

Price and stock dependent demand rate Inventory model for non-instantaneous deteriorating items having life time under the partial backlogging rate

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Abstract

This article presents mathematical model results for items that are not immediately deteriorating, and the service life depends on the item's price and availability. The main purpose of this work is to increase the business industry benefits. Here we are developing an inventory model where demand for a product is actually driven by inventory. The developed inventory model is widely applied to products such as food, electronics, cosmetics, clothing, fashion, merchandise, fruit, etc., where price and inventory play a very great role in demand. This reinforces the idea that inventory plays a key role in helping shoppers select a particular product from an assortment. Through numerical exploration and sensitivity analysis of the proposed model, we describe the behavior of the stock model using the particle swarm optimization method by determining the optimal order quantity, optimal order time and selling price, as well as determining and maximizing the gross profit.

Keywords: Deterioration, Inventory, Price and Stock-dependent demand. Purchasing Cost, Sales Revenue Cost.

1. Introduction

Today, trend to influence the consumers' interest by displaying the inventoried items at the shopping centre, mall, stores etc. ., is essential for every industry and business organization. The inventory level on the shopping behavior of fashion apparel products has a very effective role for any business organization to exist in the market to maintain their values. The advanced inventory levels may attract more attractive displays and thus increase sales of the items. Managing the stored inventory is a key method in trading. It is necessary to all business organizations to maintain the sufficient inventory items so as to change prospective demand into sales. The study of inventory depreciation models began when Gare and Schroeder

(1963) established a classic inventory model with a constant depreciation rate without a deficit. Covert and Philip (1973) extended the inventory model described above to a two-parameter Weibull distribution.

Dave (1988) discusses data on finite and infinite charged air emissions in more detail. It corrects Murdeshwar and Seth (1985) error and resolves the model proposed by Sarma (1983). Wear events are not included in this article. Sarma (1987) extended his original model to a state of endless iteration in the presence of scarcity, i.e. two stores in decline. Pakkala and Achary (1992) discuss the decision-making model for two-store products where value added is limited, demand equals value added, and scarcity is sufficient. Pakkala and Achary (1994) proposed a two-stage discount model where demand is inelastic, surplus is limited, and scarcity is sufficient. Singh et al. (2008) proposed the optimal policy for ordering intangible assets with inventory-driven demand in a financial market environment. Ahmad M. Alshamrani (2013) determined the stochastic fit of the inventory model used to determine the wear price of the product. Tsu-Pang Hsieh and Chung-Yuan Dye (2013) discuss the production of clothing and goods with different demand times and payback costs, showing that the savings in technology and Product code are different.

Gupta et al (2013) stated that the recommended solution for assessing product quality is the level of inventory required for non-perishable products. The impact of optimal decision making in product-based products are (Hartely (1976); Dave and Patel (1981); Sarma (1983); Bhunia and Maiti (1994); Sachan (1984); Goswami and Chaudhuri (1998); Raafat (1991) and Goyal (2001); Lee and Hsu (2009), Yong et al (2010), Hui-Ling Yang (2012), Liao et al. (2012), Hsieh and Dye (2013), Kumar et al. 2016, 2017, 2019; Mathur et al. 2019; Malik 2016, 2017, 2018; Singh 2008, 2009, 2010); when viewing data. Malik et al. (2012), Yadav and Malik (2014) discuss optimizing inventory management. Sarkar and Sarkar (2013) presented an improved inventory model where demand is different from inventory. Singh et al. (2014) discussed an equity-driven demand model in terms of acceptable late fees and inflation. Chang et al. (2015) proposed an optimal price and order policy for products that are not immediately damaged when payment is delayed according to order volume.

