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SUPER STOLARSKY 3 MEAN LABELING OF LINE GRAPHS

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ABSTRACT

Let G be a graph with p vertices and q edges. Let $f: V(G) \to \{1,2,\dots,p+q\}$ be an injective function. For a vertex labeling f, the induced edge labeling $f(e=uv)$ is defined by $f(e) = \left\lceil\sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}}\right\rceil$ or $\left\lfloor\sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}}\right\rfloor$,then the resulting edge labels are distinct. In this case f is called Super Stolarsky-3 Mean labeling of G. In this paper we investigate Super Stolarsky-3 Mean labeling of line graph.

Key words- Graph, Stolarsky-3 mean graph, Line Graph, Diamond graph, Fork graph, Bull graph, Fish graph, Cricket graph, Butterfly graph.

I INTRODUCTION

The graph considered here will be finite, undirected and simple. The vertex set is denoted by V(G) and the edge set is denoted by E(G). For all detailed survey of graph labeling, we refer to Galian [1]. For all other standard terminology and notations we follow Harary[2].S.S.Sandhya, S.Somasundaram and S.Kavitha introduced the concept of Stolarsky-3 Mean labeling of graphs in [3]. In this paper, we investigate the Line graphs of Super Stolarsky-3 Mean graphs, we will provide a brief summary of definitions and other information which are necessary for our present investigation.

Definition : 1.1

A graph G with p vertices and q edges is called a Stolarsky-3 Mean graph,if each vertices $x \in V$ with distinct labels $f(x)$ from $1,2,\dots\dots,q+1$ and eachedge $e = uv$ is assigned the distinct labels $f(e=uv) = \left\lceil\sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}}\right\rceil$ or $\left\lfloor\sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}}\right\rfloor$**t**hen the resulting edge labels are distinct. In this case f is called Stolarsky 3 Mean labeling of G.

Definition: 1.2

Let G = (V, E) be a non-trivial graph. Now each edge in E can be considered as a set of two elements of

V. So E is a non-empty collection of distinct non-empty subsets of v, such that their union is V. So there is a intersection graph Ω(E). The graph Ω(E) is called the Line graph of G and is denoted by L(G).We observe that the vertices of L(G) are the edges of G. Further two vertices of L(G) are adjacent iff their corresponding edges are adjacent in G. Thus the vertices a, b in L(G) are adjacent iff a = uv and b = vw are in G

Definition: 1.3:The Diamond graph is a planar undirected graph with 4 vertices and 5 edges.

Definition: 1.4:The Fork graph is a tree 5vertices and 4 edges. It is also called the chair graph.
Definition: 1.5:The Cross graph is the 6 vertex tree and it has 5 edges.
Definition: 1.6 :The Bull graph is a planar undirected graph with 5 vertices and 5 edges in the form of the triangle with two disjoint pendent edges.
Remarks: 1.10
Let u gets label '1' then any edge incident with u must get label 1 (or) 2 (or) 3. Hence this vertex must have a degree ≤ 3.

II Main Results

Theorem 2.1. Line graph of Diamond graph $L(G_d)$ is a Super Stolarsky-3 mean graph.
Proof. The graph $L(G_d)$ is shown below.
Let $G = L(G_d)$. Let the vertex set of G be $\{u_i; 1 \le i \le 5\}$ and the edges set of G be $\{u_i u_{i+1}; 1 \le i \le 4\} \cup \{u_1 u_i; 3 \le i \le 5\} \cup \{u_2 u_5\}$.
Define a function $f: V(G) \to \{1,2, \dots, p+q\}$ by

$$f(u_1 u_{i+2}) = 4i + 1;\ i \le i \le 2,$$
$$f(u_1 u_5) = 10$$
$$f(u_2 u_5) = 7$$

Thus f admits Super Stolarsky-3 mean labeling of G.
Hence $L(G_d)$ is a Super Stolarsky-3 mean graph.
Example 2.2. Super Stolarsky-3 mean labeling of $L(G_d)$ is shown below.

