646424733	0.000000
0.7	0.6497653470	0.6595779319	0.6557008685	0.6442176872	0.644224	0.6442176872	0.000000
0.8	0.6962210863	0.6933764384	0.7219893212	0.7173560908	0.717371	0.7173560908	0.000000
0.9	0.7524838427	0.7188656840	0.7800598993	0.7833269095	0.783361	0.7833269096	0.000000



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Example2:Let us take linear Fractional delay differential equation:

$$D_0^c f(t) = \frac{2}{\Gamma(3-\alpha)} t^{2-\alpha} - \frac{t^2}{9} + f\left(\frac{t}{3}\right);\ 0<\alpha\le 1;\ 0\le t\le 1 \tag{15}$$

$$f(0)=0 \tag{16}$$

Exact solution is

$$f(t)=t^2 \tag{17}$$

Solution: On taking Tarig transform and with the help of initial condition we get :

$$\frac{1}{u^{2\alpha}} T\left[f(t)\right] = T\left[\frac{2}{\Gamma(3-\alpha)} t^{2-\alpha}\right] - T\left[\frac{t^2}{9}\right] + T\left[f\left(\frac{t}{3}\right)\right]$$

$$\frac{1}{u^{2\alpha}} F_m(u) = T\left[\frac{2}{\Gamma(3-\alpha)} t^{2-\alpha}\right] - T\left[\frac{t^2}{9}\right] + T\left[f\left(\frac{t}{3}\right)\right]$$

By using Eq. (7), we obtain:

$$F_{m+1}(u) = F_m(u) + \lambda(u)\left[\frac{1}{u^{2\alpha}} F_m(u) - T\left[\frac{2}{\Gamma(3-\alpha)} t^{2-\alpha}\right] + T\left[\frac{t^2}{9}\right] - T\left[f\left(\frac{t}{3}\right)\right]\right]$$

$$F_{m+1}(u) = u^{2\alpha} T\left[\frac{2}{\Gamma(3-\alpha)} t^{2-\alpha}\right] - u^{2\alpha} T\left[\frac{t^2}{9}\right] + u^{2\alpha} T\left[f\left(\frac{t}{3}\right)\right]$$

Taking inverse of Tarig transform yields:

$$f_{m+1}(u) = T^{-1}\left[u^{2\alpha} T\left[\frac{2}{\Gamma(3-\alpha)} t^{2-\alpha}\right]\right] - T^{-1}\left[u^{2\alpha} T\left[\frac{t^2}{9}\right]\right] + T^{-1}\left[u^{2\alpha} T\left[f\left(\frac{t}{3}\right)\right]\right]$$

By, using equation (12) we obtain;

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$$f_0(t) = T^{-1}\left[u^{2\alpha}T\left[\frac{2}{\Gamma(3-\alpha)}t^{2-\alpha}\right]\right] - T^{-1}\left[u^{2\alpha}T\left[\frac{t^2}{9}\right]\right] = t^2 - \frac{2}{9\Gamma(3+\alpha)}t^{2+\alpha}$$

$$f_{m+1}(t) = t^2 - \frac{2}{9\Gamma(3+\alpha)}t^{2+\alpha} + T^{-1}\left[u^{2\alpha}T\left[f_m\left(\frac{t}{3}\right)\right]\right] \;\; ; \; n \geq 0$$

We will calculate the components as;

$$f_1(t) = t^2 - \frac{2}{9\Gamma(3+\alpha)}t^{2+\alpha} + T^{-1}\left[u^{2\alpha}T\left[f_0\left(\frac{t}{3}\right)\right]\right]$$

$$f_1(t) = t^2 - \frac{2}{9\Gamma(3+\alpha)}t^{2+\alpha} + T^{-1}\left[u^{2\alpha}T\left[\left(\frac{t}{3}\right)^2 - \frac{2}{9\Gamma(3+\alpha)}\left(\frac{t}{3}\right)^{2+\alpha}\right]\right]$$