There are many research papers developing inventory models in which demand is constant, exponential, linearly increasing and decreasing, changing over time, and increasing directly with inventory availability. Among these strategies, Kumar et al.'s inventory management system makes extensive use of inventory-dependent demand, linear demand, quadratic demand, partial balance, two warehouses, instantaneous demand, and uncertainty. (2017, 2022); Malik et al b); Sharma et al. (2013, 2022a, 2022b); Verma et al (2022); Yadav et al. (2022a, 2022b); Singh and Malik (2009, 2010a, 2010b), Singh et al. (2011a, 2011b, 2014a, 2014b), Tyagi et al. (2022a, 2022b), Vashisth et al. (2016). However, holding large amounts of inventory is costly because it requires increased working capital and can pose significant obsolescence risks, especially for some innovative products. Most inventory items become unusable or wear out over time. Most of the above publications ignore inventory and price-dependent demand, which is very important for seasonal and fashion products. Supermarkets now have a wide range of products that are influenced by consumer demand, price and wear factors. Therefore, we have developed a model in which demand depends on inventory as prices fall. This study discusses a retailer's optimal seasonal restocking policy using a model that depends on availability and price. Our goal is to optimize retailers' overall

bottom line. Numerical examples are shown and optimal results are graphically displayed using sensitivity analysis.

2. Notations

The key notations used to obtain the solution of this model are listed below:

p	Selling price per unit of the item
t_1	Length of time in which there is no inventory shortage
Q	Ordering quantity
M	Maximum amount of demand shortage level
A	Ordering cost per order
S_c	Shortage cost per unit
L_c	Lost sale cost per unit
P_c	Purchase cost per unit
C_c	Holding cost per unit
D_c	Cost of deteriorated unit
T	Length of the replenishment cycle time
$TPF(p, t_2, T)$	Optimal total profit per unite time

3. Assumptions

The assumptions in this presents study are as follows:

- (1) The inventory level of the proposed model is due to the effects of its demand and deterioration is continuously depleted.
- (2) The demand rate is deterministic and considerably depends on the numberof the product displayed and selling price. Sell of the productsis depends their availability (displays), if a big number of product is available (displays) then sell is very high otherwise low. The demand $D(t)$ rate at time t , considered as stock and price dependent function is denoted as:

$$D(t) = \begin{cases} a + bI(t) - cp, & I(t) \geq 0 \\ a - cp, & I(t) \leq 0 \end{cases} \text{ where } a, b, c \geq 0 \text{ and } p \text{ is selling price.}$$

- (3) In the present model, no deterioration equippedin the time $[0, t_1]$ and after this period the deterioration rate $\theta(t) = \frac{1}{1 + R - t}$, where R is the maximum life time.

- (4) Shortages are permitted and partially backlogging rate is $B(t) = \frac{1}{1 + \eta(T - t)}$, where $\eta(\eta > 0)$.

- (5) In proposed model assuming the lead time is zero.

4. Mathematical Analysis

This paper proposes the Particle swarm optimization techniques for the inventory system. Here we proposed a model in which the inventory level is continuously depleted due to the shared effects of its demand and deterioration. The deviation of inventory level $I_1(t)$ changes with respect to t due to effects of demand as well as there is no deterioration; the variation of inventory level $I_2(t)$ changes with respect to t due to effects of demand and deterioration and the variation of inventory level $I_3(t)$ changes with respect to t due to effects of shortage. From the above assumptions (1) and (2), the inventory level, $I_1(t)$, at any time t can be expressed by the following differentialequation:

$$\frac{dI_1(t)}{dt} = -D(t), \quad 0 \leq t \leq t_1 \text{ or } \frac{dI_1(t)}{dt} = -(a + I_1(t) - cp), \quad 0 \leq t \leq t_1 \dots(1)$$

With boundary condition $I_1(t) = I_0$ when at $t = 0$, solution of (1) is

$$I_1(t) = -\frac{(a - cp)}{b}(1 - e^{-bt}) + I_0 e^{-bt} \dots(2)$$

The inventory levels in $[t_1, t_2]$ can be described as

$$\frac{dI_2(t)}{dt} + \theta(t)I_2(t) = -D(t), \quad t_1 \leq t \leq t_2$$

$$\text{or } \frac{dI_2(t)}{dt} + \frac{1}{1+R-t}I_2(t) = -(a + I_2(t) - cp), \quad t_1 \leq t \leq t_2 \dots(3)$$

With boundary condition $I_2(t_2) = 0$, the solution of (3), is given by

$$I_2(t) = -(a - cp)(1 + R - t)e^{-bt} \left\{ \frac{b^2}{4}(t_2^2 - t^2) + x_1(t_2 - t) + x_2 \log\left(\frac{1 + R - t_2}{1 + R - t}\right) \right\} \dots(4)$$

where $x_1 = \frac{b^2}{2}(1 + R) + b$, $x_2 = x_1(1 + R) + 1$.