Theorem 2.3. Line graph of Fork graph $L(G_f)$ is a Super Stolarsky-3 mean graph.
Proof. The graph $L(G_f)$ is shown below.
Let $G = L(G_f)$. Let the vertex set of G be $\{u_i; 1 \le i \le 4\}$ and the edges set of G be $\{u_i u_{i+1}; 1 \le i \le 3\} \cup \{u_1 u_3\}$.
Define a function $f: V(G) \to \{1,2, \dots, p+q\}$ by

$$f(u_i) = 2i - 1; 1 \le i \le 2$$
$$f(u_i) = 2i; 1 \le i \le 4$$

In the above figure, the vertices and edges together get labels from {1,2,…,p+q}.
Hence $L(G_f)$ is a Super Stolarsky-3 mean graph.
Example 2.4. Super Stolarsky-3 mean labeling of $L(G_f)$ is shown below.

Theorem 2.5. Line graph of Cross graph $L(G_c)$ is a Super Stolarsky-3 mean graph.
Proof. The graph $L(G_c)$ is shown below.

Let $G = L(G_c)$. Let the vertex set of G be $\{u_i, 1 \leq i \leq 5\}$ and the edges set of G be $\{u_i u_{i+1}, 1 \leq i \leq 4\} \cup \{u_2 u_4, u_3 u_5\}$.
Define a function $f: V(G) \to \{1,2, \dots, p+q\}$ by
By the above labeling pattern, the vertices and edges together get labels from {1,2,…,p+q}.
Hence $L(G_c)$ is a Super Stolarsky-3 mean graph.

Theorem 2.7. Line graph of Bull graph $L(G_b)$ is a Super Stolarsky-3 mean graph.-
Proof.
Let $G = L(G_b)$. Let the vertex set of G be $\{u_i, 1 \leq i \leq 5\}$ and the edges set of G be $\{u_i u_{i+1}, 1 \leq i \leq 4\} \cup \{u_1 u_i, 3 \leq i \leq 5\}$.
Define a function $f: V(G) \to \{1,2, \dots, p+q\}$ by
Then the edges are labeled with
$f(u_i u_{i+1}) = 3i - 1;\ 1 \leq i \leq 2$ $f(u_i u_{i+1}) = 3i;\ 3 \leq i \leq 4$
$f(u_1 u_{i+2}) = 2i;\ 2 \leq i \leq 4$
Theorem 2.8. Line graph of Fish graph $L(G_{fh})$ is a Super Stolarsky-3 mean graph.
Proof. The graph $L(G_{fh})$ is shown below.

Let $G = L(G_{fh})$. Let the vertex set of G be $\{u_i, 1 \leq i \leq 7\}$ and the edges set of G be $\{u_i u_{i+1}; 1 \leq i \leq 6\} \cup \{u_3 u_6, u_3 u_7, u_4 u_6, u_4 u_7, u_1 u_7\}$.
Define a function $f: V(G) \to \{1,2, \dots, p+q\}$ by $f(u_i) = 2i - 1; 1 \leq i \leq 3 f(u_i) = 4i - 8; 4 \leq i \leq$
Then the edges are labeled with
$f(u_1 u_{i+1}) = 2i; 1 \leq i \leq 2$
By the above labeling pattern, the vertices and edges together get labels from {1,2,…,p+q}.
Hence $L(G_{fh})$ is a Super Stolarsky-3 mean graph.

Remark: Graph G and line graph of G are isomorphic to each other.

III CONCLUSION

The study of Super Stolarsky-3 mean labeling of line graphs is important due to its diversified applications. Line graphs of all Stolarsky-3 mean graphs are not Stolarsky 3 mean graphs. It is very interesting to investigate graphs which admit Stolarsky 3 mean labeling. In this paper, we proved that Line graph of Diamond graph, Fork graph, Bull graph, Fish graph, Cricket graph, Butterfly graph are Super Stolarsky-3 mean graphs. The derived results are demonstrated by means of sufficient illustration which provide better understanding. It is possible to investigate similar results for several other graphs.

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