THE CONNECTED RESTRAINED DETOUR EDGE MONOPHONIC DOMINATION NUMBER OF A GRAPH

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ABSTRACT- In this paper the concept of connected restrained detour edge monophonic domination number $M$of a graph G is introduced. For a connected graph $G = (V, E)$ of order at least two, a connected restrained detour edge monophonic dominating set M of a graph G is a detour edge monophonic dominating set such that either $M = V$ or the sub graph induced by $V - M$ has no isolated vertices. The minimum cardinality a minimal restrained detour edge monophonic dominating set of G is the minimal restrained detour edge monophonic domination number of G and is denoted by $\gamma_{demc_r}(G)$. We determine bounds for it and characterize graphs which realize these bounds. It is shown thatIf p, a, b, c and d are positive integers such that $3 \leq a \leq b \leq c \leq d \leq p - 2$, then there exists a connected graph G of order p,$dm(G) = a, \gamma_{dm}(G) = b, \gamma_{dm_r}(G) = c$ and $\gamma_{dmc_r}(G) = d$.

Keywords :**Connected detour edge monophonic dominating set,connected detour edge monophonic domination number, connected restrained edge detour monophonic dominating set and connected restrained detour edge monophonic domination number.**

I. INTRODUCTION

By a graph $G = (V, E)$ we mean a finite undirected connected graph without loops or multiple edges. The order and size of G are denoted by p and q, respectively. The neighborhood of a vertex v of G is the set $N(v)$ consisting of all vertices u which are adjacent with v. A vertex v of G is an extreme vertex if the sub graph induced by its neighborhood is complete. A vertex with degree one is called an end vertex. A vertex v of a connected graph G is called a support vertex of G if it is adjacent to an end vertex of G. A vertex v in a connected graph G is a cut vertex of G, if $G - v$is disconnected. A chord of a path $u_1, u_2, u_3, \ldots, u_k$ in G is an edge $u_i u_j$ with $j \geq i + 2$. A path P is called a monophonic path if it is a chordless path. A set M of vertices of G is aedge monophonic set of G if each vertex of G lies on a u-v monophonic path for some u and v in M. The minimum cardinality of aedge monophonic set of G is the edge monophonic number of G and is denoted by e(G). For a subset D of vertices, we call D a dominating set for each $x \in V(G) - D$, x is adjacent to at least one vertex of D. The domination number of D is the minimum cardinality of a dominating set of G and is denoted by $\gamma_m(G)$ [4]. A set of vertices M in G is called a monophonic dominating set if M is both edge monophonic set and a dominating set. The minimum cardinality of aedge monophonic dominating set of G is the edge monophonic domination number of G and is denoted by $\gamma_{em}(G)$ [5]. A longest $x - y$monphonic path is called an $x - y$ detour monophonic path. A set M of a graph G is a detour edge monophonic set of G if each vertex v of G is lies on an $x - y$ detour monophonic path, for some x and y in M. The minimum cardinality of a detour edge monophonic set of G is the detour edge monophonic number of G and is denoted by $dem(G)$ [6]. A minimalrestrained detour edge monophonic dominating set of G is a detour edge monophonic dominating set M such that either $M = V$ or the sub graph induced by $V - M$ has no isolated vertices. The minimum cardinality of a connectedrestrained detour edge monophonic dominating set of G is the connectedrestrained detour edge monophonic domination number of G and is denoted by$\gamma_{demc_r}(G)$.

Theorem 1.1. [6] Each extreme vertex of a connected graph G belongs to every detour monophonic set of G. Moreover, if the set M of all extreme vertices of G is a detour monophonic set, then M is the unique minimum detour monophonic set of G.
Theorem 1.2. [6] Let G be a connected graph with cut – vertex v and let M be a detour monophonic set of G. Then every component of $G - v$ contains an element of M.
Theorem 1.3. [3] No cut vertex of a connected graph G belongs to any minimum monophonic set of G.
Theorem 1.4. [5] Each extreme vertex of a connected graph G belongs to every monophonic dominating set of G.

II. THE CONNECTED RESTRAINED DETOUR EDGE MONOPHONIC DOMINATION NUMBER OF A GRAPH

Definition 2.1 A subset M of $V(G)$ is a connected restrained detour edge monophonic dominating set if (i) M is a detour edge monophonic set (ii) the induced subgraph$< M >$ is connected (iii) the subgraph $V - M$ has no isolated vertices and (iv) M is a dominating set of G.
Definition 2.2 The connected restrained detour edge monophonic domination number is the minimum counting number with all the connected restrained detour edge monophonic dominating sets and is denoted by $\gamma_{demc_r}(G)$.
Theorem 2.4 Each extreme vertex of a connected graph G belongs to every connected restrained detour edge monophonic dominating set of G.