At $t = t_1$, the inventory levels $I_1(t) = I_2(t)$, then we have

$$-\frac{(a - cp)}{b}(1 - e^{-bt_1}) + I_0 e^{-bt_1} = -(a - cp)(1 + R - t_1)e^{-bt_1} \left\{ \frac{b^2}{4}(t_2^2 - t_1^2) + x_1(t_2 - t_1) + x_2 \log\left(\frac{1 + R - t_2}{1 + R - t_1}\right) \right\}$$

$$\text{or } I_0 = -(a - cp) \left[\frac{(1 + R - t_1) \left\{ \frac{b^2}{4}(t_2^2 - t_1^2) + x_1(t_2 - t_1) + x_2 \log\left(\frac{1 + R - t_2}{1 + R - t_1}\right) \right\}}{1 - e^{-bt_1}} + \frac{1}{b} \right] \dots(5)$$

Using Eqns.(2) and (5), we get

$$I_1 = -(a - cp)e^{-bt} \left[\frac{(1 + R - t_1) \left\{ \frac{b^2}{4} (t_2^2 - t_1^2) + x_1(t_2 - t_1) + x_2 \log \left(\frac{1 + R - t_2}{1 + R - t_1} \right) \right\}}{+ \frac{(e^{bt} - e^{bt_1})}{b}} \right] \dots(6)$$

The shortage level $I_3(t)$, in $[t_2, T]$ is described by:

$$\frac{dI_3(t)}{dt} = -\frac{D(t)}{1 + \eta(T - t)}, \quad t_2 \leq t \leq T \text{ or } \frac{dI_3(t)}{dt} = -\frac{(a - cp)}{1 + \eta(T - t)}, \quad t_2 \leq t \leq T \dots(7)$$

With the boundary condition $I_3(t_2) = 0$, solution of (7) is

$$I_3(t) = -\frac{(a - cp)}{\eta} \log \left(\frac{1 + \eta(T - t_2)}{1 + \eta(T - t)} \right) \dots (8)$$

When we put $t = T$ in Eqn. (8), we get $M = -I_3(T) = \frac{(a - cp)}{\eta} \log(1 + \eta(T - t_2)) \dots (9)$

Order quantity can be determined from Eqns. (5) and (9) is given by: $Q = I_0 + M$

$$Q = -(a - cp) \left[\frac{(1 + R - t_1) \left\{ \frac{b^2}{4} (t_2^2 - t_1^2) + x_1(t_2 - t_1) + x_2 \log \left(\frac{1 + R - t_2}{1 + R - t_1} \right) \right\}}{+ \frac{(1 - e^{-bt})}{b} - \frac{1}{\eta} \log(1 + \eta(T - t_2))} \right] \dots(10)$$

Here A is the setup cost per cycle.(11)

Purchase cost for the total order quantity is

$$PC = P_c \times Q = -P_c(a - cp) \left[\frac{(1 + R - t_1) \left\{ \frac{b^2}{4} (t_2^2 - t_1^2) + x_1(t_2 - t_1) + x_2 \log \left(\frac{1 + R - t_2}{1 + R - t_1} \right) \right\}}{+ \frac{(1 - e^{-bt})}{b} - \frac{1}{\eta} \log(1 + \eta(T - t_2))} \right] \dots(12)$$

The inventory carrying cost of the system is given by:

$$CC = C_c \left(\int_0^{t_1} I_1(t) dt + \int_{t_1}^{t_2} I_2(t) dt \right) \\ = -C_c(a - cp) \left[\frac{(1 - e^{-bt_1})}{b} \left\{ \frac{b^2}{4} (t_2^2 - t_1^2) + x_1(t_2 - t_1) + x_2 \log \left(\frac{1 + R - t_2}{1 + R - t_1} \right) \right\} + \left(\frac{t_1}{b} + \frac{1}{b^2} - \frac{e^{bt_1}}{b^2} \right) \right]$$

