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BIPARTITE THROUGH PRESCRIBED MEDIAN AND ANTIMEDIAN OF A COMMUTATIVE RING WITH RESPECT TO AN IDEAL

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ABSTRACT

There are plenty of ways of partners with arithmetical constructions. Some of them to make reference to are bipartite from gatherings, median and anti – median from commutative rings with reference to an ideal. Partnering a with median and anti – median of a commutative ring was presented by Beck in 1988. Similarly Beck has researched the exchange between the ring theoretic properties of a commutative ring and related median and anti – median. Further Anderson and Badawi presented the idea absolute of commutative rings with median and anti – median in the year 2008. The absolute of a commutative ring *R* is the undirected with *R* as the vertex set and two particular vertices in *R* are nearby if and provided that their total is a median and anti – median of *R*. As of late Anderson and Badawi presented and concentrated on the summed up all out the of commutative rings concerning the multiplicatively prime subset *H* of *R*. The summed up complete of a commutative ring is the undirected with all components of *R* as vertices, and for two unmistakable vertices in *R* are nearby if and provided that their total is in *H*. In this article, an endeavor has been made to learn about in hypothetical properties and different control boundaries of summed up absolute of commutative rings of median and anti – median and its supplement.

Keywords: median, anti - median, commutative ring, ideal.

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1. Introduction

Let $G = (V, E)$ be a on n vertices with vertex set V and edge set E. A is bipartite if its vertex set can be partitioned into two nonempty subsets X and Y such that each edge of G has one end in X and the other in Y, and a is k-partite if its vertex set can be partitioned into k nonempty subsets such that no edge in G has its both ends in the same subset. Degree of a vertex v, $d(v)$, is the number vertices adjacent to v and by $N(v)$ we denote the neighbor set of v. The smallest and largest degrees of vertices in G are respectively denoted by $\delta(G)$ and $\Delta(G)$.

Given a G the issue of tracking down a H to such an extent that $M(H) \cong G$ is alluded to as the median issue. In [6], it is shown that any $G = (V, E)$ is the median of some associated . In [3] the thought of against median of a was presented and demonstrated that each is the counter median of some . The issue of concurrent inserting of median and hostile to median is examined in [1]. Another development, which sums up every one of the recently referenced developments, can be found in [5].

The median vertices have the base normal distance in *a* and subsequently the median issue is huge among the improvement issues including the position of organization servers. Nonetheless, the median developments for general can't be straightforwardly applied to a huge number as their fundamental has a place with various classes of . It tends to be seen that are bipartite to basic s of a huge number. For instance, the vast majority of the examinations in network networks are finished utilizing inclination networks [4] and they are displayed utilizing bipartite .

It is notable that the median of a tree is a vertex or an edge. This administrator was additionally read up for certain classes of in [7] and [8]. In this paper we show that any bipartite is the median of another bipartite . With an alternate development, we show that the comparative outcome additionally hold for k-partite. The undifferentiated from results for against median issue on these classes are additionally acquired. Since any is a k-partite , for some k, these developments can be applied overall. For any remaining fundamental ideas and documentations not referenced in this paper we allude to [2].In variation to the concept of zero divisor, few authors [8] introduced the total of a commutative ring. Let R be a commutative ring with $Nil(R)$ its ideal of nilpotent elements, $Z(R)$ its set of zero-divisors, and

Reg(R) its set of regular elements. The total of *R*, denoted by *T (R)*, is the undirected with all elements of *R* as vertices, and for distinct x, $y \in R$, the vertices x and y are adjacent if and only if $x + y \in Z(R)$. Also they introduced the three induce subs *Nil (R) Z (R)* and *Reg (R)* of *T (R)* with vertices *Nil(R), Z(R),* and *Reg(R).*

A graph wherein each sets of particular vertices is joined by an edge is known as a total graph. We use K_n for the total graph with n vertices. A r-partite graph is a graph whose vertex set can be divided into r subsets so that no edge has the two vertices in any one subset. A total I-partite graph is one in which every vertex is joined to each vertex that isn't in a similar subset as the given vertex. The total bipartite (i.e., complete 2-partite) graph is signified by $K_{m,n}$ where the arrangement of segment has sizes m and n. The circumference of a graph G is the length of a most limited cycle in G and is meant by bigness *(G)*. We characterize a shading of a graph G to be a task of tones (components of some set) to the vertices of G, one tone to every vertex, so nearby vertices are doled out unmistakable tones. In the event that n tones are utilized, the shading is alluded to as a n-shading. On the off chance that there exists a n-shading of a graph G, G is called n-colorable. The base n for which a graph G is n-colorable is known as the chromatic number of G, and is indicated by $\chi(G)$. A club of a graph is a maximal complete sub and the quantity of vertices in the biggest inner circle of graph G, signified by $\omega(G)$, is known as the faction number of G. Clearly $\chi(G) \geq \omega(G)$ for general graph G.

