A STUDY OF STABILITY ANALYSIS OF MAGNETO CONVECTION OF A COUPLE STRESS FLUID

Dr. Irfana Begum
Lecturer in Science, S.J.Polytechnic , K R Circle Bangalore., Karnataka.

Abstract

The non autonomous Ginzburg-Landau equation with time-periodic coefficients is derived for Rayleigh Bènard convection of couple stress fluid in the presence magnetic field. Nusselt number, which is obtained as function of the slow time scale, is used to quantify heat transport.

Keywords-Couple stress fluid, Rayleigh Bènard convection, Magnetic field.

1.Introduction

The classical Rayleigh Bènard convection studies the convection in fluids that arise due to Couple stress fluids are used as a continuum model in many fluid applications that involve suspended particles. Couple stress fluids have distinct features, such as polar effects.These fluids find the in uses in a number of processes that occurring industry,such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of metallic plate in a bath, exotic lubrication and colloidal and suspension solutions. The constitutive equations for couple stress fluids were given by Stokes[1]. Since then, many authors have done some extensive studies on Rayleigh Bènard convection of couple stress fluids. Some of the notable studies performed in this area are by Sharma and Thakur [3], R.C Sharmaand Shivani Sharma [4], VivekKumar and SudhirKumar [5]. Various practical applications of couple stress fluidin industries led to the studies on Rayleigh Bènard convection in couple stress fluids in the presence of rotation and magnetic field. The works of A.S. Banyal [6],R.C. Sharma and Monika Sharma [7], A. K. Agarwal and Suman Makhija[8],PradeepKumar[9]are some of the important works in this area. All these works strongly established the stabilizing effect of magnetic field and rotation on the onset of convection in a couple stress fluid. Thereafter, extensive research is being conducted on the effects of modulation on the onset of convection in a couple stress fluid.

In the present work, we intend to investigate the onset of RayleighBènard convection in a couple stress fluid in the presence of magnetic field using weak nonlinear stability analysis with the help of Ginzburg- Landau model. The objective of this paper is to study how the onset criteria for convection is affected by Prandtl number , Magnetic Prandtl number,Taylornumber, Chandrasekhar number and Couple stressparameter.

II. MATHEMATICAL FORMULATION

Consider a layer of couple stress liquid confined between two infinite horizontal surfaces separated by a distance d. A Cartesian system is taken withorigin in the lower boundary and z-axis vertically upward. Lower surfaceis maintained at higher temperature $T_0 + \Delta T$ and upper surface is maintained at temperature.

The basic governing equationsare:

$$\nabla\vec{q} = 0 \qquad \ldots(1)$$

$$\rho_0\left[\frac{\partial\vec{q}}{\partial t} + \left(\vec{q}\nabla\right)\vec{q} + 2\vec{\Omega}(t)\times\vec{q}\right] = -\nabla P_{rm} + \rho\vec{g} + \mu\nabla^2\vec{q} - \mu'\nabla^4\vec{q} + \mu_m\left(\vec{H}.\nabla\right)\vec{H} \qquad \ldots (2)$$

$$\frac{\partial T}{\partial t} + \left(\vec{q}.\nabla\right)T = \chi_t\nabla^2 T \qquad \ldots (3)$$

$$\rho = \rho_0\left(1-\alpha_t\left(T-T_0\right)\right) \qquad \ldots (4)$$

$$\frac{\partial\vec{H}}{\partial t} + \left(\vec{q}.\nabla\right)\vec{H} = \left(\vec{H}.\nabla\right)\vec{q} + \gamma_m\nabla^2\vec{H} \qquad \ldots (5)$$

$$\nabla.\vec{H} = 0 \qquad \ldots (6)$$

The basic state is assumed to be quiescent, i.e,

$$\vec{q}_b = (0,0,0),\ \vec{\Omega} = \Omega_0 k, T = T_b(z), P_{rm.b} = P_{rm.b}(z), \rho = \rho_b(z), \vec{H}_b = H_o k \qquad \ldots (7)$$

where the subscript b denotes the basic state.

