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Research Paper

On Locating Domination Number of Some Tensor Product Graphs

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ABSTRACT

The tensor product $G_1 \otimes G_2$ of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ such that the vertex set of $G_1 \times G_2$ is the cartesian product $V(G_1) \times V(G_2)$ and two vertices sets (v_p, u_q) and (v_r, u_s), $p, q, r, s = 1, 2, \ldots$. in $V(G_1) \times V(G_2)$ are adjacent, if $v_p v_r \in E_1$ and $u_q u_s \in E_2$ the tensor product $G_1 \otimes G_2$ it is denoted by $T(G_1 \otimes G_2)$. A set D of vertices of a graph $T(G_1 \otimes G_2)$ is locating, if any two distinct vertices sets (v_p, u_q), (v_r, u_s) outside D, $N(v_p, u_q) \cap D \neq N(v_r, u_s) \cap D$, where $N(v_p, u_q)$, $N(v_r, u_s)$ denotes the open neighborhood of (v_p, u_q) and (v_r, u_s). If D is also a dominating set, it is called a locating set of $T(G_1 \otimes G_2)$ and denoted by $\gamma_{ld}(T(G_1 \otimes G_2))$. In this paper, we establish a locating domination number of tensor product $G_1 \otimes G_2$ graph. Further, we also discuss path, cycle, star, complete graph and properties of $T(G_1 \otimes G_2)$.

Keywords: Tensor *product of graphs, locating domination, locating domination number of tensor product of graphs.*

1. INTRODUCTION

For a graph G, $V(G)$ and $E(G)$ denote the vertex set and edge set of G, respectively. As an operation on binary relations, the tensor product was introduced by Alfred North Whitehead and Bertrand Russell in their Principia Mathematica (1912). It is also equivalent to the Kronecker product of the adjacency matrices of the graphs.Locating set was first introduced by Slater. The concept of a locating dominating set was introduced and first studied by Slater. A locating -dominating set is a dominating set D that locates all the vertices in $V(G) - D$ and the locating- domination number of G, denoted by $\gamma_{ld}(G)$ is the minimum cardinality of a locating-dominating set in G.John Mccoy and Michael A. Henning [2] continued the study of paired-domination in graphs. The cross symbols show visually the two edges resulting from the tensor product of two edges.the graphs G_1 and G_2 are called tensor product $G_1 \otimes G_2$.

2. Preliminaries

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Definition: 2.1

Locating domination:

A dominating set D is called a locating dominating set if for any two vertices $v, w \epsilon V - D$, $N(v) \cap D \neq N(w) \cap D$. Thus, with a locating dominating set every vertex in $V - D$ is dominated by a distinct subset of the vertices of D. The locating domination number of G, denoted by $\gamma_{ld}(G)$, is the minimum cardinally of a locating dominating set in G.

Definition: 2.2

Tensor product

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs. The tensor product of G_1 and G_2 denoted by $G = G_1 \otimes G_2$ is the graph with vertex set $V = V_1 \times V_2$ and two vertices (u, v) and (u', v') in V are adjacent in the tensor product $G_1 \otimes G_2$ if $uu' \in E_1$ and $vv' \in E_2$.

$$V \times V = \begin{cases} (\mathrm{v}_1, \mathrm{u}_1), (\mathrm{v}_1, \mathrm{u}_2), (\mathrm{v}_1, \mathrm{u}_3) \\ (\mathrm{v}_2, \mathrm{u}_1), (\mathrm{v}_2, \mathrm{u}_2), (\mathrm{v}_2, \mathrm{u}_3) \\ (\mathrm{v}_3, \mathrm{u}_1), (\mathrm{v}_3, \mathrm{u}_2), (\mathrm{v}_3, \mathrm{u}_3) \end{cases}$$

3. The locating domination number of tensor product of graphs:

Here we observed the exact values of $\gamma_{ld}(T(G_1 \otimes G_2))$ for some standard graphs and proved some standard results.

Theorem: 3. 1

For a path tensor product graph $P_m \otimes P_n$, then $\gamma_{ld}(T(P_m \otimes P_n) \geq 2$.