$$\begin{aligned}
 & + \frac{b^2}{4} \left\{ (1+R) \left(t_2^2 t_1 - \frac{t_1^3}{3} \right) - y_1 \left(t_2^2 \frac{t_1^2}{2} - \frac{t_1^4}{4} \right) + y_2 \left(t_2^2 \frac{t_1^3}{3} - \frac{t_1^5}{5} \right) - \frac{b^2}{2} \left(t_2^2 \frac{t_1^4}{4} - \frac{t_1^6}{6} \right) \right\} \\
 & + x_1 \left\{ (1+R) \left(t_2 t_1 - \frac{t_1^2}{2} \right) - y_1 \left(t_2 \frac{t_1^2}{2} - \frac{t_1^3}{3} \right) + y_2 \left(t_2 \frac{t_1^3}{3} - \frac{t_1^4}{4} \right) - \frac{b^2}{2} \left(t_2 \frac{t_1^4}{4} - \frac{t_1^5}{5} \right) \right\} \\
 & + x_2 \log \frac{(1+R-t_2)}{(1+R-t_1)} \left\{ (1+R)t_1 - y_1 \frac{t_1^2}{2} + y_2 \frac{t_1^3}{3} - \frac{b^2 t_1^4}{8} \right\} - x_2 \left\{ \frac{y_1 t_1^2}{4} - \frac{y_2 t_1^3}{9} + \frac{b^2 t_1^4}{32} \right\} \\
 & + x_2 (1+R)^2 \log \frac{(1+R-t_1)}{(1+R)} \left\{ 1 - \frac{y_1}{2} + y_2 \frac{(1+R)}{3} - \frac{b^2 (1+R)^2}{8} \right\} \\
 & + x_2 \left\{ (1+R) \left(t_1 - \frac{y_1 t_1}{2} + \frac{y_2 t_1^2}{6} - \frac{b^2 t_1^3}{24} \right) + (1+R)^2 \left(\frac{y_2 t_1}{3} - \frac{b^2 t_1^2}{16} - \frac{b^2 t_1 (1+R)}{8} \right) \right\} \\
 & \dots (13)
 \end{aligned}$$

where $y_1 = b(1+R)+1$, $y_2 = \frac{b^2}{2}(1+R)+b^2$.

Deterioration cost per cycle is

$$DC = D_c \left(\int_{t_1}^{t_2} \theta(t) I_2(t) dt \right)$$

$$\begin{aligned}
 & = -D_c (a - cp) \left[\frac{b^2}{4} \left\{ \frac{2e^{-bt_2}}{b^2} \left(t_2 - \frac{1}{b} \right) + \frac{e^{-bt_1}}{b} \left(t_2^2 - t_1^2 - \frac{2t_1}{b} + \frac{2}{b^2} \right) \right\} \right. \\
 & + x_1 \left\{ \frac{e^{-bt_2}}{b^2} + \frac{e^{-bt_1}}{b} \left(t_2 - t_1 - \frac{1}{b} \right) + x_2 \log(1+R-t_2) \frac{(e^{-bt_1} - e^{-bt_2})}{b} \right\} \\
 & \left. - x_2 \left\{ \frac{(z_2 - e^{-bt_2})}{b} \log(1+R-t_2) - \frac{(e^{-bt_1} - z_1)}{b} \log(1+R-t_1) - \frac{b}{4} (t_2^2 - t_1^2) + \frac{z_1}{b} (t_2 - t_1) \right\} \right]
 \end{aligned}$$

Where $z_1 = \frac{b^2}{2}(1+R)-b$, $z_2 = z_1(1+R)+1$(14)

Lost sales cost is

$$LC = L_c \left(\int_{t_2}^T D(t) \left(1 - \frac{1}{1+\eta(T-t)} \right) dt \right) = L_c (a - cp) \left((T - t_2) - \frac{1}{\eta} \log(1 + \eta(T - t_2)) \right) \dots (15)$$

Shortage cost is

$$SC = S_c \left(\int_{t_2}^T -I_3(t) dt \right) = S_c \frac{(a - cp)}{\eta} \left\{ (T - t_2) - \frac{1}{\eta} \log(1 + \eta(T - t_2)) \right\} \quad \dots(16)$$