Now we introduce infinitesimally small perturbations to the basic state to get

$$\vec{q} = \vec{q}\,',\ \rho_b + \rho',\ P_{rm} = P_{rm.b} + P_{rm}',\ T = T_b + T',\ \vec{H} = H_0 k + \vec{H}' \qquad \ldots (8)$$

Substituting (8) in (1) to (6), eliminating pressure by cross differentiation, introducing the stream functions

$u' = \frac{\partial\psi}{\partial z}, w' = -\frac{\partial\psi}{\partial x}, H_x = \frac{\partial\phi}{\partial z}, H_z = -\frac{\partial\phi}{\partial x}$, we get

$$\rho_0\left[\frac{\partial}{\partial t}\left(\nabla^2\Psi\right) + J\left(\nabla^2\Psi,\Psi\right)\right] = 2\rho_0\vec{\Omega}(t)\frac{\partial v'}{\partial z} + \mu\nabla^4\Psi - \mu'\nabla^6\Psi + \mu_m H_0\frac{\partial}{\partial z}\nabla^2\phi + \mu_m J\left(\nabla^2\phi,\phi\right) - \rho_0\alpha_t\frac{\partial T'}{\partial x}g \qquad \ldots(9)$$

$$\rho_0\left[\frac{\partial v'}{\partial t} + J\left(v',\psi\right) + 2\vec{\Omega}(t)\frac{\partial\psi}{\partial z}\right] = \mu\nabla^2 v' - u'\nabla^4 v' \qquad \ldots (10)$$

$$\frac{\partial T'}{\partial t} + J\left(T',\Psi\right) = \chi_t\nabla^2 T' - \frac{\nabla T}{d}\frac{\partial\Psi}{\partial x} \qquad \ldots (11)$$

$$\frac{\partial\phi}{\partial t} - J\left(\Psi,\phi\right) = H_0\frac{\partial\Psi}{dt} + \gamma_m\nabla^2\phi \qquad \ldots (12)$$

Equations (9) to (12) are rendered dimensionless using the following transformations.

$$\left(x^*,z^*\right) = \left(\frac{x}{d},\frac{z}{d}\right), \Psi^* = \frac{\Psi}{\chi_t}, \nabla^* = \frac{\nabla}{(1/d)}, t^* = \frac{t}{\left(d^2/\chi_t\right)}, T^* = \frac{T}{\Delta T}, u^* = \frac{u}{\left(\chi_t/d\right)}, w^* = \frac{w}{\left(\chi_t/d\right)}, H^* = \frac{H}{H_0}$$

The dimensionless equations are given by

$$\begin{bmatrix} -\nabla^4 + C\nabla^6 & R_0\frac{\partial}{\partial x} & -QPm\frac{\partial}{\partial z}\nabla^2 & -Ta^{1/2}\left(1+\varepsilon^2\delta\cos\omega t\right)\frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & -\nabla^2 & 0 & 0 \\ -\frac{\partial}{\partial z} & 0 & -Pm\nabla^2 & 0 \\ Ta^{1/2}\left(1+\varepsilon^2\delta\cos\omega t\right)\frac{\partial}{\partial z} & 0 & 0 & -\nabla^2 + C\nabla^4 \end{bmatrix}\begin{bmatrix}\Psi \\ T \\ \phi \\ v\end{bmatrix} =$$

ISSN PRINT 2319 1775 Online 2320 7876
 UGC CARE Listed (Group -I) Journal Volume 11, Iss 12, 2022

$$\begin{bmatrix} -\dfrac{1}{\Pr}\left[\dfrac{\partial}{\partial t}\nabla^2 + J\left(\nabla^2\Psi,\Psi\right)\right] + QPmJ\left(\nabla^2\phi,\phi\right) \\ -\dfrac{\partial T}{\partial t} + J\left(\Psi,T\right) \\ -\dfrac{\partial \phi}{\partial t} + J\left(\Psi,\phi\right) \\ -\dfrac{1}{\Pr}\left[\dfrac{\partial v}{\partial t} - J\left(\Psi,v\right)\right] \end{bmatrix} \ldots(13)$$