$$= t^2 - \frac{2}{9\Gamma(3+\alpha)}t^{2+\alpha} + T^{-1}\left[u^{2\alpha}T\left(\frac{t^2}{9} - \frac{2}{9\Gamma(3+\alpha)}\frac{t^{2+\alpha}}{3^{2+\alpha}}\right)\right]$$

$$= t^2 - \frac{2}{9\Gamma(3+\alpha)}t^{2+\alpha} + T^{-1}\left[u^{2\alpha}\left(\frac{2}{9}u^5 - \frac{2}{3^{4+\alpha}}u^{2\alpha+5}\right)\right]$$

$$= t^2 - \frac{2}{9\Gamma(3+\alpha)}t^{2+\alpha} + T^{-1}\left[\left(\frac{2}{9}u^{2\alpha+5} - \frac{2}{3^{4+\alpha}}u^{4\alpha+5}\right)\right]$$

$$= t^2 - \frac{2}{9\Gamma(3+\alpha)}t^{2+\alpha} + \frac{2}{9}\frac{t^{2+\alpha}}{\Gamma(3+\alpha)} - \frac{2}{3^{4+\alpha}}\frac{t^{2\alpha+2}}{\Gamma(3+2\alpha)}$$

$$f_2(t) = t^2 - \frac{2}{9\Gamma(3+\alpha)}t^{2+\alpha} + T^{-1}\left[u^{2\alpha}T\left[f_1\left(\frac{t}{3}\right)\right]\right]$$

$$= t^2 - \frac{2}{9\Gamma(3+\alpha)}t^{2+\alpha} + T^{-1}\left[u^{2\alpha}T\left[\left(\frac{t}{3}\right)^2 - \frac{2}{9\Gamma(3+\alpha)}\left(\frac{t}{3}\right)^{2+\alpha} + \frac{2}{9\Gamma(3+\alpha)}\left(\frac{t}{3}\right)^{2+\alpha} - \frac{2}{3^{4+\alpha}\Gamma(3+2\alpha)}\left(\frac{t}{3}\right)^{2+2\alpha}\right]\right]$$

$$= t^2 - \frac{2}{9\Gamma(3+\alpha)}t^{2+\alpha} + T^{-1}\left[u^{2\alpha}\left(\frac{2u^5}{9}\right)\right] - T^{-1}\left[u^{2\alpha}\frac{2}{3^{4+\alpha}\Gamma(3+2\alpha)}T\left(\frac{t^{2+2\alpha}}{3^{2+2\alpha}}\right)\right]$$

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$$=t^2-\frac{2}{9\Gamma(3+\alpha)}t^{2+\alpha}+T^{-1}\left[\frac{2}{9}u^{2\alpha+5}\right]-T^{-1}\left[\frac{2\Gamma(3+2\alpha)}{3^{4+\alpha}\Gamma(3+2\alpha)3^{2\alpha+2}}\left(u^{6\alpha+5}\right)\right]$$

$$=t^2-\frac{2}{9\Gamma(3+\alpha)}t^{2+\alpha}+\frac{2}{9\Gamma(3+\alpha)}t^{2+\alpha}-\frac{2}{3^{6+3\alpha}\Gamma(3+3\alpha)}t^{2+3\alpha}$$

