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SUM OF POWER n DIVISOR CORDIAL LABELING OF CONNECTED GRAPHS

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Abstract:

A Sum of Power n Divisor Cordial labeling of a graph G with a collection of vertex V is a bijection f from V to $\{1, 2, 3, \dots, |V(G)|\}$, where an edge uv is assigned the value 1 if 2 divides $(f(u) + f(v))^n$ and the edge uv is assigned the value 0 if 2 does not divides $(f(u) + f(v))^n$. The number of edges labeled with 0 and the number of edges labeled with 1 differ by no more than 1. A graph with a sum of power n divisor cordial labeling is called a Sum of Power n Divisor Cordial Graph.

A graph G is said to be *connected* if every pair of vertices are joined by a path. Otherwise, it is *disconnected*. In this paper, we investigate sum of power n divisor cordial labeling of corona related connected graphs such as $C_n\hat{o}K_{1,5}$, $\mathrm{C_n\tilde{o}K_{1,m}}$, $(P_n \odot K_1) \odot K_{1,3}$, $P_n \odot K_{1,2}$ respectively.

Keywords: Divisor cordial labeling, Sum divisor cordial labeling, Sum of Power n Divisor Cordial Labeling, Ladder graphs.

1. Introduction

Here simple, finite, connected and undirected graphs are all that are being taken inti consideration. Harary is used for all other accepted terms and notations [1]. We refer to Gallian [2] for a comprehensive analysis of graph labeling. A labeling of a graph is a map that carries the graph elements to the set of numbers, usually to the set of non-negative or positive integers. If the domain is the set of vertices, the labeling is called vertex labeling. If the domain is the set of edges, the labeling is called edge labeling. If the labels are assigned to both vertices and edges then the labeling is called total labeling. A. Lourdswamy and F. Patrik introduced the concept of sum divisor cordial labeling in [3]. Preetha lal and M. Jaslin Melbha [4,5] introduced the concept of sum of power n divisor cordial labeling. Corona product of graphs was introduced by Frucht and Harary in 1970. By using the above result, we introduce Sum of Power n Divisor Cordial Labeling of Connected Graphs.

Definition 1.1. A Sum of Power n Divisor Cordial labeling of a graph G with a collection of vertex V is a bijection f from V to $\{1, 2, 3, \dots, |V(G)|\}$, where an edge uv is assigned the value 1 if 2 divides $(f(u) + f(v))^n$ and the edge uv is assigned the value 0 if 2 does not divides $(f(u) + f(v))^n$. The number of edges labeled with 0 and the number of edges labeled with 1 differ by no more than 1. A graph with a sum of power n divisor cordial labeling is called a *Sum of Power n Divisor Cordial Graph.*

Definition 1.2. Path refers to a walk where each of the vertices $\mathrm{u_0, u_1, \dots, u_n}$ are distinct. A path on n vertices is denoted by $\mathrm{P_n}$.

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Definition 1.3.

A graph obtained from a path P_n by attaching a pendant edge to every internal vertex of the path is called Hurdle graph. It is denoted by Hd_n and has $n-2$ hurdles.

Definition 1.4.

A graph obtained from an open ladder by joining each u_i with v_{i+1} for $1 \leq i \leq n-1$ and each u_{i+1} with v_{i+1} for $1 \leq i \leq n-2$ is called an open triangular ladder and is denoted by $O(TL_n)$.

Definition 1.5. The double ladder graph is the graph obtained by using cartesian product of path graph P_n with n vertices and P_3. It is denoted by $P_n \times P_3$.

Definition 1.6. Circular ladder graph is a simple graph obtained by using cartesian product of cycle graph C_n with n vertices and path graph P_2. It is denoted by CL_n.

2. Main Results

Theoren 2.1. A Hurdle graph Hd_n is a sum of power n divisor cordial graph.