Where $\Pr = \dfrac{u}{\rho_0\chi_t}, Ta = \left(\dfrac{2\rho_0 d^2\Omega_0}{\mu}\right)^2, Pm = \dfrac{\gamma_m}{\chi_1}, Q = \dfrac{\mu_m H_0^2 d^2}{\mu\gamma_m}, C = \dfrac{\mu'}{\mu d^2}$ and $R = \dfrac{\alpha_1 g d^3 \Delta T \rho_0}{\mu\chi_1}$

The system of equations (13) are solved for free- free isothermal boundary conditions

$\Psi = \nabla^2\Psi = T = \phi = D\phi = 0$ at $z = 0,1$.

Finite Amplitude Equation, Heat and Mass Transfer

We now introduce the following asymptotic expansions in (13).

$$R = R_0 + \varepsilon^2 R_2 + \varepsilon^4 R_4 + \ldots$$

$$\Psi = \varepsilon\Psi_1 + \varepsilon^2\Psi_3 + \varepsilon^3\Psi_3 + \ldots.$$

$$v = \varepsilon v_1 + \varepsilon^2 v_2 + \varepsilon^3 v_3 + \ldots$$

$$T = \varepsilon T_1 + \varepsilon^2 T_2 + \varepsilon^3 T_3 + \ldots.$$

$$\phi = \varepsilon\phi_1 + \varepsilon^2\phi_2 + \varepsilon^3\phi_3 + \ldots.$$

At the lowest mode, we have

$$\begin{bmatrix} -\nabla^4 + C\nabla^6 & R_0\dfrac{\partial}{\partial x} & -QPm\dfrac{\partial}{\partial z}\nabla^2 & -Ta^{1/2}\dfrac{\partial}{\partial z} \\ \dfrac{\partial}{\partial x} & -\nabla^2 & 0 & 0 \\ -\dfrac{\partial}{\partial z} & 0 & -Pm\nabla^2 & 0 \\ Ta^{1/2}\dfrac{\partial}{\partial z} & 0 & 0 & -\nabla^2 + C\nabla^4 \end{bmatrix} \begin{bmatrix} \Psi_1 \\ T_1 \\ \phi_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \ldots(14)$$

The solution to the lowest order system is given as

$$\Psi_1 = A(\tau)\sin(K_C x)\sin(\pi z)$$

$$T_1 = -\frac{K_c}{\delta^2}A(\tau)\cos(K_C x)\sin(\pi z)$$

$$\phi_1 = \frac{\pi}{Pm\delta^2}A(\tau)\sin(K_C x)\cos(\pi z)$$

$$v_1 = -\frac{\pi Ta^{1/2}}{\delta^2\left(1 + C\delta^2\right)}A(\tau)\sin(K_C x)\cos(\pi z)$$

where $\delta^2 = K_C^2 + \pi^2$

The critical Rayleigh number is given by $R_0 = \frac{Ta\pi^2}{K_c^2\left(1+C\delta^2\right)} + \frac{Q\pi^2\delta^2}{K_c^2} + \frac{\delta^6\left(1+C\delta^2\right)}{K_c^2}$

At the second order, we have,

$$\begin{bmatrix} -\nabla^4 + C\nabla^6 & R_0\frac{\partial}{\partial x} & -QPm\frac{\partial}{\partial z}\nabla^2 & -Ta^{1/2}\frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & -\nabla^2 & 0 & 0 \\ -\frac{\partial}{\partial z} & 0 & -Pm\nabla^2 & 0 \\ Ta^{1/2}\frac{\partial}{\partial z} & 0 & 0 & -\nabla^2 + C\nabla^4 \end{bmatrix} \begin{bmatrix} \Psi_2 \\ T_2 \\ \phi_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} R_{21} \\ R_{22} \\ R_{23} \\ R_{24} \end{bmatrix}$$

where

$$R_{21} = 0$$

$$R_{22} = -\frac{A(\tau)^2 K_c^2 \pi}{2\delta^2}\sin(2\pi z)$$

$$R_{23} = -\frac{A(\tau)^2 K_c^2 \pi^2}{2Pm\delta^2}\sin(2K_c x)$$

$$R_{24} = -\frac{A(\tau)^2 K_c \pi^2 Ta^{1/2}}{2Pm\delta^2\left(1+C\delta^2\right)}\sin(2K_c x)$$