$$\therefore f(t)=\lim_{r\to\infty}f_r(t)$$

$$=t^2-\frac{2}{3^{6+3\alpha}\Gamma(3+3\alpha)}t^{2+3\alpha}-\cdots$$

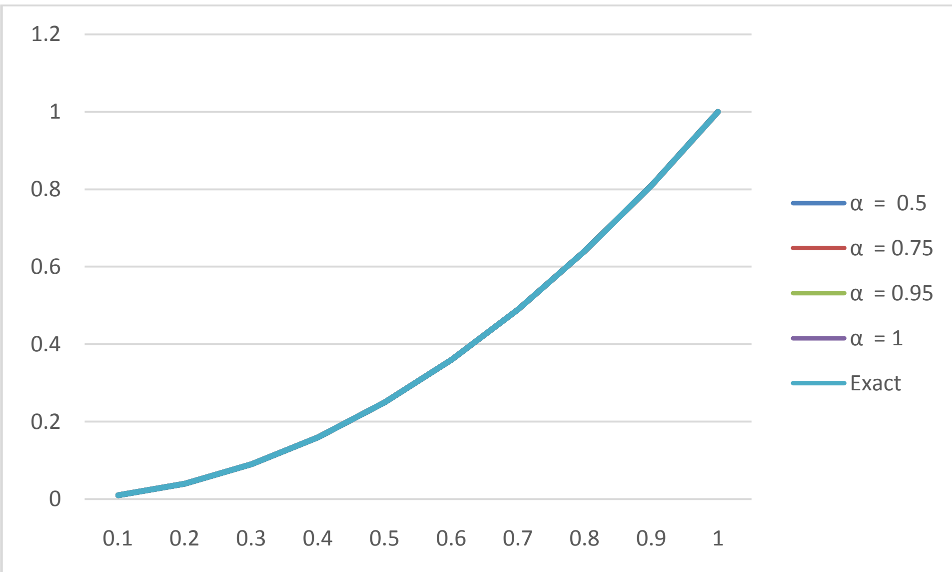
For $\alpha=1$, approximate solution in the series form of (15)-(16) is expressed as:

$\therefore f(t)=\lim_{r\to\infty}f_r(t)=t^2$, which is the exact solution of (15) -(16).

Table 2: Table 2 represents the approximate solution of Eqn. (15-16) using Tarig transform for different values of $\alpha$and the exact solution when α = 1.

T	At $\alpha=0.5$	At $\alpha=0.75$	A $\alpha=0.95$	$\alpha=1$			
	TVIM	TVIM	TVIM	TVIM	MVIM	Exact	Error
0.1	0.0099999856	0.0099999996	0.0099999999	0.0099999999	0.01	0.01	0.000000
0.2	0.0399998376	0.0399999929	0.0399999994	0.0399999997	0.04	0.04	0.000000
0.3	0.0899993287	0.0899999605	0.0899999962	0.0899999979	0.09	0.09	0.000000
0.4	0.1599981626	0.1599998660	0.1599999848	0.1599999913	0.16	0.16	0.000000
0.5	0.2499959879	0.2499996542	0.2499999553	0.2499999735	0.25	0.25	0.000000
0.6	0.3599924053	0.3599992496	0.3599998919	0.3599999415	0.36	0.36	0.000000

0.7	0.4899869737	0.4899985553	0.4899997718	0.4899998576	0.49	0.49	0.000000
0.8	0.6399792130	0.6399974518	0.6399995639	0.6399997225	0.64	0.64	0.000000
0.9	0.8099686075	0.8099957963	0.8099992279	0.8099995	0.81	0.81	0.000000



Example3: Let us take linearFracional Delay Differerntial Equation:

$$D_0^c f(t) = \frac{3}{4} f(t) + f\left(\frac{t}{2}\right) - t^2 + 2\,;\; 1 < \alpha \le 2;\; 0 \le t \le 1 \qquad (17)$$

$$f(0)=0;\; f'(0) = 0 \qquad (18)$$

Exact solution is

$$f(t) = t^2 \qquad (19)$$

Solution: On taking Tarig transform and with the help of initial condition we get :

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$$\frac{1}{u^{2\alpha}}T\left[f(t)\right]=T\left[2-t^2\right]+T\left[\frac{3}{4}f(t)+f\left(\frac{t}{2}\right)\right]$$

$$\frac{1}{u^{2\alpha}}T\left[f(t)\right]=2u-2u^5+T\left[\frac{3}{4}f(t)+f\left(\frac{t}{2}\right)\right]$$

$$\frac{1}{u^{2\alpha}}F_m(u)=2u-2u^5+T\left[\frac{3}{4}f(t)+f\left(\frac{t}{2}\right)\right]$$

