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Anti Skolem Mean Labeling of Alternate Triangular Snake Graphs

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Abstract:

A graph G = (V, E) with p vertices and q edges where p < q + 1 is said to be an Anti Skolem Mean graph if it is possible to label the vertices x ∈ V with distinct labels f(x) from {1, 2, …, q + 1} in such a way that when each edge e = uv is labeled with $f(e = uv) = \frac{f(u)+f(v)}{2}$ if f(u) + f(v) is even and $\frac{f(u)+f(v)+1}{2}$ if f(u) + f(v) is odd then the resulting edges are distinct labels from the set *{2, 3, . . . , p}*. In this case *f* is called an Anti Skolem Mean labeling of G. In this paper, we prove that Alternate Triangular Snake Graphs are Anti Skolem Mean graphs.

Keywords: Anti Skolem Mean Graph, Anti Skolem Mean labeling, Alternate Triangular Snake Graphs.

1. Introduction

All graphs with p vertices and q edges are finite, simple and undirected graph without loops or parallel edges. Detailed survey for all graph labeling we refer to Gallian [1]. For all other standard terminology and notations, we follow Harary [2]. The concept of Skolem Mean Labeling was introduced by A. Subramanian, D. S.T. Ramesh and V. Balaji [4].

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We already investigate the new concept Anti Skolem Mean labeling of Triangular Snake Graphs. In this paper we investigate the Anti Skolem Mean labeling of Alternate Triangular Snake Graphs.

2. Preliminaries

Definition: 2.1

A graph G = (V, E) with p vertices and q edges where $p < q + 1$ is said to be an Anti Skolem Mean graph if it is possible to label the vertices $x \in V$ with distinct labels f(x) from {1, 2, … ,q + 1} in such a way that when each edge e = uv is labeled with $f(e = uv) = \frac{f(u)+f(v)}{2}$ if f(u) + f(v) is even and $\frac{f(u)+f(v)+1}{2}$ if f(u) + f(v) is odd then the resulting edges are distinct labels from the set *{2, 3, . . . , p}*.

Result 2.2

This condition $p < q + 1$ does not satisfies path and star graphs.

Definition 2.3

A Triangular Snake graph T_n is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} to a vertex v_i for $1 \le i \le n - 1$. That is every edge of a path is replaced by a triangle C_3.

Theorem: 2.4

Triangular Snake graph T_n is an Anti Skolem Mean graph.

Definition: 2.5

A Double triangular Snake graph is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} to a new vertex v_i, w_i, $1 \le i \le n - 1$.

Theorem: 2.6

The Double triangular Snake graph $D(T_n)$ is an Anti Skolem Mean graph.

Definition: 2.7

An Alternate Triangular Snake graph $A(T_n)$ is obtained from a path $u_1, u_2, \ldots, u_n$ by joining each of the vertices u_i and u_{i+1} (Alternatively) to a new vertex v_i. That is, every alternate edge of a path is replaced by C_3.

Definition: 2.8

An Alternate Double Triangular Snake $A[D(T_n)]$ is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} (Alternatively) to a new vertex v_i, w_i, consists of two Alternate Triangular snake that have a common path.

3. Main Results

Theorem: 3. 1

Alternate Triangular Snakes graph $A(T_n)$ are Anti Skolem Mean graph.

Proof:

Let P_n be the path with vertices $u_1, u_2, \ldots, u_n$.

Let $A(T_n)$ be the Alternate Triangular Snake graph obtained from the path P_n by joining u_i and u_{i+1}(Alternatively)to new vertex v_i.

Here we consider two different cases.

Case (i):

If an Alternate Triangle Snake graph $A(T_n)$ starts from u_1, then we consider two subcases.

Subcase (i) (a): If n is even

Define a function $f: V(G) \rightarrow \{1,2,\ldots, q+1\}$ by

$f(u_{2i-1}) = 4i - 3, \ 1 \leq i \leq \frac{n}{2}$

$f(u_{2i}) = 4i, \ 1 \leq i \leq \frac{n}{2}$

$f(v_i) = 4i - 1, \ 1 \leq i \leq \frac{n}{2}$.

Then the edges are labeled with

$f(u_{2i-1}u_{2i}) = 4i - 1, \ 1 \leq i \leq \frac{n}{2}$

$f(u_{2i}u_{2i+1}) = 4i + 1, \ 1 \leq i \leq \frac{n-2}{2}$,

$f(u_{2i-1}v_i) = 4i - 2, \ 1 \leq i \leq \frac{n}{2}$

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$f((u_{2i}v_i) = 4i, \quad 1 \le i \le \frac{n}{2}.$

Then the edge labels are distinct.

