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SUM DIVISOR EDGE CORDIAL GRAPHS

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ABSTRACT : Graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. A graph $G = (V, E)$ with p vertices and q edges is said to admit sum divisor edge cordial labeling if the edge labeling h from $E(G)$ to $\{1,2,\ldots,q\}$ induces the mapping h^* from $V(G)$ to $\{0,1\}$ as $h^*(v)$ is 1 if 2 is a divisor of $\sum_{uv\in E(G)} h(uv)$ and 0 otherwise with the condition that the number of vertices having label 0 and the number of vertices having label 1 differ by at most 1. A graph with a sum divisor edge cordial labeling is called a sum divisor edge cordial graph. In this paper, we prove that path graph, complete bipartite graph, gear graph, wheel graph, helm graph, closed helm graph, friendship graph, fan graph and double fan graph are sum divisor edge cordial graphs.

Keywords : Cordial labeling, Divisor Cordial labeling, Sum Divisor Cordial labeling.

1. INTRODUCTION

Labeling of a graph is a map that carries the graph elements to a set of numbers. The concept of cordial labeling was introduced by Cahit [2] in 1987 as a weaker version of graceful and harmonious labeling. The notion of sum divisor cordial labeling was introduced by A. Lourdusamy and F. Patrick [5]. Motivated by the concept of sum divisor cordial labeling, we define the sum divisor edge cordial labeling of graphs. In this section we provide a summary of definitions required for our investigation.

Definition 1.1. The wheel graph W_n is defined to be the join $K_1 + C_n$.

Definition 1.2. The gear graph G_n is the graph obtained from a wheel W_n by subdividing each of its rim edges. The helm H_n is the graph obtained from a wheel W_n by attaching a pendent edge to each rim vertex. The closed helm CH_n is the graph obtained from a helm H_n by joining each pendant vertex to form a cycle.

Definition 1.3. The fan graph $F_{m,n}$ is defined as the join $\overline{K_m} + P_n$, where $\overline{K_m}$ is the trivial graph on m vertices and P_n is the path graph on n vertices.

Notation 1.4. Let h be a labeling of a graph G. Then we denote
$v_h(k)$- Number of vertices which has label k under the map h.
$e_h(k)$- Number of edges which has label k under the map h.

Definition 1.5. Let $G = (V(G), E(G))$ be a simple graph and let $h: E(G) \to \{1, 2, \ldots, |E(G)|\}$ be a bijection. For each vertex v, assign

$$h^*(v) = \begin{cases} 1 & if\ \ 2\ divides \displaystyle\sum_{uv\in E(G)} h(uv) \\ 0 & otherwise \end{cases}$$

The function h is called a sum divisor edge cordial labeling if $|v_{h^*}(0) - v_{h^*}(1)| \leq 1$. A graph is said to be a sum divisor edge cordial graph if it admits a sum divisor edge cordial labeling.

2. MAIN RESULTS

Theorem 2.1. The path graph P_n is a sum divisor edge cordial graph if $n \not\equiv 2(mod\ 4)$.

Proof. Let $x_j (1 \leq j \leq n)$ be the vertices of P_n, where $n \not\equiv 2(mod\ 4)$. Then $E(P_n) = \{x_j x_{j+1} : 1 \leq j \leq n-1\}$. Define $h : E(P_n) \to \{1,2, \ldots, n-1\}$ by

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$$h(x_jx_{j+1}) = \begin{cases} j & if\ \ j \equiv 0,1(mod\ 4) \\ j+1 & if\ \ j \equiv 2(mod\ 4) \\ j-1 & if\ \ j \equiv 3(mod\ 4) \end{cases} \text{ for } 1 \le j \le n-2 \text{ and}$$

$$h(x_{n-1}x_n) = \begin{cases} n-2 & if\ n\ is\ even \\ n-1 & if\ n\ is\ odd \end{cases}$$

Here, the labeling h will induce the mapping $h^*: V(P_n) \to \{0,1\}$ and the number of vertices labeled with 0 and 1 are as follows:

