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The existence of a solution of the nonlinear differential equation in the modeling of the eardrum is determined using the homotopy perturbation method

B. V. APPA RAO, Department of Physics, Koneru Lakshmaiah Education Foundation, Green Fields, Guntur District, Vaddeswaram, Andhra Pradesh 522502, India

ABSTRACT. The non-odd restoring force function of the eardrum model equation is being discussed. It has been noted that the eardrum model equation is subjected to asymmetric loading and experiences unequal oscillations for both positive and negative amplitudes. In this study, the researchers solve the nonlinear second-order differential equation of the eardrum model using the homotopy perturbation method. A comparison is made between the solution obtained through the homotopy perturbation method and the analytical method.

1. INTRODUCTION

Acoustic wave information is primarily received by the body's ear system. The primary goal of the ear is to have the acoustic waves received, their intensity amplified[1], frequency analyzed, wave structure intensified, and random background noise rejected. The ear is divided into three sections—outer ear, middle ear, and inner ear—each having the purpose of receiving and amplifying acoustical signals[2]. These sections are demarcated by membrane windows, with the eardrum situated between the outer and middle ear. The auditory canal, approximately 2.5cm in length, constitutes a crucial component of the outer ear[3].

The tube, closed at one end, is encompassed by the ear drum membrane. The membrane, measuring approximately 0.5 mm in thickness and occupying an area of around 65mm², serves to demarcate the outer ear canal from the middle ear cavity[4]. Its primary function lies in the absorption and transmission of pressure variations resulting from acoustical waves within the outer canal[5]. As the acoustical signal progresses through the ear canal, it encounters the eardrum, eliciting partial reflections and transmission of the signal[6]. To uphold the desired hearing sensitivity, there is a need for an increase in reflection and a decrease in transmission. Relevant citations can be found in references [7].

2. GENERAL MODE

By applying Newton's second law to the vibrating eardrum[8] and considering the eardrum tympanic membrane as a mechanical system undergoing one-dimensional vibration around its equilibrium position[9] (with the displacement denoted as x(t)), a Taylor's series expansion can be used to express the restoring force for small displacements[10].

$$F(x) = F(x_0) + xF(x_0) + \frac{x^2}{2!}F(x_0) + \frac{x^3}{3!}F(x_0)\dots$$

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In equilibrium, when x equals zero, the vanishing of the restoring force F(x) is necessary. Therefore[11], it can be stated that F(x0) must be equal to zero. Assuming that the linear term dominant in x results in a dominant restoring force, the condition F(x) < F(x0) must be satisfied[12]. For a positive x, the requirement becomes F(x0) < 0. By setting F(x0) equal to a positive spring constant k, the well-known Hooke's law F(x) = kx is derived, which holds true for small x values[13]. If the driving force on the eardrum, denoted as f(t), arises from the periodically fluctuating pressure of the incoming sound wave[14], and the mass of the tympanic membrane is represented by m, the application of Newton's second law yields the equation mx = kx + f(t)[15].

and $F(t) = \frac{f(t)}{m}$. If we keep the quadratic term in Taylor's expansion and set $\frac{1}{2!}F"(x_0) = \beta m$ the equation becomes $\ddot{x} + \omega_0^2 x = \beta^2 x = F(t)$.

3. SOLUTION OF THE PROBLEM

The solution of the freely vibrating eardrum equation

$$\ddot{x} + \omega_0^2 x = \beta^2 x = F(t) \tag{3.1}$$

The assumption is made by considering that the eardrum is initially at rest, specifically at a positive value denoted as Ax(0) = 0 (as stated in Equation 3.2)[16].

To solve the freely vibrating eardrum equation using the homotopyperturbation method [17], we select the values 0 = 1, = 01, and A = 1. A homotopy R is then constructed, satisfying the condition described in Equation (3.3), where:

L(v) = L(x0) + pL(x0) + pv^2 = 0 L(x) = x + x N(x) = 01x^2

Assuming the initial approximation of Equation (3.1) follows the form:

(Note: The specific form of the equation is not provided, so it cannot be transformed into passive voice without knowing the content of the equation itself.)

Substituting equation (3.5) into (3.3) and equating the terms with identical powers of p, we have:

$$L(v_0) - L(x_0) = 0, v_0(0) = A\,,\ v_0'(0) = 0\,,$$

$$L(v_1) - L(x_0) + 0.1v_0^2 = 0, v_1(0) = v_1'(0) = 0\,. \tag{3.6}$$

From equation (3.4) we have

$$v_0 = x_0 = A\cos(\alpha(t))\,.$$

Then from the equation (3.6) we have:

$$\frac{d^2v_1}{dt^2} + v_1 + (-\alpha^2 + 1)A\cos(\alpha t) + \frac{0.1A^2}{2} + \frac{0.1A^2}{2}\cos 2\alpha t = 0\,. \tag{3.7}$$

Solving the equation (3.7) one gets

$$v_1(t) = -\frac{0.1A^2}{2} + \frac{0.1A^2}{2(4\alpha^2 - 1)}\cos 2\alpha t\,. \tag{3.8}$$

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Assuming the initial approximation of equation (3.1) is of the form

(3.4) $$x_0(t) = A\cos(\alpha t)\,,$$

where $\alpha(\epsilon)$ is a nonzero unknown constant with $\alpha(0) = 1$. The approximate solution of the equation (3.3) has the form