Hence, the Alternate Triangular Snake graph $A(T_n)$ is an Anti Skolem Mean graph.

Example: 3.2 Anti Skolem Mean labeling of $A(T_6)$ is given below.

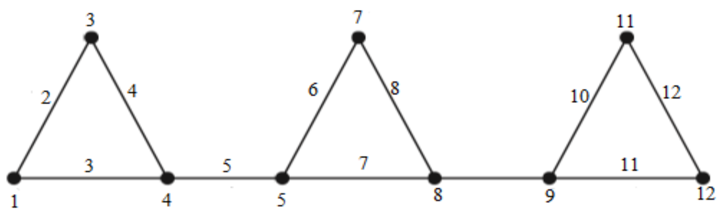

Figure: 3.1

Subcase (i)(b): If n is odd

Define a function f: V(G) → {1,2,…, q + 1} by

$f(u_{2i-1}) = 4i - 3, \ 1 \le i \le \frac{n+1}{2}$

$f(u_{2i}) = 4i, \ 1 \le i \le \frac{n-1}{2}$

$f(v_i) = 4i - 1, \ 1 \le i \le \frac{n-1}{2}.$

Then the edges are labeled with

$f(u_{2i-1}u_{2i}) = 4i - 1, \quad 1 \le i \le \frac{n-1}{2}$

$f(u_{2i}u_{2i+1}) = 4i + 1, \quad 1 \le i \le \frac{n-1}{2},$

$f(u_{2i-1}v_i) = 4i - 2, \quad 1 \le i \le \frac{n-1}{2}$

$f((u_{2i}v_i) = 4i, \quad 1 \le i \le \frac{n-1}{2}.$

Then the edge labels are distinct.

Hence, the Alternate Triangular Snake graph $A(T_n)$ is an Anti Skolem Mean graph.

Example: 3. 3 Anti Skolem Mean labeling of $A(T_6)$ is given below.

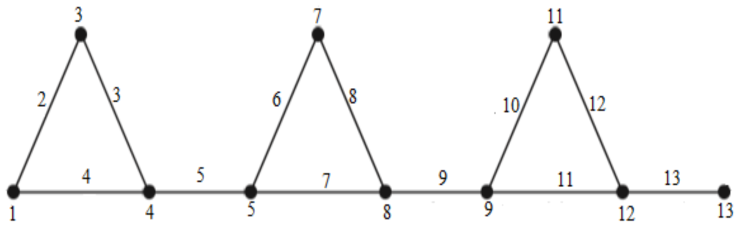

Figure: 3.2

Case (ii):

If an Alternate Triangular Snake graph $A(T_n)$ starts from u_2, then we consider two subcases:

Subcase (ii) (a): If n is even

Define a function f: V(G) → {1,2,…, q + 1} by

$f(u_{2i-1}) = 4i - 3,\ 1 \leq i \leq \frac{n}{2}$

$f(u_{2i}) = 4i - 2,\ 1 \leq i \leq \frac{n}{2}$

$f(v_i) = 4i,\ 1 \leq i \leq \frac{n-2}{2}$.

Then the edges are labeled with

$f(u_{2i-1}u_{2i}) = 4i - 2,\quad 1 \leq i \leq \frac{n}{2}$

$f(u_{2i}u_{2i+1}) = 4i,\quad 1 \leq i \leq \frac{n-2}{2}$,

$f(u_{2i}v_i) = 4i - 1,\quad 1 \leq i \leq \frac{n-2}{2}$

$f((u_{2i+1}v_i) = 4i + 1,\quad 1 \leq i \leq \frac{n-2}{2}$.

Then the edge labels are distinct.

Hence, the Alternate Triangular Snake graph $A(T_n)$ is an Anti Skolem Mean graph.

Example: 3. 4 Anti Skolem Mean labeling of $A(T_6)$ is given below.