If $n \equiv 0(mod\ 4)$, $v_{h^*}(0) = v_{h^*}(1) = \frac{n}{2}$ and if $n \equiv 1,3(mod\ 4)$,

$$v_{h^*}(0) = \begin{cases} \left\lceil \frac{n}{2} \right\rceil & if\ \ n \equiv 1(mod\ 4) \\ \left\lfloor \frac{n}{2} \right\rfloor & if\ \ n \equiv 3(mod\ 4) \end{cases} \quad v_{h^*}(1) = \begin{cases} \left\lfloor \frac{n}{2} \right\rfloor & if\ \ n \equiv 1(mod\ 4) \\ \left\lceil \frac{n}{2} \right\rceil & if\ \ n \equiv 3(mod\ 4) \end{cases}$$

Here,$|v_{h^*}(0) - v_{h^*}(1)| \le 1$. Hence P_n admits sum divisor edge cordial labeling for $n \equiv 0,1,3(mod\ 4)$.

Theorem 2.2. The complete bipartite graph $K_{2,n}$ is a sum divisor edge cordial graph if $n \not\equiv 0(mod\ 4)$.

Proof. Let $V(K_{2,n}) = V_1 \cup V_2$ be the bi-partition of $K_{2,n}$ such that $V_1 = \{x,y\}$ and $V_2 = \{z_j : 1 \le j \le n\}$. Then
$E(K_{2,n}) = \{xz_j, yz_j : 1 \le j \le n\}$.
Define $h: E(K_{2,n}) \to \{1,2,\dots,2n\}$ as follows

Case(i): $n \equiv 1,3(mod\ 4)$

$$h(xz_j) = \begin{cases} \left\lceil \frac{j}{2} \right\rceil & if\ j\ is\ odd \\ n + \frac{j}{2} & if\ \ j\ is\ even \end{cases} \quad h(yz_j) = \begin{cases} \left\lceil \frac{3n+j-1}{2} \right\rceil & if\ j\ is\ odd \\ \left\lceil \frac{n+j}{2} \right\rceil & if\ \ j\ is\ even \end{cases}$$

Here, the labeling h will induce the mapping $h^*: V(K_{2,n}) \to \{0,1\}$ and we have

$$v_{h^*}(0) = \begin{cases} \left\lceil \frac{n}{2} \right\rceil + 1 & if\ \ n \equiv 1(mod\ 4) \\ \left\lceil \frac{n}{2} \right\rceil & if\ \ n \equiv 3(mod\ 4) \end{cases} \text{ and } v_{h^*}(1) = \begin{cases} \left\lceil \frac{n}{2} \right\rceil & if\ \ n \equiv 1(mod\ 4) \\ \left\lceil \frac{n}{2} \right\rceil + 1 & if\ \ n \equiv 3(mod\ 4) \end{cases}$$

Case(ii): $n \equiv 2(mod\ 4)$ $h(xz_j) = \begin{cases} 4\left\lceil \frac{j}{2} \right\rceil - 3 & if\ j\ is\ odd \\ 2j & if\ \ j\ is\ even \end{cases}$

for $1 \le j \le \frac{n}{2}$ $h(yz_j) = \begin{cases} 4\left\lceil \frac{j}{2} \right\rceil - 2 & if\ j\ is\ odd \\ 2j-1 & if\ \ j\ is\ even \end{cases}$

for $1 \le j \le \frac{n}{2}$, $h(xz_j) = n+1$ and $h(yz_j) = n+2$ for $j = \frac{n}{2}+1$,

$h(xz_j) = \frac{n}{2} + j + 1$ and $h(yz_j) = n + j$ for $\frac{n}{2} + 2 \le j \le n$.

Here, we have $v_{h^*}(0) = v_{h^*}(1) = \frac{n}{2} + 1$.In each case, $|v_{h^*}(0) - v_{h^*}(1)| \le 1$. Hence $K_{2,n}$ admits sum divisor edge cordial labeling for $n \equiv 1,2,3(mod\ 4)$.

Theorem 2.4. The gear graph G_n is a sum divisor edge cordial graph.