Assume that S is a commutative semigroup with nothing. For ideal hypothesis in commutative semigroup, we allude to the overview of median and anti – median [3] (additionally see [2]). Here we simply review a portion of the ideas. A non-void subset I of S is called ideal if $xS \subseteq I$ for any $x \in I$. An optimal p of a commutative semigroup is known as an excellent ideal of S if for any two component $x, y \in S$, $xy \in p$ infers $x \in p$ or $y \in p$. Let $Z(S)$ be its arrangement of zero-divisors of S. All together that $\Gamma(S)$ be non void, we generally expect S generally contains somewhere around one nonzero zero divisor. In [14] we can view that $\Gamma(S)$ (as in the ring case) is generally associated, and the breadth of $\Gamma(S) \leq 3$. In the event that $\Gamma(S)$ has a cycle bigness $(\Gamma(S)) \leq 4$. They additionally show that the quantity of insignificant beliefs of S gives a lower bound to the coterie number of S. In [26] authors concentrated on a graph Γ(S) where the vertex set of this chart is $Z(S)*$ and for particular components $x, y \in Z(S)*$, in the event that $xSy = 0$, there is an edge interfacing x and y. Note

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that *Γ(S)* is a subgraph of *Γ(S).* As of late, several authors concentrated on additional the graph *Γ(S)* and its augmentation to a simplicial complex, cf. [13]. Obviously for any superb ideal p in the event that x and y are nearby in *Γ(S),* $x \in p$ or $y \in p$. So, for each superb ideal p and each edge e, one of the end points of e has a place with p,

Building graphs from commutative rings was started by Ivan Beck through his work on zero-divisor charts and from that point a few graphs developments were made by a few creators. Through the development of charts from commutative rings, exchange between mathematical properties of commutative rings and graphs hypothetical properties of determined charts are contemplated. A portion of the charts characterized out of gatherings are Cayley graphs from bunches [25], non-commutating chart of a gathering [2], power chart of a limited gathering [29]. A chart is characterized out of non no divisors of a ring and is called zero-divisor graphs of a ring [12]. Intriguing varieties are likewise characterized like all out graphs[8], unit charts [15] and co maximal charts [33] related with rings. Additionally, charts are characterized out of standards of a ring, to be specific obliterating ideal graphs of a ring [23], convergence chart of beliefs of rings [30, 31] and so forth.

Connecting a graphs with zero-divisors of a commutative ring was presented [24] in 1988, where the creator discussed shading of such graphs. Subsequent to presenting zero-divisor graphs, I. Beck made a guess that the faction number and chromatic number of the zero-divisor graphs are equivalent. In 1993, few authors settled Beck's guess in negative by giving a counter model [11]. Additionally, they have explored the interchange between the ring hypothetical properties of a commutative ring and graphs hypothetical properties of the zero-divisor chart. The definition alongside name for zero-divisor chart (R) was first presented in 1999, subsequent to altering the meaning of D. D. Anderson [11, 12].

Example of a zero-divisor graph for $R = Z_2 X \frac{Z_2(x)}{<x^2>}$ is shown in fig 1.1

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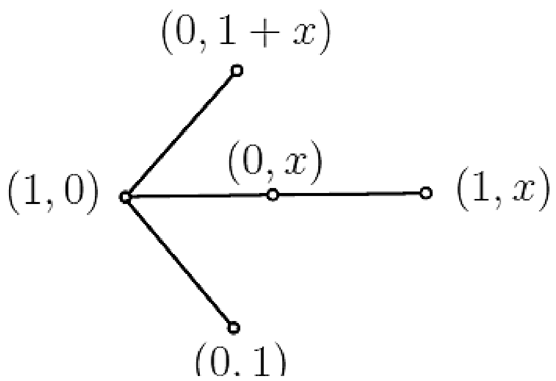