The second order solutions are as follows.

$$\Psi_2 = 0$$

$$T_2 = -\frac{K_c^2}{8\pi\delta^2}A(\tau)^2\sin(2\pi z)$$

$$\phi_2 = -\frac{\pi^2}{8K_c Pm^2\delta^2}A(\tau)^2\sin(2K_c x)$$

$$v_2 = \frac{\pi^2 Ta^{1/2}}{8K_c \Pr\delta^2\left(1+C\delta^2\right)\left(1+4K_c^2\right)}A(\tau)^2\sin(2K_c x)$$

The horizontally averaged Nusselt number is given by $Nu(\tau) = 1 + \frac{K_c^2}{8\pi\delta^2}A(\tau)^2$ …(15)

At the third order, we have

$$\begin{bmatrix} -\nabla^4 + C\nabla^6 & R_0\frac{\partial}{\partial x} & -QPm\frac{\partial}{\partial z}\nabla^2 & -Ta^{1/2}\frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & -\nabla^2 & 0 & 0 \\ -\frac{\partial}{\partial z} & 0 & -Pm\nabla^2 & 0 \\ Ta^{1/2}\frac{\partial}{\partial z} & 0 & 0 & -\nabla^2 + C\nabla^4 \end{bmatrix} \begin{bmatrix} \Psi_3 \\ T_3 \\ \phi_3 \\ v_3 \end{bmatrix} = \begin{bmatrix} R_{31} \\ R_{32} \\ R_{33} \\ R_{34} \end{bmatrix}$$

where

$$R_{31}=-\frac{1}{\Pr}\frac{\partial}{\partial\tau}\nabla^2\Psi_1+\frac{1}{\Pr}\left[J\left(\Psi_1,\nabla^2\Psi_1\right)+J\left(\Psi_2,\nabla^2\Psi_1\right)\right]+QPm\left[J\left(\nabla^2\phi_2,\phi_1\right)+J\left(\nabla^2\phi_1,\phi_2\right)\right]-R_2\frac{\partial T_1}{\partial x}=Ta^{1/2}\delta\cos\omega t\frac{\partial v_1}{\partial z}$$

$$R_{32}=J\left(\Psi_1,T_2\right)+J\left(\Psi_2,T_1\right)-\frac{\partial T_1}{\partial\tau}$$

$$R_{33}=J\left(\Psi_1,\phi_2\right)+J\left(\Psi_2,\phi_1\right)-\frac{\partial\phi_1}{\partial\tau}$$

$$R_{34}=-\frac{1}{\Pr}\frac{\partial v_1}{\partial\tau}-Ta^{1/2}\delta\cos\omega t\frac{\partial\Psi_1}{\partial z}$$

Now we apply the solvability condition for the criterion of the third order solution of the system (16) to arrive at the non-autonomous Ginzburg Landau equation for stationary convection with time periodic coefficients in theform

$$A_1A'(\tau)-A_2A(\tau)+A_3A(\tau)^3=0$$

where

$$A_1=\frac{\delta^2}{\Pr}+\frac{R_0K_c^2}{\delta^4}-\frac{Q\pi^2}{\delta^2Pm}-\frac{Ta\pi^2}{\delta^4\left(1+C\delta^2\right)^2\Pr},A_2=\frac{K_c^2R_2}{\delta^2}-\frac{2\mathrm{Ta}\,\pi^2\delta\cos\omega t}{\delta^2\left(1+C\delta^2\right)}$$

$$A_3=\frac{Q\pi^4K_c^2}{2Pm^2\delta^4}-\frac{Q\pi^4}{4Pm^2\delta^2}+\frac{K_c^4R_0}{8\delta^4}$$

The Ginzburg-Landau equation given in (17) is Bernoulli equation and hence, obtaining its analytical solution is difficult as it is autonomous in nature. Inview of this, it has been solved numerically using Mathematica, subject tothe initial condition A(0) = a0 , where a0 is the chosen initial amplitude ofconvection.

