

TRIPLE EQUAL SUMLABELING OF SOME CYCLE RELATED GRAPH

EBI SUJITHA, Assistant Professor, Department of Mathematics, Sarah Tucker College and Research Scholar (19121172092016) of PG & Research Department of Mathematics, Rani Anna Govt College for Women,Manonmaniam Sundaranar University,Tirunelveli, Tamilnadu, India, ebisujitha@gmail.com

DR.J.GOLDEN EBENEZER JEBAMANI, Assistant Professor & Head, Department of Mathematics, Sarah Tucker College, Manonmaniam Sundaranar University,Tirunelveli, Tamilnadu, India, goldensambeth@gmail.com

DR.D.PREMALATHA, Associate Professor & Head, PG & Research Department of Mathematics, Rani Anna Govt College for Women,Manonmaniam Sundaranar University,Tirunelveli, Tamilnadu, India, lathaaedward@gmail.com

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ABSTRACT : In this paper we defineTriple Equal Sum labeling and investigate the triple equal sum labeling graphs obtained by some graph operations on cycle related graphs.

Keywords: Triple Equal Sum labeling, Triple Equal Sum graph

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1. INTRODUCTION

Let G = G(V,E) be a simple, finite, connected and undirected graph. A graph labeling is an assignment of integers to vertices or edges or both,based on certain conditions.Graph labeling have enormous applications within mathematics as well as to several areas of computer science and communication networks. A dynamic survey on graph labeling is regularly updated by Gallian[1] and it is published by Electronic Journal of Combinatorics.Vast amount of literature is available on different types of graph labeling and more than 1000 research papers have been published so far in past three decades.The concept of Prime Cordial labeling was introduced by Sundaram[2] et al and they have investigated several results on Prime Cordial labeling. S.K Vaidya and P.L Vihol[3] have investigated that the prime cordial labeling on some cycle related graphs.

Definition 1.1[6] For a graph $G = (V(G), E(G))$, a vertex labeling function $f: V(G) \rightarrow \{-1,0,1\}$ such that no two adjacent vertices have the same label which induces an edge labeling function $f^*: E(G) \rightarrow \{-1,0,1\}$ defined as $f^*(e = uv) = f(u) + f(v)$.Then f is called a Triple Equal Sum labeling if the sum of the labels of all the vertices in the graph G is equal to the sum of the labels of all the edges in the graph G.A graph G is called **Triple Equal Sumgraph** if it admits a Triple Equal Sum labeling.

Definition 1.2[4] For a graph G,the splitting graph $S'$of G is obtained by adding to each vertex v, a new vertex v' so that v' is adjacent to every vertex that is adjacent to v in G ; that is N(v) = N(v').

Definition 1.3 The helm H_n is the graph obtained from a wheel W_n by attaching a pendant edge to each rim vertex.

Definition 1.4[5] The Web Wb_n is the graph obtained by joining the pendant vertices of a helm H_n to form a cycle and then adding a pendent edge to each vertex of outer cycle.

Definition 1.5[5] The flower Fl_n is the graph obtained from a helm H_n by joining each pendant vertex to the apex of the helm.

2.MAIN RESULTS

Theorem 2.1 Splitting graph of a Theta graph admits Triple Equal Sum labeling.

Proof: Let $v_0,v_1,v_2,v_3,v_4,v_5,v_6$ be the vertices of the Theta graph T_α with centre v_0 and $u_0,u_1,u_2,u_3,u_4,u_5,u_6$ be the newly added vertices corresponding to v_i , $0\le i \le 6$ to obtain the splitting graph of a Theta graph.

$|V(S'(T_\alpha)| = 14$

We define a vertex labeling function f: $V(S'(T_\alpha)) \to \{-1,0,1\}$ as follows.

$f(v_0) = 0 = f(u_0)$;$f(v_1) = f(v_3) = f(v_5) = f(u_1) = f(u_3) = f(u_5) =1$

$f(v_2) = f(v_4) = f(v_6) = f(u_2) = f(u_4) = f(u_6) = -1$

In view of this labeling pattern, we have $\sum_{u\epsilon V(S'(T_\alpha))} f(u) = \sum_{e\epsilon E(S'(T_\alpha))} f^*(e) = 0$

Hence the Splitting graph of a Theta graph admits Triple equal Sum labeling.

