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SOME STUDIES ON FUZZY SUBLATTICES THROUGH INTUITIONISTIC FUZZY SETTING

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ABSTRACT:

In this article, author has made an attempt to study more about fuzzy sublattices with special emphasis on their properties in the intuitionistic fuzzy setting. Also deals with the concepts of Intuitionistic fuzzy ideals of an intuitionistic fuzzy sublattice, $(\in, \in \vee q)$ - intuitionistic fuzzy sublattices and obtained some properties of intuitionistic fuzzy sublattices correspondence between intuitionistic fuzzy ideals and their lattice homomorphism and lattice epimorphism. Inclusion principle is also discussed.

Keywords: intuitionistic fuzzy sublattice, lattice homomorphism, lattice epimorphism.

2010 Mathematics Subject Classification: Primary 05C50, Secondary 05C31

1.0 Introduction

Application of fuzzy sets in Lattice theory have been found to be very useful in diversely applied areas of science and technology. Among various branches of pure and applied mathematics abstract algebra was one of the first few subjects where research was carried out using the notion of IFS.

The hypothesis of fuzzy sets was presented by L. A. Zadeh in 1965.This hypothesis has given a helpful numerical apparatus to portraying the way of behaving of frameworks that are excessively complicated or ill characterized to concede exact numerical investigation by old style strategies and instruments. Broad utilizations of the fuzzy set hypothesis have been viewed as in different fields. However, the portrayal gave by the customary [0, 1] - based fuzzy set isn't satisfactory all the time. For instance it doesn't recognize the circumstances where we don't know anything about a specific explanation and a circumstance in which we have precisely however many contentions for this assertion as we have against it.

Intuitionistic fuzzy set (in short IFS) presented by Atanassov [1] in 1983 as a speculation of fuzzy set hypothesis empowers us to portray this distinction. Intuitionistic fuzzy sets give us the likelihood to show wavering and vulnerability by utilizing an extra degree. In intuitionistic fuzzy sets to each element of the universe. We assign both a degree of membership and a non membership degree with the requirement that sum of these two less than or equal to one.

L. A. Zadeh [8] in 1965 presented the thought of fuzzy set to depict ambiguity numerically in its actual relevancy and attempted to take care of such issues by relegating to every conceivable person in the universe of talk a worth addressing its grade of participation in the fuzzy set. This grade compares to how much that individual is comparative or viable with the idea addressed by the fuzzy set. Along these lines, people might have a place in the fuzzy set to a more noteworthy or lesser degree as shown by a bigger or more modest participation grade. These enrolment grades are frequently addressed by genuine numbers running in the shut span somewhere in the range of 0 and 1.

K. Hur et.al [3, 4] presented the thought of intuitionistic fuzzy congruences on a semigroup and cross section and concentrated on their grid structure. Too they concentrated on the connection between intuitionistic fuzzy standards and intuitionistic fuzzy congruences and laid out an isomorphism between them. In this postulation, we completely examined the ideas of intuitionistic fuzzy proportionality and compatibility relations by summing up their standard definition. We laid out an isomorphism between cross section of intuitionistic fuzzy congruences and standards with the assistance of solid level subsets. Additionally we have concentrated on the idea of intuitionistic fuzzy quotient lattice and characterized the quotient of an intuitionistic fuzzy lattice relative to ordinary congruence relation and proved the intuitionistic fuzzy version of fundamental homomorphism theorem.

In 1975, Zadeh [8] made an augmentation of the idea of fuzzy set by an span esteemed fuzzy set, where the upsides of the enrolment capacities are the timespans rather than the actual numbers. In [5] P.P .Ming and P.Y.Ming presented the ideas of 'belongingness' and 'semi occurrence' of a fuzzy point with a fuzzy subset. In view of these thoughts S.K.Bhakat and P. Das [2] presented another kind of fuzzy subring called $(\in, \in \vee q)$ - fuzzy subring (we peruse as having a place or quasi coincident fuzzy subring). The thought of span esteemed intuitionistic fuzzy set (IVIFS) was presented by K.T. Atanassov and, G. Gargov [1]. In this article we characterize the intuitionistic fuzzy lattices and standards and a few properties of their level subsets. After that we characterize another sort of span esteemed intuitionistic fuzzy sublattices and goals called (∈, ∈∨q) - intuitionistic fuzzy sublattices (beliefs, prime goals). Additionally we concentrate on certain properties and portrayals of these intuitionistic fuzzy lattices with ideals