Sales revenue costis

$$\begin{aligned} SRC &= p \left(\int_0^{t_2} D(t) dt - I_3(T) \right) \\ &= -p(a - cp) \left[-t_2 + \frac{(1 - e^{-bt_1})}{b} \left\{ \frac{b^2}{4} (t_2^2 - t_1^2) + x_1(t_2 - t_1) + x_2 \log \left(\frac{1 + R - t_2}{1 + R - t_1} \right) \right\} + \left(\frac{t_1}{b} + \frac{1}{b^2} - \frac{e^{-bt_1}}{b^2} \right) \right. \\ &\quad + \frac{b^2}{4} \left\{ (1 + R) \left(t_2^2 t_1 - \frac{t_1^3}{3} \right) - y_1 \left(t_2^2 \frac{t_1^2}{2} - \frac{t_1^4}{4} \right) + y_2 \left(t_2^2 \frac{t_1^3}{3} - \frac{t_1^5}{5} \right) - \frac{b^2}{2} \left(t_2^2 \frac{t_1^4}{4} - \frac{t_1^6}{6} \right) \right\} \\ &\quad + x_1 \left\{ (1 + R) \left(t_2 t_1 - \frac{t_1^2}{2} \right) - y_1 \left(t_2 \frac{t_1^2}{2} - \frac{t_1^3}{3} \right) + y_2 \left(t_2 \frac{t_1^3}{3} - \frac{t_1^4}{4} \right) - \frac{b^2}{2} \left(t_2 \frac{t_1^4}{4} - \frac{t_1^5}{5} \right) \right\} \\ &\quad + x_2 \log \left(\frac{1 + R - t_2}{1 + R - t_1} \right) \left\{ (1 + R) t_1 - y_1 \frac{t_1^2}{2} + y_2 \frac{t_1^3}{3} - \frac{b^2 t_1^4}{8} \right\} - x_2 \left\{ \frac{y_1 t_1^2}{4} - \frac{y_2 t_1^3}{9} + \frac{b^2 t_1^4}{32} \right\} \\ &\quad + x_2 (1 + R)^2 \log \left(\frac{1 + R - t_1}{1 + R} \right) \left\{ 1 - \frac{y_1}{2} + y_2 \frac{(1 + R)}{3} - \frac{b^2 (1 + R)^2}{8} \right\} \\ &\quad + x_2 \left\{ (1 + R) \left(t_1 - \frac{y_1 t_1}{2} + \frac{y_2 t_1^2}{6} - \frac{b^2 t_1^3}{24} \right) + (1 + R)^2 \left(\frac{y_2 t_1}{3} - \frac{b^2 t_1^2}{16} - \frac{b^2 t_1 (1 + R)}{8} \right) \right\} \\ &\quad \left. - \frac{1}{\eta} \log(1 + \eta(T - t_2)) \right] \quad \dots(17) \end{aligned}$$

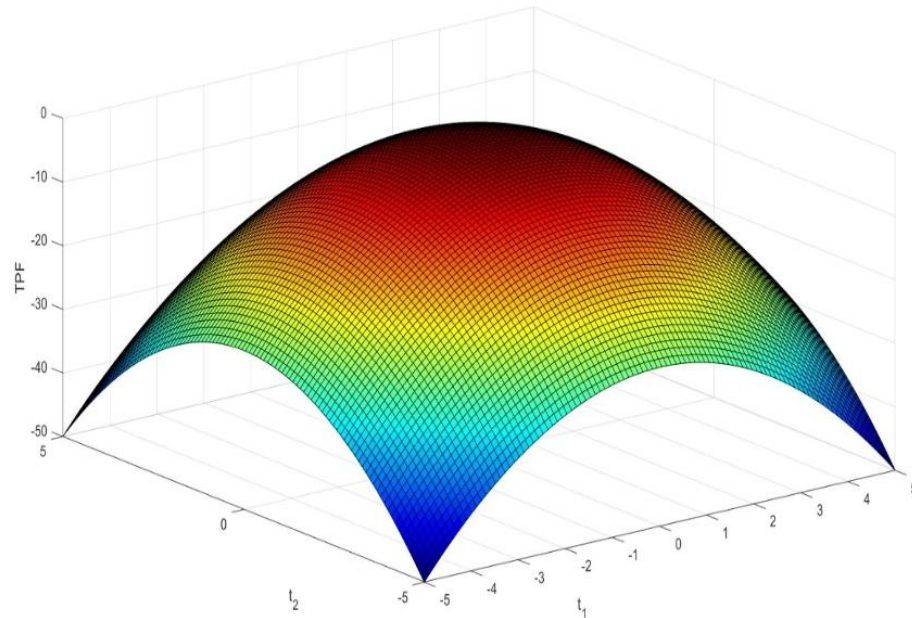
Hence, the profit is given by

$$TPF = \frac{1}{T} [SRC - A - PC - CC - DC - LC - SC] \quad \dots(18)$$

V. NUMERICAL EXAMPLES

In this section presents, the numerical example to illustrate the developed model, additionally, here the sensitivity analysis is applied to obtain the sensitivity of the proposed constraints to the optimum solution. For the numerical illustration, consider the following value:

$D_c=3/$ unit, $L_c=1.5 /$ per unit, $C_c=6/$ per unit / per unit time, $S_c=2/$ per unit / per unit time, $P_c=40 /$ per unit, $R=0.4$ year, $\eta=0.1$, $t_1=0.2$ year, $a=200$ units / year, $b=0.06$, $c=1.5$, $A=40 /$ per order. When $t_2= 0.5$ year, $T = 0.791$ year are fixed then the optimal price $p^* =0.236$, Optimal order quantity $Q^* =26.83$ and the Optimal profit $TPF^* =124.705$.



PSO strategy, a particularly look method that was built up on the behavior of running of winged creature. For the most part, the directors depend upon numerical calculation and computer structures advanced through mathematicians, factual, and commercial engineers to bargain with the state of issues of problem-answer of stock control and administration of the framework. There are as numerous styles reasonable as there are businesses since each incorporates an unmistakable cost shape and imperatives.

Iteration	Particle	Position	Velocity	p-Best	p-Best Value	g-Best	g-Best Value
1	1	[0.2, 0.4, 0.6]	[0.1, 0.3, 0.5]	[0.2, 0.4, 0.6]	10.5	[0.2, 0.4, 0.6]	10.5
	2	[0.5, 0.3, 0.8]	[0.2, 0.4, 0.6]	[0.5, 0.3, 0.8]	8.2	[0.2, 0.4, 0.6]	10.5
	3						
	N	[0.1, 0.7, 0.9]	[0.3, 0.2, 0.4]	[0.1, 0.7, 0.9]	9.8	[0.2, 0.4, 0.6]	10.5
2	1	[0.3, 0.2, 0.7]	[0.05, 0.4, 0.2]	[0.3, 0.2, 0.7]	12.1	[0.3, 0.2, 0.7]	12.1
	2	[0.6, 0.1, 0.4]	[0.3, 0.05, 0.1]	[0.4, 0.2, 0.6]	11.2	[0.3, 0.2, 0.7]	12.1
	...						
	N	[0.4, 0.6, 0.8]	[0.2, 0.1, 0.3]	[0.2, 0.4, 0.6]	10.5	[0.3, 0.2, 0.7]	12.1
...
T	1	[0.1, 0.5, 0.9]	[0.1, 0.1, 0.1]	[0.1, 0.5, 0.9]	11.9	[0.3, 0.2, 0.7]	12.1
	2	[0.7, 0.4, 0.2]	[0.2, 0.2, 0.2]	[0.4, 0.2, 0.6]	11.2	[0.3, 0.2, 0.7]	12.1
	...						
	N	[0.2, 0.8, 0.6]	[0.05, 0.05, 0.05]	[0.2, 0.8, 0.6]	9.5	[0.3, 0.2, 0.7]	12.1

Conclusions and Further Research

An inventory model is suggested in this study to identify the best restocking strategy in accordance with seasonal demand during a limited time period (seasonal period) that maximizes the retailer's total profit. We pay particular attention to retailers selling special items for showcases to meet customer needs. Most physical objects fail or wear out over time. We also considered the case where inventory levels were continually depleted due to the collective effects of demand and wear and tear. It was discovered that an optimal replenishment policy exists that maximizes the retailer's overall profit. Generally, retail stores often place mirrors in display windows or display products on faux floors to increase the amount of external display. These factors would be explained by an interesting addition. There are several extensions you can explore that include real-world features we may have overlooked, such as multiple stores using common inventory with limited inventory, multiple time periods, fixed restocking costs, and more. PSO and BAT algorithms were also applied. Applies individually to the proposed model. For example, you need to study dynamic programs where stocks of attractive products must be distributed across stores and time periods.

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