Proof:

Let $V(P_m) = \{v_1, v_2, ..., v_m\}$ and $V(P_n) = \{u_1, u_2, ... u_n\}$ be the vertex sets in two path graphs. Let $V(T(P_m \otimes P_n)) = \{(v_i, u_j), i = 1,2 \ldots .m$ and $j = 1, 2, \ldots .. n\}$ be the vertex set of Tensor product $P_m \otimes P_n$ graphs. The degree of every vertex of $T(P_m \otimes P_n)$ graph is greater than or equal to 1.

Theorem: 3. 2

Let G be a cycle tensor product graph $C_m \otimes C_n$, then $\gamma_{ld}(T(C_m \otimes C_n)) \geq 3$.

Proof:

Let $V(C_m) = \{v_1, v_2, \ldots\ldots. v_m\}$ and $V(C_n)$ $\{u_1, u_2, \ldots\ldots. u_n\}$, be vertex sets in two cycle graphs. Let $V(T(C_m \otimes C_n)) = \{(v_i, u_j), i = 1, 2\ldots.m$ and $j = 1,2\ldots..n\}$ be the vertex set of Tensor product $C_m \otimes C_n$ graphs. The degree of every vertex of $T(C_m \otimes C_n)$ graph is 4. Then the graph $T(C_m \otimes C_n)$ is 4 – regular graph.

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Case (i): m, n = 3The collection of (v_i, u_j) ∀ i,j = 1,2,3 is the vertex set of dominating set D of T ($C_3 \otimes C_3$) graph. If any two vertex sets (v_p, u_q), $(v_r, u_s)$where $p,q,r,s = 1,2,3$ in V((T ($C_3 \otimes C_3$)) – D) satisfy the condition that $N(v_p, u_q) \cap D \neq N(v_r, u_s) \cap D$, $N(v_p, u_q)$and $N(v_r, u_s)$are the neighborhood of (v_p, u_q), (v_r, u_s).

Theorem: 3. 3

For a path and cycle tensor product graph $P_m \otimes C_n$, then γ_{ld} (T ($P_m \otimes C_n$)) ≥ k if $k \geq 2, m, n \geq 2$.

Proof:

Let V(P_m) = {v_1, v_2, ……. v_m}and V(C_n) ={u_1, u_2, ……. u_n}, be the vertex sets in path and cycle graphs. Let V(T($C_m \otimes C_n$)) = {(v_i, u_j), $i = 1, 2 \dots . m$ and $j = 1, 2 \dots . . n$} be the vertex set of tensor product $P_m \otimes C_n$ graphs. The degree of every vertex of T ($P_m \otimes C_n$) graph is greater than or equal to m.

D|=3

γ_{ld} (T ($S_3 \otimes S_4$)) = 3

4. RESULTS

Corollary 4.1

Let G be a path and star tensor product graph then

$$\gamma_{ld} \ (T\ (P_m \otimes S_n)) = \begin{cases} \geq 2 \text{ if } n = 2 \\ m \text{ if } n \geq 3 \end{cases}$$

Corollary 4.2

For a complete and cycle tensor product graph $K_m \otimes C_n$ then,

$$\gamma_{ld}\ (T\ (K_m \otimes C_n)) = \begin{cases} n \\ (or) \quad \text{if } n \geq 2 \\ n-1 \end{cases}$$

5. Conclusion

In this paper, we discussed about the tensor product of graphs for some special graphs and also found the locating domination number for tensor product of those graphs.

REFERENCES

1. A. Finbow, B.L. Hartnell, On locating dominating sets and well – covered graphs, congr. Number, 65 (1988), PP. 191 – 200.
2. John Mccoy, Michael A. Henning, Locating and Paired-dominating sets in graphs, Discrete Applied Mathematics, 157 (2009), PP. 3268 - 3280
3. H.P. Patil, V. Raja, On tensor product of graphs, Girth and Triangles, Iranian Journal of Mathematical sciences and Informatics, Vol. 10, No. 1 (2015), PP 139 – 147.
4. H. P Patil, Isomorphisms of duplicate graphs with some graph valued functions, Joul. of combinatorics Inf. and system sciences, 10 (1-2), (1985), 36-40