By using Eq. (7), we obtain:

$$F_{m+1}(u)=F_m(u)+\lambda(u)\left(\frac{1}{u^{2\alpha}}F_m(u)-2u+2u^5-T\left[\frac{3}{4}f(t)+f\left(\frac{t}{2}\right)\right]\right)$$

$$F_{m+1}(u)=2u^{2\alpha+1}-2u^{5+2\alpha}+u^{2\alpha}T\left[\frac{3}{4}f(t)+f\left(\frac{t}{2}\right)\right]$$

Taking inverse of Tarig transform yields:

$$f_{m+1}(u)=T^{-1}\left[2u^{2\alpha+1}-2u^{5+2\alpha}\right]+T^{-1}\left[u^{2\alpha}T\left[\frac{3}{4}f(t)+f\left(\frac{t}{2}\right)\right]\right]$$

By using Eq. (12), we obtain:

$$f_0(t)=T^{-1}\left[2u^{2\alpha+1}-2u^{5+2\alpha}\right]=\frac{2t^{\alpha}}{\Gamma(\alpha+1)}-\frac{2t^{\alpha+2}}{\Gamma(\alpha+3)}$$

$$\Rightarrow f_{m+1}(t)=\frac{2t^{\alpha}}{\Gamma(\alpha+1)}-\frac{2t^{\alpha+2}}{\Gamma(\alpha+3)}+T^{-1}\left[u^{2\alpha}T\left[\frac{3}{4}f_m(t)+f_m\left(\frac{t}{2}\right)\right]\right]$$

We will calculate the components as;

$$\Rightarrow f_1(t)=\frac{2t^{\alpha}}{\Gamma(\alpha+1)}-\frac{2t^{\alpha+2}}{\Gamma(\alpha+3)}+T^{-1}\left[u^{2\alpha}T\left[\frac{3}{4}f_0(t)+f_0\left(\frac{t}{2}\right)\right]\right]$$

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$$= \frac{2t^{\alpha}}{\Gamma(\alpha+1)} - \frac{2t^{\alpha+2}}{\Gamma(\alpha+3)} + T^{-1}\left[u^{2\alpha}T\left[\frac{3}{4}\left(\frac{2t^{\alpha}}{\Gamma(\alpha+1)} - \frac{2t^{\alpha+2}}{\Gamma(\alpha+3)}\right)\right] + \left(\frac{2\left(\frac{t}{2}\right)^{\alpha}}{\Gamma(\alpha+1)} - \frac{2\left(\frac{t}{2}\right)^{\alpha+2}}{\Gamma(\alpha+3)}\right)\right]$$

$$= \frac{2t^{\alpha}}{\Gamma(\alpha+1)} - \frac{2t^{\alpha+2}}{\Gamma(\alpha+3)} + T^{-1}\left[\frac{3}{4}u^{2\alpha}\left(2u^{2\alpha+1} - 2u^{2\alpha+5}\right) + u^{2\alpha}\left(\frac{2u^{2\alpha+1}}{2^{\alpha}} - \frac{2u^{2\alpha+5}}{2^{\alpha+2}}\right)\right]$$

$$= \frac{2t^{\alpha}}{\Gamma(\alpha+1)} - \frac{2t^{\alpha+2}}{\Gamma(\alpha+3)} + T^{-1}\left[\frac{3}{4}\left(2u^{4\alpha+1} - 2u^{4\alpha+5}\right) + \left(\frac{2u^{4\alpha+1}}{2^{\alpha}} - \frac{2u^{4\alpha+5}}{2^{\alpha+2}}\right)\right]$$

$$= \frac{2t^{\alpha}}{\Gamma(\alpha+1)} - \frac{2t^{\alpha+2}}{\Gamma(\alpha+3)} + \frac{3}{2}\left(\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{t^{2\alpha+2}}{\Gamma(2\alpha+3)}\right) + \frac{2}{2^{\alpha}}\left(\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{t^{2\alpha+2}}{4\Gamma(2\alpha+3)}\right)$$