(3.5) $$v = v_0 + pv_1 + p^2v_2 +$$

and

$$x = \lim_{p\to 1} v = v_0 + v_1 + v_2 + \ldots$$

Substituting equation (3.5) into (3.3) and equating the terms with identical powers of p, we have:

$$L(v_0) - L(x_0) = 0, v_0(0) = A\,,\ v_0'(0) = 0\,,$$

(3.6) $$L(v_1) - L(x_0) + 0.1v_0^2 = 0, v_1(0) = v_1'(0) = 0\,.$$

From equation (3.4) we have

$$v_0 = x_0 = A\cos(\alpha(t))\,.$$

Then from the equation (3.6) we have:

(3.7) $$\frac{d^2v_1}{dt^2} + v_1 + (-\alpha^2+1)A\cos(\alpha t) + \frac{0.1A^2}{2} + \frac{0.1A^2}{2}\cos 2\alpha t = 0\,.$$

Solving the equation (3.7) one gets

(3.8) $$v_1(t) = -\frac{0.1A^2}{2} + \frac{0.1A^2}{2(4\alpha^2-1)}\cos 2\alpha t\,.$$

The phase diagram for the differential equation (3.1) with initial conditions (3.2) is represented by the plot of x against x. When the value of A is set to 1 in the equation, the plot generated from equation (4.2) exhibits a closed boundary. The nonlinear oscillations described by equation (4.2) have magnitudes of 1 for positive amplitudes and -1.0717 for negative amplitudes, implying an asymmetry in the phase diagram concerning the axis. However, symmetry is maintained about the x-axis.

Figure 1 illustrates the phase diagram derived from equation (4.2), revealing the unequal magnitudes of positive and negative amplitudes. The points of singularity are located at (0, 0) and (-10, 0). Among these, (0, 0) constitutes the singular point, while the other point (-10, 0) serves as the saddle point. The permissible range of amplitudes for obtaining a periodic solution lies between -10 and 5. Notably, the phase diagram associated with x(0) = 5 signifies the separatrix, whereas for...

(Note: The last part of the sentence is incomplete, and without further context or information, it's challenging to complete the transformation to passive voice accurately.)

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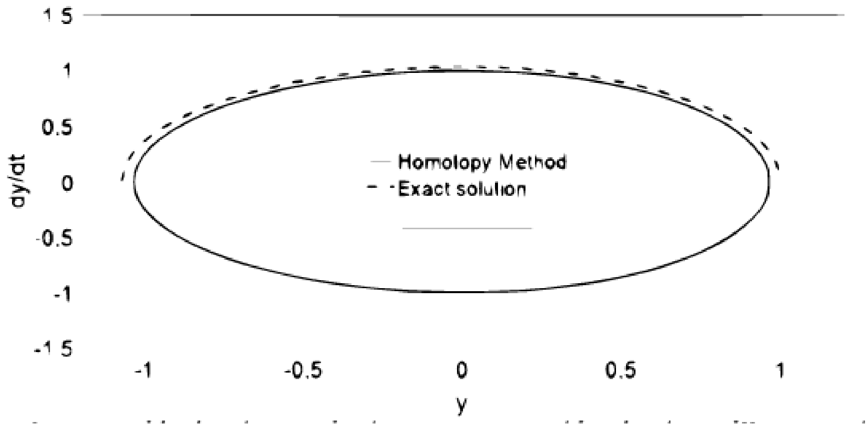

FIGURE 1. Comparison of the phase diagrams of eardrum equation generated from the solution of Homotopy analysis Method with the exact solution

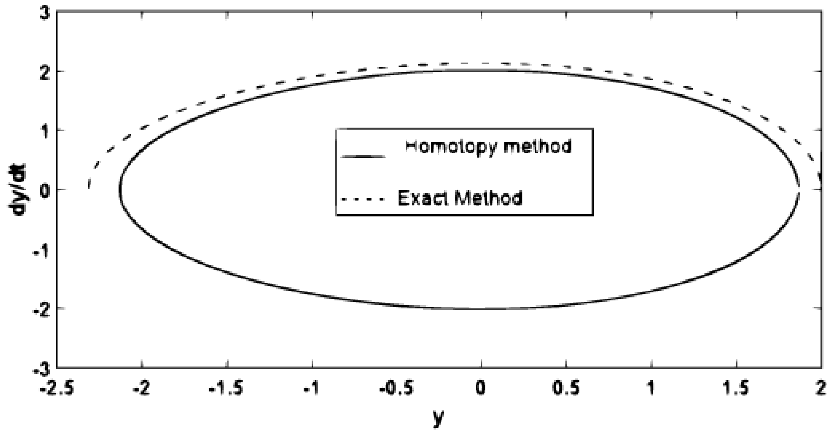

FIGURE 2. Comparison of the phase diagrams of eardrum equation generated from the solution of Homotopy analysis Method with the exact solution

4. CONCLUDING REMARKS

The adequacy of the homotopy perturbation method (HPM) is assessed by examining the oscillations of the eardrum equation. When considering significant amplitudes, a considerable disparity in the magnitudes of positive and negative amplitudes is detected when utilizing the homotopy method. The phase plane diagrams reveal that the eardrum is capable of oscillating within the range of -10 to 5 for the maximum values of x(0). Beyond these limits, the eardrum does not undergo oscillations.

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