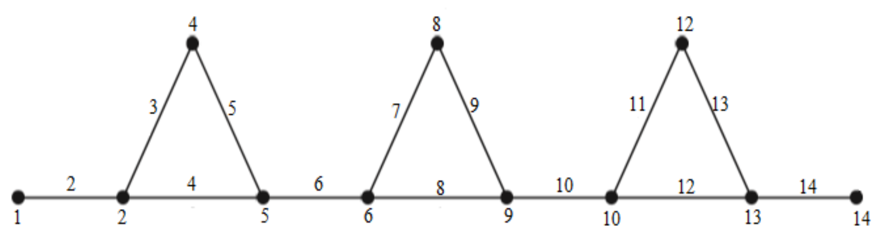

Figure: 3.3

Subcase (ii)(b): If n is odd

Define a function f: V(G) → {1,2,…, q + 1} by

$f(u_{2i-1}) = 4i - 3, \ 1 \leq i \leq \frac{n+1}{2}$

$f(u_{2i}) = 4i - 2, \ 1 \leq i \leq \frac{n-1}{2}$

$f(v_i) = 4i, \ \ 1 \leq i \leq \frac{n-1}{2}$.

Then the edges are labeled with

$f(u_{2i-1}u_{2i}) = 4i - 2, \ \ 1 \leq i \leq \frac{n-1}{2}$

$f(u_{2i}u_{2i+1}) = 4i, \ \ 1 \leq i \leq \frac{n-1}{2}$,

$f(u_{2i}v_i) = 4i - 1, \ \ 1 \leq i \leq \frac{n-1}{2}$

$f((u_{2i+1}v_i) = 4i + 1, \ \ 1 \leq i \leq \frac{n-1}{2}$.

Then the edge labels are distinct.

Hence, the Alternate Triangular Snake graph $A(T_n)$ is an Anti Skolem Mean graph.

Example: 3. 5 Anti Skolem Mean labeling of $A(T_6)$ is given below.

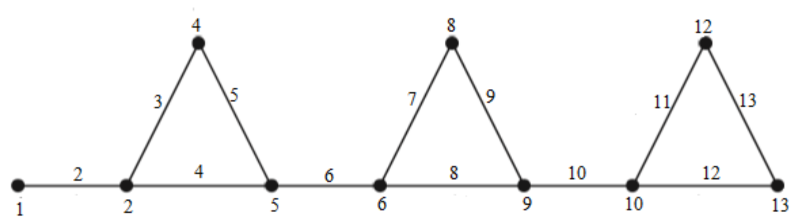

Figure: 3.4

Hence from case (i) and case (ii), we conclude that Alternate Triangular Snakes graph $A(T_n)$ are Anti Skolem Mean graph.

Theorem: 3. 6

Alternate Double Triangular Snakes $A[D(T_n)]$ are Anti Skolem Mean Graph.

Proof:

Let P_n be the path with vertices $u_1, u_2,\ldots,u_n$.

Let $A[D(T_n)]$ be the Alternate Double Triangular Snake graph obtained from the path P_n by joining u_i and u_{i+1}(Alternatively)to two new vertices v_i, w_i.

Here we consider two different cases.

Case (i):

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If an Alternate Double Triangle Snake graph $A[D(T_n)]$ starts from u_1, then we consider two subcases.

Subcase (i) (a): If n is even

Define a function f: V(G) → {1,2,…, q + 1} by

$f(u_{2i-1}) = 6i - 5, \ 1 \le i \le \frac{n}{2}$

$f(u_{2i}) = 6i, \ 1 \le i \le \frac{n}{2}$

$f(v_i) = 6i - 3, \ 1 \le i \le \frac{n}{2}$

$f(w_i) = 6i - 1, \ 1 \le i \le \frac{n}{2}$.

Then the edges are labeled with

$f(u_{2i-1}u_{2i}) = 6i - 2, \ 1 \le i \le \frac{n}{2}$

$f(u_{2i}u_{2i+1}) = 6i + 1, \ 1 \le i \le \frac{n-2}{2}$,

$f(u_{2i-1}v_i) = 6i - 4, \ 1 \le i \le \frac{n}{2}$

$f(u_{2i}v_i) = 6i - 1, \ 1 \le i \le \frac{n}{2}$

$f(u_{2i-1}w_i) = 6i - 3, \ 1 \le i \le \frac{n}{2}$

$f(u_{2i}w_i) = 6i, \ 1 \le i \le \frac{n}{2}$.

Then the edge labels are distinct.

Hence, the Alternate Double Triangular Snake graph $A[D(T_n)]$ is an Anti Skolem Mean graph.

Example: 3. 7 Anti Skolem Mean labeling of $A[D(T_6)]$ is given below.