Definition 1.1. [6] An IVIFS A in L is called an interval valued intuitionistic fuzzy sublattice (IVIFSL) of L if and only if $\forall x, y \in L$ the following conditions holds:

(i) $\hat{\mu}(x \vee y) \geq r\min\{\hat{\mu}(x), \hat{\mu}(y)\}$,

(ii) $\underset{A}{\hat{\mu}}(x \wedge y) \geq r\min\{\hat{\mu}(x), \hat{\mu}(y)\}$

(iii) $\hat{v}(x \vee y) \leq r\max\{\hat{v}(x), \hat{v}(y)\}$,

(iv) $\hat{v}(x \wedge y) \leq r\max\{\hat{v}(x), \hat{v}(y)\}$

Definition 1.2. [6,7] An IVIFS A in L is called an interval valued intuitionistic fuzzy ideal (IVIFI) of L if and only if $\forall x, y \in L$ the following conditions holds:

. (i) $\hat{\mu}(x \vee y) \geq r\min\{\hat{\mu}(x), \hat{\mu}(y)\}$,

(ii) $\underset{A}{\hat{\mu}}(x \wedge y) \geq r\max\{\hat{\mu}(x), \hat{\mu}(y)\}$

(iii) $\hat{v}(x \vee y) \leq r\max\{\hat{v}(x), \hat{v}(y)\}$,

(iv) $\hat{v}(x \wedge y) \leq r\min\{\hat{v}(x), \hat{v}(y)\}$

Remark 1.1. Consider the lattice L of "divisors of 10" That is L= {1, 2, 5, 10}

Let A $= \{\langle x, \hat{\mu}(x), \hat{A}(x)\rangle / x \in L\}$ is given by,

$$\{\langle 1, [.5, .7], [.1, .2]\rangle, \langle 2, [.3, .4], [.3, .5]\rangle,$$

$$\langle 5, [.4, .5], [.2, .3]\rangle, \langle 10, [.5, .7], [.2, .3]\rangle\}.$$

Then A is an IVIFL of L. Let B $= \{\langle x, \hat{\mu}(x), \hat{v}_B(x)\rangle / x \in L\}$ is given by,

$$\{\langle 1, [.5, .7], [.1, .2]\rangle, \langle 2, [.4, .5], [.2, .3]\rangle,$$

$$\langle 5, [.4, .5], [.2, .3]\rangle, \langle 10, [.4, .5], [.2, .3]\rangle\}$$

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Definition 1.3.[9] Let $A = \{\langle x, \hat{\mu}(x), \hat{v}(x)\rangle / x \in L\}$ be an IVIFS in L. Then for $\hat{\alpha}, \hat{\beta} \in D[0,1]$ the set $A^{[\hat{\alpha},\hat{\beta}]} = \{x \in X / \hat{\mu}(x) \geq \hat{\alpha}$ and $\hat{v}_A(x) \leq \hat{\beta}\}$, is called the $(\hat{\alpha}, \hat{\beta})$ level subset of A. The sets $U(\hat{\mu}_A, \hat{\alpha}) = \{x \in X / \hat{\mu}_A(x) \geq \hat{\alpha}\}$ and $L(\hat{v}, \hat{\beta}) = \{x \in X / \hat{v}(x) \leq \hat{\beta}\}$ are called the upper and lower level subsets of A, respectively. Clearly $A^{[\hat{\alpha},\hat{\beta}]} = U(\hat{\mu}, \hat{\alpha}) \cap L(\hat{v}_A, \hat{\beta})$. Also the set $A^{(\hat{\alpha},\hat{\beta})} = \{x \in X / \hat{\mu}(x) > \hat{\alpha}$ and $\hat{v}(x) < \hat{\beta}\}$, is called the strong level subset of A.