Proof. Let $G = Hd_n$ be hurdle graph with $\alpha_1, \alpha_2, \dots, \alpha_n$ as the vertices of path and let $\beta_1, \beta_2, \dots, \beta_{n-2}$ be the vertices of the pendant edges attached to the internal vertices of the path. Let $V(G) = \{\alpha_i : 1 \leq i \leq n\} \cup \{\beta_i : 1 \leq i \leq n-2\}$ and $E(G) = \{\alpha_i\alpha_{i+1}; 1 \leq i \leq n-1\} \cup \{\beta_i\alpha_{i+1} ; 1 \leq i \leq n-2\}$. Then G has $2n-2$ vertices and $2n-3$ edges. Define a function $f: V(G) \to \{1, 2, \dots, |V(G)|\}$ by $f(\alpha_i) = i, 1 \leq i \leq n$, $f(\beta_i) = f(\alpha_n) + i, 1 \leq i \leq n-2$. Then the induced edge labels are $f^*(\alpha_i\alpha_{i+1}) = 0, 1 \leq i \leq n-1$, $f^*(\beta_i\alpha_{i+1}) = 1, 1 \leq i \leq n-2$. Hence $|e_f(0) - e_f(1)| \leq 1$. Thus, hurdle graph Hd_n is a sum of power n divisor cordial graph.

Example 2.2. The sum of power n divisor cordial labeling of Hd_7 as shown in the figure.

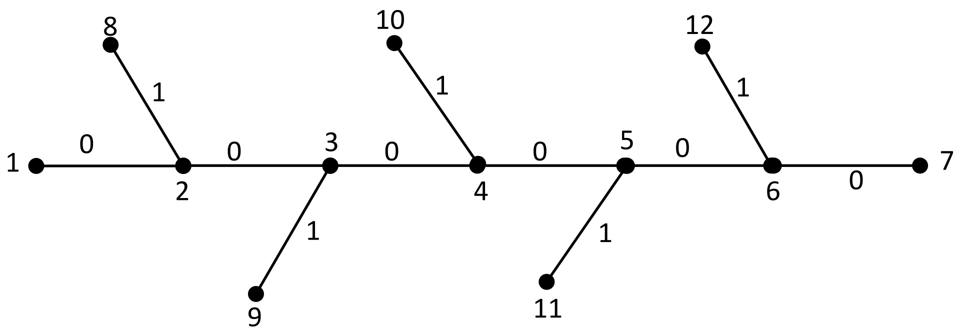

Figure 1. Hd_7

Theorem 2.3. A graph obtained by attaching P_4 at each vertex of P_n is a sum of power n divisor cordial graph.

Proof: Let P_n be a path $\alpha_1, \alpha_2, \dots, \alpha_n$. Let $\alpha_i, \beta_i, \gamma_i, \delta_i$ be the i[th] copy of P_4 ; $1 \leq i \leq n$. The resultant graph G is the required graph is with $V(G) = \{\alpha_i, \beta_i, \gamma_i, \delta_i : 1 \leq i \leq n\}$ and $E(G) = \{\alpha_i\alpha_{i+1}: 1 \leq i \leq n-1\} \cup \{\alpha_i\beta_i, \beta_i\gamma_i, \gamma_i\delta_i : 1 \leq i \leq n\}$. Then G has $4n$ vertices and $4n-1$ edges. Define a function $f: V(G) \to \{1, 2, \dots, |V(G)|\}$ by $f(\alpha_i) = 3i-2, 1 \leq i \leq n$, $f(\beta_i) = 3i, 1 \leq i \leq n$, $f(\gamma_i) = 3i-1, 1 \leq i \leq n$, $f(\delta_i) = f(\beta_n) + i, 1 \leq i \leq n$. Then the induced edge labels are $f^*(\alpha_i\alpha_{i+1}) = 0, 1 \leq i \leq n-1$, $f^*(\alpha_i\beta_i) = 1, 1 \leq i \leq n$, $f^*(\beta_i\gamma_i) = 0, 1 \leq i \leq n$, $f^*(\gamma_i\delta_i) = 1, 1 \leq i \leq n$. Hence $|e_f(0) - e_f(1)| \leq 1$. Thus, the graph G is a sum of power n divisor cordial graph.