III. RESULTS AND DISCUSSIONS

In this paper, a nonlinear analysis is performed to study the effect of magnetic field and rotational modulation on Rayleigh Bènard convection ofa couple stress fluid. Autonomous Ginzburg-Landau equation is derived to study the effects of parameters of theproblem on heat and mass transfer.

Fig (2) shows the variation of *Nu* with for different values of couplestress parameter *C*. It is observed that as *C* increases *Nu* decreases. The presence of couple stress increases the viscosity of the fluid and hence as *C* increases, more heating is required to make the system unstable, indicating the stabilising effect of *C*. Fig 3 show the variation of *Nu* with for different values of *Pm* and *Q* respectively. It is found that *Nu* decreases with increase in *Pm* and *Q*, showing the delayed onset of convection.

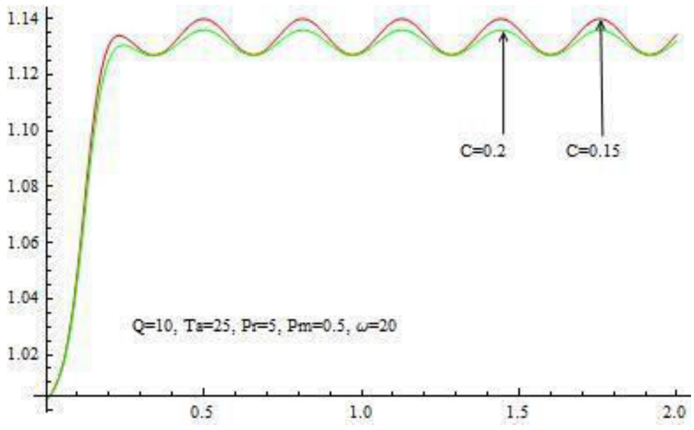

Fig. 2. Variation of Nu for different values of$C$

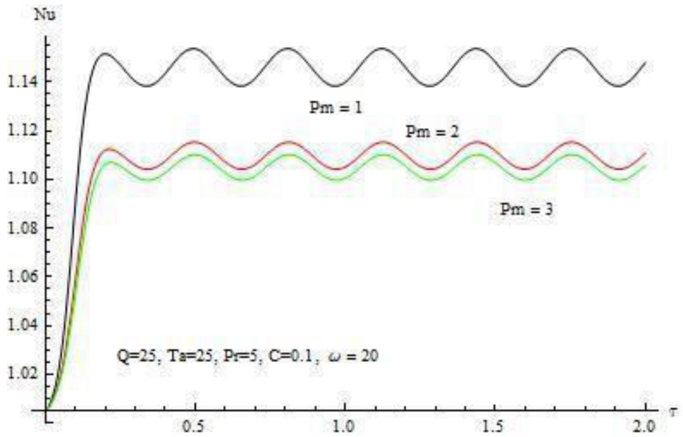

Fig. 3. Variation of Nu with for different values ofPm

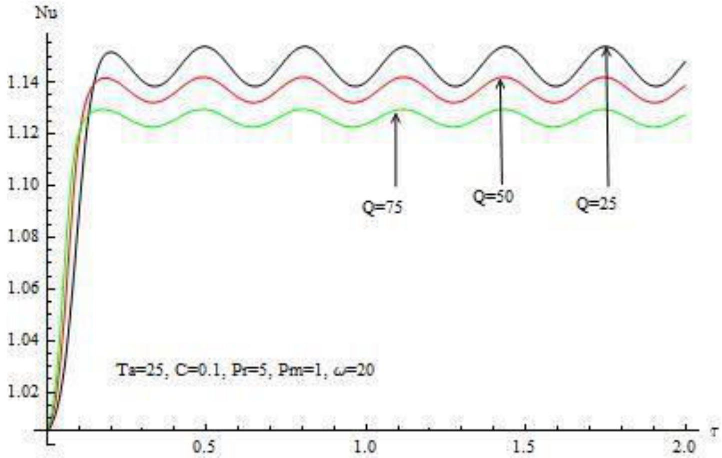

Fig. 4. Variation of Nu with for different values of$Q$

IV. CONCLUSIONS

The combined effect of magnetic field and rotational modulation on Rayleigh Bènard convection of a couple stress fluid has been studied by employing nonlinear analysis using Ginzburg- Landau model. The results have been obtained in terms of Nusselt number, and the effects of various parameters have been obtained graphically. The following are the observations.

1. The effect of increasing Couple stress parameter is to decrease the Nusselt number, thus indicating stabilising effect.

4. The effect of increasing Magnetic Prandtl number is to decrease Nusselt number, thus indicating stabilising effect.

5. The effect of increasing Chandrasekhar number is to decrease Nusselt number, hence delaying the onset of convection.

REFERENCES

[1] Stokes V.K., "Couple stresses in fluids", *Phys. Fluids*, vol. 9, pp.1079-1715, 1996.

[2] S. Chandrashekhar, "Hydrodynamic and hydromagnetic stability", Oxford: Clarendon Press