Proof. Let $x, x_j, y_j (1 \le j \le n)$ be the vertices of G_n and let
$E(G_n) = \{xx_j, x_jy_j: 1 \le j \le n\} \cup \{y_jx_{j+1} : 1 \le j \le n-1\} \cup \{y_nx_1\}$ be the edge set of G_n. Define
$h: E(G_n) \to \{1,2,\dots,3n\}$ by

$h(xx_j) = 2j-1$,
$h(x_jy_j) = 2j$ for $1 \le j \le n$, $h(y_jx_{j+1}) = 2n+j$ for $1 \le j \le n-1$ and $h(y_nx_1) = 3n$.

The labeling h will induce the mapping $h^*: V(G_n) \to \{0,1\}$ and we have

$v_{h^*}(0) = \begin{cases} n+1 & if\ n\ is\ odd \\ n & if\ n\ is\ even \end{cases}$ and $v_{h^*}(1) = \begin{cases} n & if\ n\ is\ odd \\ n+1 & if\ n\ is\ even \end{cases}$

Here,$|v_{h^*}(0) - v_{h^*}(1)| \le 1$. Hence G_n admits sum divisor edge cordial labeling.

Theorem 2.5. The wheel graph W_n is a sum divisor edge cordial graph if $n \not\equiv 1(mod\ 4)$

Proof. Let x,$x_j (1 \le j \le n)$ be the vertices of W_n. Then $E(W_n) = \{xx_j : 1 \le j \le n\} \cup \{x_jx_{j+1} :\ 1 \le j \le n-1\} \cup \{x_nx_1\}$. Define $h: E(W_n) \to \{1,2,\dots,2n\}$ by

$h(xx_j) = j$ for $1 \le j \le n$

$h(x_jx_{j+1}) = n+j$ for $1 \le j \le n-1$ and $h(x_nx_1) = 2n$

The labeling h will induce the mapping $h^*: V(W_n) \to \{0,1\}$ and the number of vertices labeled with 0 and 1 are as follows:

If $n \equiv 3(mod\ 4)$, $v_{h^*}(0) = v_{h^*}(1) = \frac{n+1}{2}$ and if $n \equiv 0,2(mod\ 4)$,

$v_{h^*}(0) = \begin{cases} \frac{n}{2} & if\ \ n \equiv 0(mod\ 4) \\ \frac{n}{2}+1 & if\ \ n \equiv 2(mod\ 4\ ) \end{cases}$ $v_{h^*}(1) = \begin{cases} \frac{n}{2}+1 & if\ \ n \equiv 0(mod\ 4) \\ \frac{n}{2} & if\ \ n \equiv 2(mod\ 4) \end{cases}$

In each case, $|v_{h^*}(0) - v_{h^*}(1)| \le 1$. Hence, W_n admits sum divisor edge cordial labeling for$n \equiv 0,2,3(mod\ 4)$.

Theorem 2.6. The helm graph H_n is a sum divisor edge cordial graph.

Proof. Let x, x_j,$y_j (1 \le j \le n)$ be the vertices of H_n. Then $E(H_n) = \{xx_j, x_jy_j: 1 \le j \le n\} \cup \{x_jx_{j+1} :\ 1 \le j \le n-1\} \cup \{x_nx_1\}$. Define $h: E(H_n) \to \{1,2,\dots,3n\}$ by $h(xx_j) = 2j-1$, $h(x_jy_j) = 2j$ for $1 \le j \le n$, $h(x_jx_{j+1}) = 2n+j$ for $1 \le j \le n-1$ and $h(x_nx_1) = 3n$.

The labeling h will induce the map$h^*: V(H_n) \to \{0,1\}$ and we have

$v_{h^*}(0) = \begin{cases} n+1 & if\ n\ is\ odd \\ n & if\ n\ is\ even \end{cases}$ and $v_{h^*}(1) = \begin{cases} n & if\ n\ is\ odd \\ n+1 & if\ n\ is\ even \end{cases}$

Here,, $|v_{h^*}(0) - v_{h^*}(1)| \le 1$. Hence H_n admits sum divisor edge cordial labeling.

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