$$\Rightarrow f_2(t) = \frac{2t^{\alpha}}{\Gamma(\alpha+1)} - \frac{2t^{\alpha+2}}{\Gamma(\alpha+3)} + T^{-1}\left[u^{2\alpha}T\left[\frac{3}{4}f_1(t) + f_1\left(\frac{t}{2}\right)\right]\right]$$

$$= \frac{2t^{\alpha}}{\Gamma(\alpha+1)} - \frac{2t^{\alpha+2}}{\Gamma(\alpha+3)} + T^{-1}\left[u^{2\alpha}T\left[\frac{3}{4}\left(\frac{2t^{\alpha}}{\Gamma(\alpha+1)} - \frac{2t^{\alpha+2}}{\Gamma(\alpha+3)} + \frac{3}{2}\left(\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{t^{2\alpha+2}}{\Gamma(2\alpha+3)}\right) + \frac{2}{2^{\alpha}}\left(\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{t^{2\alpha+2}}{4\Gamma(2\alpha+3)}\right)\right)\right]\right]$$

$$+T^{-1}\left[u^{2\alpha}\left(\frac{2\left(\frac{t}{2}\right)^{\alpha}}{\Gamma(\alpha+1)} - \frac{2\left(\frac{t}{2}\right)^{\alpha+2}}{\Gamma(\alpha+3)} + \frac{3}{2}\left(\frac{\left(\frac{t}{2}\right)^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{\left(\frac{t}{2}\right)^{2\alpha+2}}{\Gamma(2\alpha+3)}\right) + \frac{2}{2^{\alpha}}\left(\frac{\left(\frac{t}{2}\right)^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{\left(\frac{t}{2}\right)^{2\alpha+2}}{4\Gamma(2\alpha+3)}\right)\right)\right]$$

$$= \frac{2t^{\alpha}}{\Gamma(\alpha+1)} - \frac{2t^{\alpha+2}}{\Gamma(\alpha+3)} + \frac{3}{2}\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{3}{2}\frac{t^{2\alpha+2}}{\Gamma(2\alpha+3)} + \frac{9}{8}\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} - \frac{9}{8}\frac{t^{3\alpha+2}}{\Gamma(3\alpha+3)}$$

$$+\frac{3}{2^{\alpha+1}}\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} - \frac{3}{2^{\alpha}}\frac{t^{3\alpha+2}}{8\Gamma(3\alpha+3)} + \frac{2t^{2\alpha}}{2^{\alpha}\Gamma(2\alpha+1)} - \frac{2t^{2\alpha+2}}{2^{\alpha+2}\Gamma(2\alpha+3)} + \frac{3}{2^{2\alpha+1}}\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} - \frac{3}{2^{2\alpha+3}}\frac{t^{3\alpha+2}}{\Gamma(3\alpha+3)}$$

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$$+\frac{2}{2^{3\alpha}}\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}-\frac{2}{2^{3\alpha+4}}\frac{t^{3\alpha+2}}{4\Gamma(3\alpha+3)}$$

$$\therefore\ f(t)=\lim_{r\to\infty} f_r(t)$$

$$=\frac{2t^{\alpha}}{\Gamma(\alpha+1)}-\frac{2t^{\alpha+2}}{\Gamma(\alpha+3)}+\frac{3}{2}\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}-\frac{3}{2}\frac{t^{2\alpha+2}}{\Gamma(2\alpha+3)}+\frac{9}{8}\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}-\frac{9}{8}\frac{t^{3\alpha+2}}{\Gamma(3\alpha+3)}$$

$$+\frac{3}{2^{\alpha+1}}\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}-\frac{3}{2^{\alpha}}\frac{t^{3\alpha+2}}{8\Gamma(3\alpha+3)}+\frac{2t^{2\alpha}}{2^{\alpha}\Gamma(2\alpha+1)}-\frac{2t^{2\alpha+2}}{2^{\alpha+2}\Gamma(2\alpha+3)}+\frac{3}{2^{2\alpha+1}}\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}-\frac{3}{2^{2\alpha+3}}\frac{t^{3\alpha+2}}{\Gamma(3\alpha+3)}$$

$$+\frac{2}{2^{3\alpha}}\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}-\frac{2}{2^{3\alpha+4}}\frac{t^{3\alpha+2}}{4\Gamma(3\alpha+3)}$$

For $\alpha=2$, approximate solution in the series form of (17)-(18) is expressed as:

$\therefore\ f(t)=\lim_{r\to\infty} f_r(t)=t^2$; is the exact solution of (17) - (18).