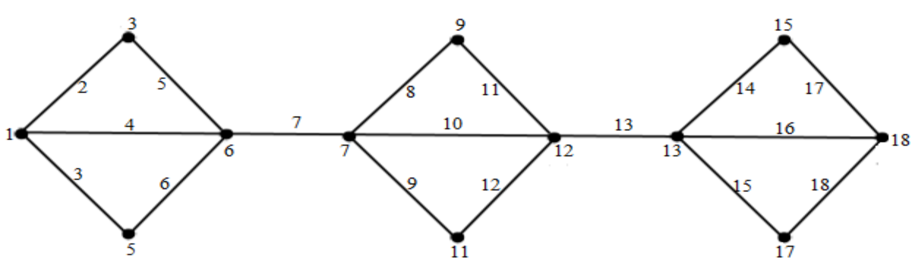

Figure: 3.5

Subcase (i)(b): If n is odd

Define a function f: V(G) → {1,2,…, q + 1} by

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$f(u_{2i-1}) = 6i - 5,\ 1 \le i \le \frac{n+1}{2}$

$f(u_{2i}) = 6i,\ \ 1 \le i \le \frac{n-1}{2}$

$f(v_i) = 6i - 3,\ \ 1 \le i \le \frac{n-1}{2}$

$f(w_i) = 6i - 1,\ \ 1 \le i \le \frac{n-1}{2}$.

Then the edges are labeled with

$f(u_{2i-1}u_{2i}) = 6i - 2,\ \ 1 \le i \le \frac{n-1}{2}$,

$f(u_{2i}u_{2i+1}) = 6i - 1,\ \ 1 \le i \le \frac{n-1}{2}$,

$f(u_{2i-1}v_i) = 6i - 4,\ \ 1 \le i \le \frac{n-1}{2}$,

$f((u_{2i}v_i) = 6i - 1,\ \ 1 \le i \le \frac{n-1}{2}$,

$f(u_{2i-1}w_i) = 6i - 3,\ \ 1 \le i \le \frac{n-1}{2}$,

$f((u_{2i}w_i) = 6i,\ \ 1 \le i \le \frac{n-1}{2}$.

Then the edge labels are distinct.

Hence, the Alternate Double Triangular Snake graph $A[D(T_n)]$ is an Anti Skolem Mean graph.

Example: 3. 8 Anti Skolem Mean labeling of $A[D(T_6)]$ is given below.

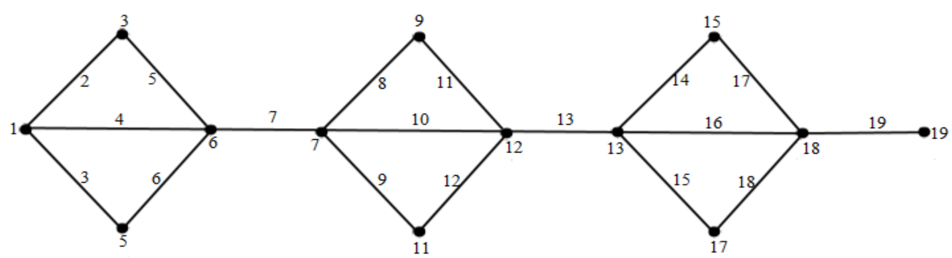

Figure: 3.6

Case (ii):

If an Alternate Double Triangular Snake graph $A[D(T_n)]$ starts from u_2, then we consider two subcases:

Subcase (ii) (a): If n is even

Define a function f: V(G) → {1,2,…, q + 1} by

$f(u_{2i-1}) = 6i - 5,\ \ 1 \le i \le \frac{n}{2}$,

$f(u_{2i}) = 6i - 1, \; 1 \le i \le \frac{n}{2},$

$f(v_i) = 6i - 2, \; 1 \le i \le \frac{n-2}{2},$

$f(w_i) = 6i, \; 1 \le i \le \frac{n-2}{2}.$

Then the edges are labeled with

$f(u_{2i-1}u_{2i}) = 6i - 4, \; 1 \le i \le \frac{n}{2},$

$f(u_{2i}u_{2i+1}) = 6i - 5, \; 1 \le i \le \frac{n-2}{2},$

$f(u_{2i+1}v_i) = 6i - 3, \; 1 \le i \le \frac{n-2}{2},$

$f((u_{2i}v_i) = 6i, \; 1 \le i \le \frac{n-2}{2},$

$f(u_{2i+1}w_i) = 6i - 2, \; 1 \le i \le \frac{n-2}{2},$

$f((u_{2i}w_i) = 6i + 1, \; 1 \le i \le \frac{n-2}{2}.$

Then the edge labels are distinct.