Proof. Since every connected restrained detour edge monophonic dominating set of G is a restrained detour edge monophonic dominating set of G, it follows from Theorem 1.2.

Corollary 2.5 For the complete graph $K_p (p \geq 2), \gamma_{demc_r}(K_p) = p$.

Theorem 2.5 Let G be a connected graph with cut vertices and let M be a connected restrained detour edge monophonic dominating set of G. If v is a cutvertex of G, then every component of $G - v$ contains an element of M.

Proof. Since every connected restrained detour edge monophonic dominating set of G is a restrained detour edge monophonic dominating set of G, it follows from Theorem 1.2.

Theorem 2.6 Every cut vertex of a connected graph G belongs to every connected restrained detour edge monophonic dominating set of G.

Proof. Let v be any cutvertex of G and let $G_1, G_2, \dots, G_r (r \geq 2)$ be the components of $G - v$. Let$M$ be any connected restrained detour edge monophonic dominating set of G. Then by Theorem 2.5, M contains at least one element from each $G_i (1 \leq i \leq r)$. Since $G[M]$ is connected, it follows that $v \in M$. For a cutvertex v in a connected graph G and a component H of $G - v$, the subgraph H and the vertex v together with all edges joining v and $V(H)$ is called a branch of G at v. Since every endblock B is a branch of G at some cut vertex, it follows from Theorem 2.5 that every minimum connected restrained detour edge monophonic dominating set of G contains at least one vertex from B that is not a cutvertex.

Corollary 2.7 For any nontrivial tree T of order p, $\gamma_{demc_r}(T) = p$.

Proof. It follows from Theorems 2.4 and 2.6.

Theorem 2.8 For any connected graph G of order $p \geq 2, 2 \leq \gamma_{demc_r}(G) \leq p$.

Proof. Since $V(G)$ is a connected restrained detour edge monophonic dominating set of G, it follows that $\gamma_{demc_r}(G) \leq p$. Also it is clear that $\gamma_{demc_r}(G) \geq 2$ and so $2 \leq \gamma_{demc_r}(G) \leq p$

Theorem 2.9 For a connected graph G of order $p \geq 2, 2 \leq \gamma_{dem_r}(G) \leq \gamma_{demc_r}(G) \leq p$.

Proof. Any restrained detour edge monophonic dominating set needs at least two vertices and so $\gamma_{dem_r}(G) \geq 2$. Since every connected restrained detour edge monophonic dominating set of G is also a restrained detour edge monophonic dominating set of G, it follows that $\gamma_{dem_r}(G) \leq \gamma_{demc_r}(G)$. Also, since $V(G)$ induces a connected restrained detour edge monophonic dominating set of G, it is clear that $\gamma_{demc_r}(G) \leq p$.

Theorem 2.10 For a connected graph G of order $p \geq 2, 2 \leq \gamma_{mec_r}(G) \leq \gamma_{demc_r}(G) \leq p$.

Proof. Any connected restrained edge monophonic dominating set needs at least two vertices and so $\gamma_{emc_r}(G) \geq 2$. Since every connected restrained detour edge monophonic dominating set is also a connected restrained edge monophonic dominating set, it follows that $\gamma_{emc_r}(G) \leq \gamma_{demc_r}(G)$. Also, since $V(G)$ induces a connected restrained detour edge monophonic dominating set of G, it is clear that $\gamma_{demc_r}(G) \leq p$.

Theorem 2.11 Let G be a connected graph of order $p \geq 2$. Then $G = K_2$if and only if $\gamma_{demc_r}(G) = 2$.

Proof. If $G = K_2$, then $\gamma_{demc_r}(G) = 2$. Conversely, let $\gamma_{demc_r}(G) = 2$. Let $M = \{u, v\}$ be a minimum connected restrained detour edge monophonic dominating set of G. Then uv is an edge. If $G \neq K_2$, there exists a vertex w different from u and v. Then w can not lie on any $u - v$ restrained detour edge monophonic dominating path, so that M is not a restrained detour edge monophonic dominating set, which is a contradiction. Thus $G = K_2$.

Corollary2.12 For the Wheel $C_p(p \geq 6), \gamma_{demc_r}(C_p) = 3$.

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