Table 3: Table 3 represents the approximate solution of Eqn. (17-18) using tarigtransformfor different values of $\alpha$and the exact solution when α = 2.

T	At $\alpha=1.5$	At $\alpha=1.75$	At $\alpha=1.95$	$\alpha=2$			
	TVIM	TVIM	TVIM	TVIM	MVIM	Exact	Error
0.1	0.0478911616	0.0221485038	0.0117463805	0.0099999999	0.01	0.01	0.000000
0.2	0.1369166614	0.0747358303	0.0454042243	0.0399999999	0.04	0.04	0.000000

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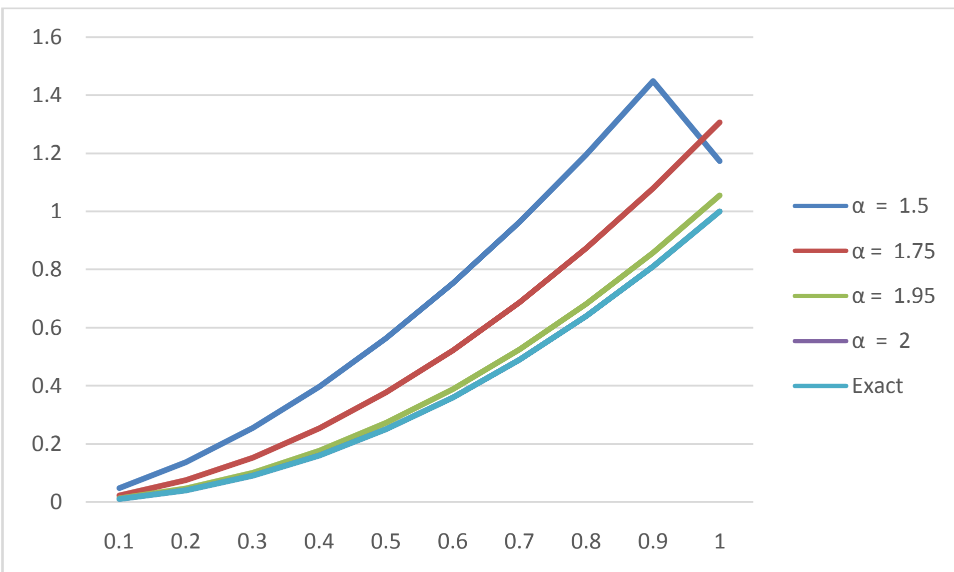
0.3	0.2547342094	0.1525727531	0.1001676558	0.0899999979	0.09	0.09	0.000000
0.4	0.3976484191	0.2536271460	0.1756567280	0.1599999797	0.16	0.16	0.000000
0.5	0.5638929635	0.3767722433	0.2716368975	0.2499998794	0.25	0.25	0.000000
0.6	0.7525107016	0.5213157746	0.3879505549	0.3599994817	0.36	0.36	0.000000
0.7	0.9629558757	0.6868137946	0.5244875808	0.4899982211	0.49	0.49	0.000000
0.8	1.1949098050	0.8729775461	0.6811694618	0.6399948231	0.64	0.64	0.000000
0.9	1.4481798337	1.0796195735	0.8579393238	0.8099867172	0.81	0.81	0.000000



5. Conclusions:

In this study, Tarig transform is combined with variational iteration method for solving linear and non-linear fractional delay differential equations. Finally we conclude that this method has a great role in solving FDDE.

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