Hence, the Alternate Double Triangular Snake graph $A[D(T_n)]$ is an Anti Skolem Mean graph.

Example: 3. 9 Anti Skolem Mean labeling of $A[D(T_6)]$ is given below.

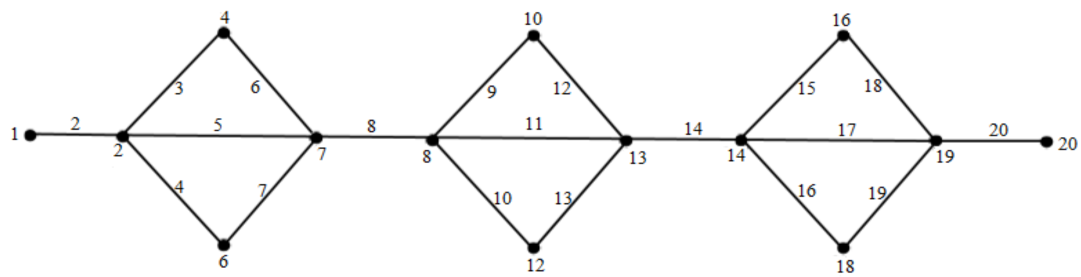

Figure: 3.7

Subcase (ii)(b): If n is odd

Define a function $f: V(G) \rightarrow \{1,2,\ldots, q + 1\}$ by

$f(u_{2i-1}) = 6i - 5, \; 1 \le i \le \frac{n+1}{2}$

$f(u_{2i}) = 6i - 4, \; 1 \le i \le \frac{n-1}{2}$

$f(v_i) = 6i - 2, \; 1 \le i \le \frac{n-1}{2}$

$f(w_i) = 6i, \; 1 \le i \le \frac{n-1}{2}.$

Then the edges are labeled with

$f(u_{2i-1}u_{2i}) = 6i - 4, \ 1 \leq i \leq \frac{n-1}{2}$

$f(u_{2i}u_{2i+1}) = 6i - 5, \ 1 \leq i \leq \frac{n-1}{2}$,

$f(u_{2i+1}v_i) = 6i - 3, \ 1 \leq i \leq \frac{n-1}{2}$

$f((u_{2i}v_i) = 6i, \ 1 \leq i \leq \frac{n-1}{2}$

$f(u_{2i+1}w_i) = 6i - 2, \ 1 \leq i \leq \frac{n-1}{2}$

$f((u_{2i}w_i) = 6i + 1, \ 1 \leq i \leq \frac{n-1}{2}$..

Then the edge labels are distinct.

Hence, the Alternate Double Triangular Snake graph $A[D(T_n)]$ is an Anti Skolem Mean graph.

Example: 3.10 Anti Skolem Mean labeling of $A[D(T_6)]$ is given below.

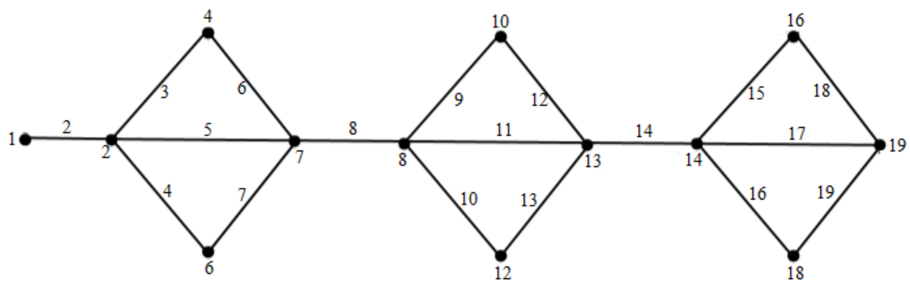

Figure: 3.8

Hence from case (i) and case (ii), we conclude that Alternate Triangular Snakes graph $A(T_n)$ are Anti Skolem Mean graph.

Conclusion

All graphs are not Anti Skolem Mean graphs. It is very interesting to investigate graphs which admit Anti Skolem Mean labeling. In this paper, we proved that Alternate Triangular Snake graphs are Anti Skolem Mean graphs. It is possible to investigate similar results for several other graphs.

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