Definition 1.4. [1,2] An interval valued intuitionistic fuzzy set [IVIFS] in X is an expression of the form $A = \{\langle x, \hat{\mu}(x), \hat{v}_A(x)\rangle / x \in X\}$ where $\hat{\mu}: X \to D[0,1]$, $\hat{v}: X \to D[0,1]$ with $0 \leq \bar{\mu}(x) + \bar{v}(x) \leq 1, \forall x \in X$ where we have $\hat{\mu}(x) = \left[\underline{\mu}_A(x), \bar{\mu}(x)\right]$ and $\underset{A}{\hat{v}}(x) = \left[\underline{v}_A(x), \underset{V}{A}^{\hat{v}}(x)\right]$

Definition 1.5. [3] Let $\mathbb{L} = \langle L, \vee, \wedge\rangle$ and $\mathbb{K} = \langle K, \vee, \wedge\rangle$ be lattices, and let $h: L \to K$. Then h is a lattice homomorphism if and only if for any $a, b \in L, h(a \vee b) = h(a) \vee h(b)$ and $h(a \wedge b) = h(a) \wedge h(b)$. Thus a lattice homomorphism is a specific kind of structure homomorphism. In other words, the mapping h is a lattice homomorphism if it is both a join-homomorphism and a meet homomorphism. Algebraic structures, homomorphisms play an important role in lattice theory.

Definition 1.6. [1-4] A mapping $f: L \to L'$ is called a homomorphism if $f(a \vee b) = f(a) \vee f(b)$ and $f(a \wedge b) = f(a) \wedge f(b), \forall a, b \in L$. In addition to this if the map f is one- one and onto we call it an isomorphism. A homomorphism from L to L' is called endomorphism and a onto homomorphism from L to L' is called epimorphism.

2.0 Main Results

In this section, we have a few properties of interval valued intuitionistic fuzzy ideals under lattice homomorphism and lattice epimorphism. Also reveals the intersection A and B are two IVIFL (IVIFI) of a lattice group. Inclusion principle is also discussed in this chapter.

Theorem 2.1. If A and B are two IVIFL (IVIFI) of a lattice L then $A \cap B$ is an IVIFL (IVIFI) of L.

Proof. We know that

(i) $\hat{\mu}(x \vee y) \geq r\min\{\hat{\mu}(x), \hat{\mu}(y)\}$,

(ii) $\underset{A}{\hat{\mu}}(x \wedge y) \geq r\min\{\hat{\mu}(x), \hat{\mu}(y)\}$

(iii) $\hat{v}(x \vee y) \leq r\max\{\hat{v}(x), \hat{v}(y)\}$,

(iv) $\hat{v}(x \wedge y) \leq r\max\{\hat{v}(x), \hat{v}(y)\}$

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Further

(i) $\hat{\mu}(x \vee y) \geq r\min\{\hat{\mu}(x), \hat{\mu}(y)\}$,

(ii) $\underset{A}{\hat{\mu}}(x \wedge y) \geq r\max\{\hat{\mu}(x), \hat{\mu}(y)\}$

(iii) $\hat{v}(x \vee y) \leq r\max\{\hat{v}(x), \hat{v}(y)\}$,

(iv) $\hat{v}(x \wedge y) \leq r\min\{\hat{v}(x), \hat{v}(y)\}$

Also one can yield an interval valued intuitionistic fuzzy set [IVIFS] in X is an expression of the form $A = \{\langle x, \hat{\mu}(x), \hat{v}_A(x)\rangle / x \in X\}$ where $\hat{\mu}: X \to D[0,1]$, $\hat{v}: X \to D[0,1]$ with $0 \leq \bar{\mu}(x) + \bar{v}(x) \leq 1, \forall x \in X$ where we have $\hat{\mu}(x) = \left[\underline{\mu}_A(x), \bar{\mu}(x)\right]$ and $\underset{A}{\hat{v}}(x) = \left[\underline{v}_A(x), \underset{V}{A}^{\hat{v}}(x)\right]$.

A and B are two IVIFL (IVIFI) of a lattice L then $A \cap B$ is an IVIFL (IVIFI) of L.

Corollary 2.1. Let A be an IVIFS in L. Then A is an IVIFI (IVIFL) of L if and only if $\forall \hat{\alpha}, \hat{\beta} \in D[0,1]$ with $\bar{\alpha} + \bar{\beta} \leq 1$, $A^{[\hat{\alpha},\hat{\beta}]}$ is an ideal (sublattice) of L.