Figure 1.1: $R = Z_2 X \frac{Z_2(x)}{<x^2>}$

Let us collect some basic definitions and results on commutative rings:

Definition 1.1.[2] A ring (R, +, ·) is a nonempty set R together with binary operations '+'and '·'defined on R, which satisfy the following conditions:

(i) $(R, +)$ is an abelian group

(ii) $a \cdot (b \cdot c) = (a \cdot b) \cdot c, \forall\, a, b, c \in R$

(iii) $a \cdot (b + c) = a \cdot b + a \cdot c, \forall\, a, b, c \in R$

(iv) $(a + b) \cdot c = a \cdot c + b \cdot c, \forall\, a, b, c \in R.$

Definition 1.2. [9] A ring R is called commutative if for every $a, b \in R$,

$\ni a \cdot b = b \cdot a.$

Definition 1.3. [10] Let R be a ring. An element $e \in R$ is called an identity element if $ea = ae = a\ \forall\, a \in R$. The identity element of a ring R is denoted by '*1*'.

Definition 1.4. [16] Let R be a ring with identity. An element $u \in R$ is called a unit element if there exists $v \in R$ such that $uv = 1 = vu$ and the inverse of u is often denoted by $u-1$. The collection of all units in R is denoted by $U(R)$ or $R\times$. It is easy to check that $U(R)$ is a group under multiplication and is called multiplicative group of R.

Definition 1.5. [17] A ring R with identity is called a division ring if every nonzero element of R is a unit. A commutative division ring R is called a field.

Definition 1.6. [18] An element $x \in R$ is said to be a zero-divisor if there exists $0 \neq y \in R \ni xy = 0$ where 0 is the additive identity. The set of all zero-divisors in R is denoted by $Z(R)$.

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Definition 1.7. [19] An ideal P of a ring R is called a prime ideal if $P \neq R$ and $\forall\, a, b \in R$, $ab \in P$ implies $a \in P$ or $b \in P$.

Definition 1.8. [20] A commutative ring R is called an integral domain if R has no non-zero zero-divisors.

Definition 1.9. [21] Let R be a ring. The characteristic of R is the least positive integer n such that $na = 0\ \forall\, a \in R$. If no such positive integer exists, then R is said to be of characteristic zero.

Let us collect some basic definitions and results on s:

Definition 1.9. [5] Given a bipartite G of n vertices, there exists a connected bipartite H' such that G is an induced sub of H' and all the vertices of G in H' have equal status in H'.

Definition 1.10. [22] Given a bipartite G there exists a bipartite H such that

$M(H) \cong G$.

Definition 1.11. [27] Given a bipartite G there exists a bipartite H such that

$AM(H) \cong G$

Definition 1.12. [28] Given a k-partite G there exists a k-partite H such that

$M(H) \cong G$.

Definition 1.13 [32] Given a k-partite G there exists a k-partite H′ such that

$AM(H') \cong G$.

2.0 Main Results

Entrenching Median and Anti - median conceptions.

Theorem 2.1. For a determinate commutative semigroup S, the set $V(\varphi(\Gamma(S))) \cup \{0\}$ is an median of S.

Proof. Let $x \in V(\varphi\ (\Gamma(S)))$, and $r \in S$. Suppose that $rx \neq 0$.

Then $e(rx) = max\{d\ (u, rx)|u \in V(G)\} \leq max\{d\ (u, x)|u \in V(G)\} = e(x)$.

Thus $e(rx) = e(x)$.

Hence, $rx \in V(\varphi\ (\Gamma(S)))\ \cup \{0\}$.

Remark 2.1. A subgraph H of a graph G is a crossing subgraph of G if $V(H) = V(G)$. On the off chance that U is a bunch of edges of a graph G, $G \setminus U$ is the crossing subgraph of G acquired by erasing the edges in U from $E(G)$. A subset U of the edge set of an associated

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chart *G* is an edge cutset of *G* if *G* \ *U* is separated. An edge cutset of *G* is negligible assuming no appropriate subset of *U* is edge cutset. Assuming *e* is an edge of *G*, with the end goal that *G* \ *{e}* is detached, then *e* is known as an extension. Note that on the off chance that *U* is an insignificant edge cutset, *G* \ *U* has precisely two associated median parts.