Theorem 2.2 The Web Wb_n , n is even admits Triple Equal Sum labeling.

Proof: Let v be an apex vertex and $v_1,v_2....v_n$ are the vertices of an inner cycle.

We denote the vertices of an outer cycle and the pendant vertices by $v_1', v_2'....v_n'$ and $v_1'', v_2''....v_n''$.

Then $|V(Wb_n)| = 3n+1$ and $|E(Wb_n)| = 5n$; Define a vertex labeling function; f : $V(Wb_n) \to \{-1,0,1\}$ as follows .When n is even f(v) = 0;$f(v_i) = (-1)^{i+1}$if $1\le i \le n$;$f(v_i') = (-1)^i$ if $1\le i \le n$;$f(v_i'') =$ 0 if $1\le i \le n$; After labeling all the vertices, we define the edge labeling function ; $f^*: E(Wb_n) \to \{-1,0,1\}$ by$f^*(e = uv) = f(u) + f(v)$; In view of this labeling pattern, we have $\sum_{u\epsilon V(Wb_n)} f(u) = \sum_{e\epsilon E(Wb_n)} f^*(e) = 0$; Hence the Web Wb_n , n is even admits Triple Equal Sum labeling.

Theorem 2.3 The Web graph Wb_n is Triple equal Sum graph iff n is even.

Proof: By theorem 2.3, Wb_n is Triple equal sum graph when n is even.

Now assume n is odd. Suppose f is a Triple equal sum labeling of Wb_n.; Here v is an apex vertex and $v_1, v_2 \ldots v_n$ are the vertices of an inner cycle of Wb_n.; We assign any one of the label from {-1,0,1} to the apex vertex v.Then we must assign the remaining two labels from {-1,0,1} to all the vertices of the inner cycle with the condition that no two adjacent vertices have the same label.; But this is not possible,since the inner cycle has odd number of vertices. So we get a contradiction to that f is a Triple equal sum labeling. Therefore the Web graph Wb_n is Triple equal Sum graph iff n is even.

Theorem 2.4 The flower Fl_n , n is even admits Triple Equal Sum labeling.

Proof: Let v be an apex vertex and $v_1, v_2 \ldots v_n$ are rim vertices. We denote the pendant vertices by $v_1', v_2' \ldots v_n'$. Then $|V(Fl_n)| = 2n+1$ and $|E(Fl_n)| = 4n$

Define a vertex labeling function f : $V(Fl_n) \rightarrow \{-1,0,1\}$ as follows. When n is even

$f(v) = 0; f(v_i) = (-1)^{i+1}$ if $1 \le i \le n$; $f(v_i') = -f(v_i)$ if $1 \le i \le n$

Then we define the edge labeling function $f^*: E(Fl_n) \rightarrow \{-1,0,1\}$ by

$f^*(e = uv) = f(u) + f(v)$; In view of this labeling pattern.

Theorem 2.5 The graph obtained by duplicating each edge by a vertex in cycle C_n admits Triple Equal Sum labeling for all n.

Proof: Let C_n' be the graph obtained by duplicating an edge by a vertex in a cycle C_n. Let $v_1, v_2, \ldots v_n$ be the vertices of the cycle C_n and $v_1', v_2' \ldots v_n'$ be the added vertices to Obtain C_n' corresponding to the vertices $v_1, v_2, \ldots v_n$ in C_n.Then $|V(C_n')| = 2n$ and $|E(C_n')| = 3n$

Define a vertex labeling function f : $V(C_n') \rightarrow \{-1,0,1\}$

3.CONCLUSION

In this paper we have proved that the splitting graph of a theta graph,the web graph ,the flower graph, the graph obtained by duplicating each edge by a vertex in cycle , the graph obtained by duplicating a vertex by an edge in cycle and the friendship graph admits Triple Equal Sum labelingand hence these graphs areTriple Equal Sum graphs.For the better understanding of the proofs of the theorems and labeling pattern defined in each theorem can be tested through illustrations.

References

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