Theorem 2.2. If A and B are IVIFI's in a lattice L, then

$$(A \cup B)^{[\hat{\alpha},\hat{\beta}]} \supseteq A^{[\hat{\alpha},\hat{\beta}]} \cup B^{[\hat{\alpha},\hat{\beta}]}$$

Proof. Since $A \subseteq A \cup B$ and $B \subseteq A \cup B$, Let $x \in A^{[\hat{\alpha},\hat{\beta}]} \Rightarrow \underset{A}{\mu}(x) \geq \hat{\alpha}$ and $\underset{A}{\hat{v}}(x) \leq \hat{\beta}$. Since $A \subseteq B$ $\hat{\mu}(x) \leq \underset{A}{\mu}(x)$ and $\hat{v}(x) \geq \hat{v}_B(x)$. So $\hat{\mu}_B(x) \geq \hat{\alpha}$ and $\underset{B}{\hat{v}}(x) \leq \hat{\beta} \Rightarrow x \in B^{[\hat{\alpha},\hat{\beta}]}$.

That is if A, B are IVIFI's in L such that $A \subseteq B$. Then $A^{[\hat{\alpha},\hat{\beta}]} \subseteq B^{[\hat{\alpha},\hat{\beta}]}$.

$A^{[\hat{\alpha},\hat{\beta}]} \subseteq (A \cup B)^{[\hat{\alpha},\hat{\beta}]}$ and $B^{[\hat{\alpha},\hat{\beta}]} \subseteq (A \cup B)^{[\hat{\alpha},\hat{\beta}]}$.

Therefore $(A \cup B)^{[\hat{\alpha},\hat{\beta}]} \supseteq A^{[\hat{\alpha},\hat{\beta}]} \cup B^{[\hat{\alpha},\hat{\beta}]}$.

Remark 2.1. The reverse inclusion in Theorem 2.2 does not hold in general.

Theorem 2.3. Let $f: L \to L'$ be a lattice epimorphism and $A = \{\langle x, \hat{\mu}(x), \hat{v}(x)\rangle / x \in L\}$, $B = \left\{\langle y, \hat{\mu}(y), \underset{B}{\hat{v}}(y)\rangle / y \in L'\right\}$ are IVIFIs (IVIFLs) of L and L′, respectively. Then, (i) $f(A)$ is an IVIFI (IVIFL) of L′ (ii) f^{-1} (B) is an IVIFI (IVIFL) of L.

Proof. (i) Let $A = \{\langle x, \hat{\mu}(x), \underset{A}{\hat{v}}(x)\rangle / x \in L\}$ be an IVIFI of L.

Let A be an IVIFS in L. Then A is an IVIFI (IVIFL) of L if and only if $\forall \hat{\alpha}, \hat{\beta} \in D[0,1]$ with $\bar{\alpha} + \bar{\beta} \leq 1, A^{(\hat{\alpha},\hat{\beta})}, [U(\hat{\mu}, \hat{\alpha})$ and $L(\hat{v}, \hat{\beta})]$ is an ideal (sublattice) of L. $A^{(\hat{\alpha},\hat{\beta})}$ is an ideal of L for any $\hat{\alpha}, \hat{\beta} \in D[0,1]$ with $\bar{\alpha} + \bar{\beta} \leq 1$.

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Let $f: L \to L'$ be a lattice epimorphism and I and I' are ideals of L and L', respectively. Then $f(I)$ and $f^{-1}(I')$ are ideals of L' and L, respectively.

Thus $f\left(A^{(\hat{\alpha},\hat{\beta})}\right)$ is an ideal of L′. If f: L → L′ is a lattice epimorphism and A is an IVIFI of L, then

$f\left(A^{(\hat{\alpha},\hat{\beta})}\right) = [f(A)]^{(\hat{\alpha},\hat{\beta})}, \forall \hat{\alpha}, \hat{\beta} \in D[0,1], f\left(A^{(\hat{\alpha},\hat{\beta})}\right) = [f(A)]^{(\hat{\alpha},\hat{\beta})}$.

So $[f(A)]^{(\hat{\alpha},\hat{\beta})}$ is an ideal of L′. Hence $f(A)$ is an IVIFI of L'.

Proof for IVIFL is similar.