Corollary 2.1. Let *T* be the minimal edge cutset of *Γ(S),* and *G1*, *G2* are two median parts of *G* \ *T*. Then the following hold.

(i) For any $i = 1, 2, (V(Gi) \cap V(T)) \cup \{0\}$ is ideal of *S* provided G_i has at least two vertices.

(ii) $V(T) \cup \{0\}$ is an ideal if G_1 or G_2 has only one vertex. A commutative semigroup is called reduced if for any $x \in S$, $x_n = 0$ implies $x = 0$. The annihilator of $x \in S$ is denoted by *Ann (x)* and it is defined as $Ann\ (x) = \{a \in S | ax = 0\}$. In Satyanarayana gave some characterization of *s*.

Theorem 2.2. Let S be a commutative and reduced semigroup in which Γ(S) does not contain a median and anti – median clique. Then S satisfies the ascending chain conditions (a.c.c) on annihilators.

Proof. Suppose that $Ann\ x_1 < Ann\ x_2 < \cdots < Ann\ x_n$ be a cumulative chain of ideals. For each$i \geq 2$, select $a_1 \in Ann x_1 \backslash Ann x_{i-1}$. Then every a_n is nonzero, for $= 2,3, \cdots$. Also $y_i y_j$ for any $i \neq j$. Since S is a commutative and condensed semigroup, we have $y_i \neq y_j$ when $i \neq j$. Thus, one can obtain an median and anti – median in *S*. This is a contradiction and so the assertion holds.

Theorem 2.3. Let S be a commutative rings of median and anti – median. Then the subsequent results hold:

(i) If $|Ass\ (S)| \geq 3$ and $\emptyset = Ann\ (x)$, $\chi = Ann\ (y)$ are two distinct elements of Ass (S), then $xy = 0$.

(ii) If $|Ass\ (S)| \geq 3$, then girth $(\Gamma(S)) = 4$.

(iii) If $|Ass\ (S)| \geq 6$, then $\Gamma(S)$ is not planar (A graph *G* is planar if it can be drawn in the plane in such a way that no two edges meet except at vertex with which they are both incident).

Proof. (i). We can assume that there exists $r \in \emptyset \setminus \chi$. Then $rx = 0$ and *so* $rSx = 0 \in \emptyset$. Since χ is a prime ideal, $x \in \chi$ and hence $xy = 0$.

(ii). Let $\emptyset_1 = Ann\ (x_1)$, $\emptyset_2 = Ann\ (x_2)$, $\emptyset_3 =$ Ann (x_3) and $\emptyset_4 =$ Ann (x_4) belong to *Ass (S)*. Then $x_1 - x_2 - x_3 - x_4 - x_1$ is a cycle of length 4.

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(iii). Since $|Ass\ (S)| \geq 6$, k_5 is a subgraph of *Γ(S),* and hence by Kuratowski's Theorem *Γ(S)* is not planar.

Corollary 2.2. Let $\emptyset_1 = Ann\ (x_1)$, $\emptyset_2 = Ann\ (x_2)$, $\emptyset_3$= Ann (x_3) and $\emptyset_4$= Ann (x_4), $\cdots$ $\emptyset_n$= Ann (x_n) belong to *Ass (S)* with reference to commutative rings of median and anti – median. Then $x_1 - x_2 - x_3 - x_4 - \cdots x_n$ is a cycle of length *n*.

Remark 2.2. Let *S* be a commutative semigroup and let *Ann a* be a maximal element of $\{Annx: 0 \neq x \in S\}$. Then *Ann a* is a prime ideal.

Theorem 2.4. Let *S* be a commutative semigroup, then the median graph of a bipartite graph is induced by the vertices of maximum degree in G.

Proof. *S* be a commutative semigroup and *G* is a bopartite median graph, thus *d(v, u) < 2* for any pair of vertices *u*, *v* of G. Let the degree of *v* in *G* be *d*. Then, these *d* vertices are at a distance 1 from *v*. So, there are *p - 1 - d* vertices u in G such that *d(v, u)* = 2 and *D(v) = d + 2(p - 1 - d) = 2(p - 1) - d*. Hence the vertices in G such that *D(v)*is minimum are those for which the degree is maximum. Hence the proof.

Remark 2.3 The median graph of a bipartite graph is also a bipartite graph.

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