[3] R.C. Sharma and K.D. Thakur, "On couple stress fluid heated from below in porous medium in hydromagnetics", *Czechoslovak J. Physics*, vol. 50, no.6, pp. 753–758, 2000.

[4] R.C Sharma and Shivani Sharma, "On couple stress fluid heated from below in porous medium", *Indian J. of Physics (B) AND Proceedings of the Indian Association of the cultivation of Science*, vol. 72, pp. 137-139, 2001.

[5] Vivek Kumar and Sudhir Kumar, "On a couple stress fluid heated from below in hydromagnetics", *Appl. Math. An Int. J.*, vol. 5, issue 10, pp. 432-445, 2010.

[6] A.S. Banyal, " A mathematical theorem on couple stress fluid in the presence of rotation", *J. Pure Appl. & Ind. Phys.*, vol. 1, issue 4, pp. 228-235, 2011.

[7] R.C. Sharma and Monika Sharma, "Effect of suspended particles on couple stress fluid heated from below in the presence of rotation and magnetic field", *Indian J. pure appl. Math.*, vol. 35, no. 8, pp. 973-989, 2004.

[8] A. K. Agarwal and SumanMakhija, "Thermal stability of couple stress fluid in presence of magnetic field and rotation", *Indian J. Biomechanics: Special Issue (NCBM 7-8 March 2009)*, 2009.

[9] Pradeep Kumar, "Magneto- rotatory thermal convection in couple stress fluid", *Int. J. Thermal Fluid Sci.*, vol. 1, issue 1, pp. 11-20, 2012.

[10] N. Rudraiah and M.S. Malashetty, "Effects of modulation on the onset of convection in porous media", *VignanaBharathi*, vol. 11, no. 20, 1988.

[11] B. S. Bhadauria, "Effect of modulation on Rayleigh Benard convection-II", *Z. Naturforsch*, vol. 59a, pp. 266-274, 2004.

[12] B. S. Bhadauria and LokenathDebnath, "Effects of modulation on Rayleigh Benard convection. Part I, *Int. J. Mathematics and Mathematical Sciences*, vol. 19, pp. 991-1001, 2004.

[13] P. G. Siddheshwar, V. Ramchandramurthy, D. Uma, "Weak nonlinear analysis of Rayleigh Benard magneto convection in Boussinesq- Stokes suspensions with gravity modulation", *Proceedings 37th National and 4th Int. Conf, Fluid Mechs. Fluid power*, December 16-18, 2010, IIT Madras, Chennai, FMFP10-AM08, 2010.