(ii) Let $B = \left\{\left\langle y, \underset{B}{\hat{\mu}}(y), \hat{v}_B(y)\right\rangle / y \in L'\right\}$ be an IVIFI of L′ · Let A be an IVIFS in L. Then A is an IVIFI (IVIFL) of L if and only if $\forall \hat{\alpha}, \hat{\beta} \in D[0,1]$ with $\bar{\alpha} + \bar{\beta} \leq 1$, $A^{(\hat{\alpha},\hat{\beta})}, [U(\hat{\mu}, \hat{\alpha})$ and $L(\hat{v}, \hat{\beta})]$ is an ideal (sublattice) of L.

$B^{(\hat{\alpha},\hat{\beta})}$ is an ideal of L', for any $\hat{\alpha}, \hat{\beta} \in$ D[0,1] with $\bar{\alpha} + \bar{\beta} \leq 1$. Thus $\left(B^{(\hat{\alpha},\hat{\beta})}\right)$ is an ideal of L.

Hence $f^{-1}\left(B^{(\hat{\alpha},\hat{\beta})}\right) = [f^{-1}(B)]^{(\hat{\alpha},\hat{\beta})}$. So $[f^{-1}(B)]^{(\hat{\alpha},\hat{\beta})}$ is an ideal of L.

Let A be an IVIFS in L. Then A is an IVIFI (IVIFL) of L if and only if $\forall \hat{\alpha}, \hat{\beta} \in D[0,1]$ with $\bar{\alpha} + \bar{\beta} \leq 1, A^{(\hat{\alpha},\hat{\beta})}, [U(\hat{\mu}, \hat{\alpha})$ and $L(\hat{v}, \hat{\beta})]$ is an ideal (sublattice) of L, hence $f^{-1}($ B) is an IVIFI of L.

Proof for IVIFL is similar.

Theorem 2.4. If L → L → L is a lattice epimorphism and A is an IVIFI of L then $f\left(A^{(\hat{\alpha},\hat{\beta})}\right) = [f(A)]^{(\hat{\alpha},\hat{\beta})}, \forall \hat{\alpha}, \hat{\beta} \in D[0,1]$.

Proof. Let $y \in f\left(A^{(\hat{\alpha},\hat{\beta})}\right)$. Then $\exists x_0 \in A^{(\hat{\alpha},\hat{\beta})}$ such that $y = f(x_0)$.

Now $\hat{\mu}(x_0) > \hat{\alpha} \Rightarrow f(\hat{\mu})(y) = \sup_{x \in f^{-1}(y)}\{\hat{\mu}(x)\} > \hat{\alpha}$ and

$$\hat{v}(x_0) < \hat{\beta} \Rightarrow f(\hat{v})(y) = \inf_{x \in f^{-1}(y)}\{\hat{v}(x)\} < \hat{\beta}.$$

Therefore $y \in [f(A)]^{(\hat{\alpha},\hat{\beta})}$.Hence $f\left(A^{(\hat{\alpha},\hat{\beta})}\right) \subseteq [f(A)]^{(\hat{\alpha},\hat{\beta})}$. For the reverse inclusion, let $y \in [f(A)]^{(\hat{\alpha},\hat{\beta})}$.

Then we have

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$$f(\hat{\mu})(y) = \text{Sup}_{x\in f^{-1}(y)}\{\hat{\mu}(x)\} > \hat{\alpha} \text{ and } f(\hat{v})(y) = \text{Inf}_{A\in f^{-1}(y)}\{\hat{v}(x)\} < \hat{\beta}.$$

So there exist $x_0 \in f^{-1}(y)$ such that $\hat{\mu}(x_0) > \hat{\alpha}$ and $\underset{A}{\hat{v}}(x_0) < \hat{\beta}$.

Hence $x_0 \in A^{(\hat{\alpha},\hat{\beta})}$ and $y = f(x_0) \in f\left(A^{(\hat{\alpha},\hat{\beta})}\right)$.

Therefore $[f(A)]^{(\hat{\alpha},\hat{\beta})} \subseteq f\left(A^{(\hat{\alpha},\hat{\beta})}\right)$.

Hence the proof.

3.0 Conclusion

Author has made an attempt to study more about fuzzy sublattices with special emphasis on their properties in the intuitionistic fuzzy setting. Author deals with the concepts of Intuitionistic fuzzy ideals of an intuitionistic fuzzy sublattice, (∈, ∈∨q) -intuitionistic fuzzy sublattices and obtained some properties of intuitionistic fuzzy sublattices and obtained the correspondence between intuitionistic fuzzy ideals and their lattice homomorphism and lattice epimorphism.

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