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Example 2.4. The sum of power n divisor cordial labeling of G when $n = 5$ as shown in the figure.

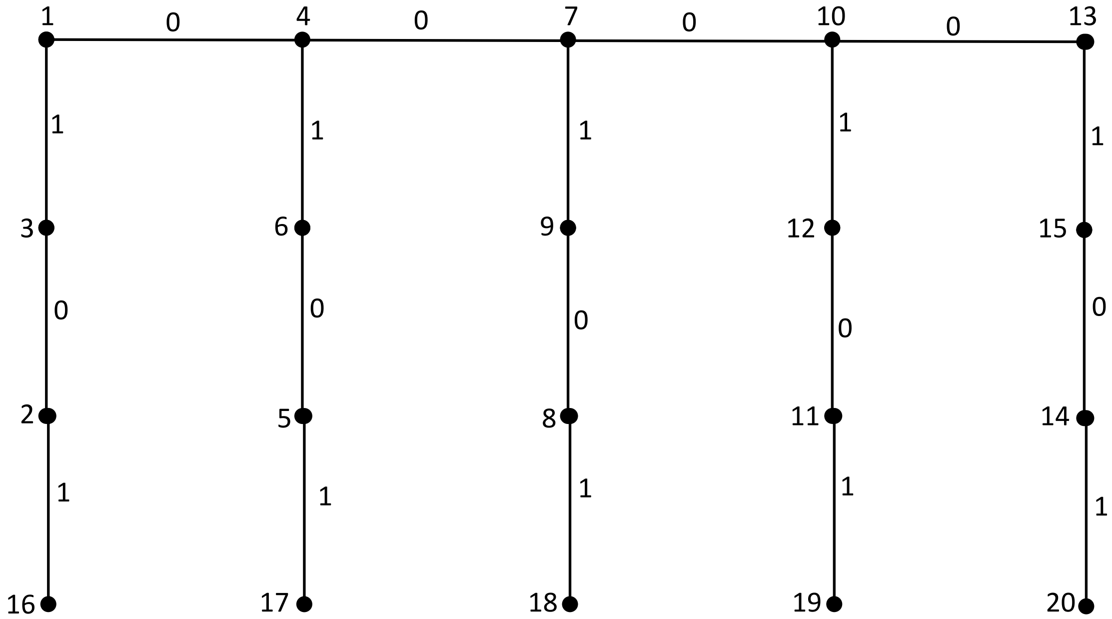

Figure 2.

Theorem 2.5. The Open Triangular Ladder $O(TL_n), n \geq 2$ is a sum of power n divisor cordial graph.

Proof: Let $G = O(TL_n)$. Let the vertices of G be $V(G) = \{\alpha_i, \beta_i : 1 \leq i \leq n\}$ and the edges are $E(G) = \{\alpha_i\alpha_{i+1}, \beta_i\beta_{i+1}, \alpha_i\beta_{i+1} : 1 \leq i \leq n-1\} \cup \{\alpha_i\beta_i : 2 \leq i \leq n-2\}$. Define a function $f: V(G) \to \{1, 2, \dots, |V(G)|\}$ by $f(\alpha_i) = 2i-1, 1 \leq i \leq n$, $f(\beta_i) = 2i, 1 \leq i \leq n$. Then the induced edge labels are $f^*(\alpha_i\alpha_{i+1}) = 1, 1 \leq i \leq n-1$, $f^*(\beta_i\beta_{i+1}) = 1, 1 \leq i \leq n-1$, $f^*(\alpha_i\beta_i) = 0, 1 \leq i \leq n-1$, $f^*(\alpha_i\beta_{i+1}) = 0, 2 \leq i \leq n-1$. Hence $|e_f(0) - e_f(1)| \leq 1$. Thus, the graph G is a sum of power n divisor cordial graph.

Example 2.6. The sum of power n divisor cordial labeling of triangular ladder $O(TL_4)$ as shown in the figure.

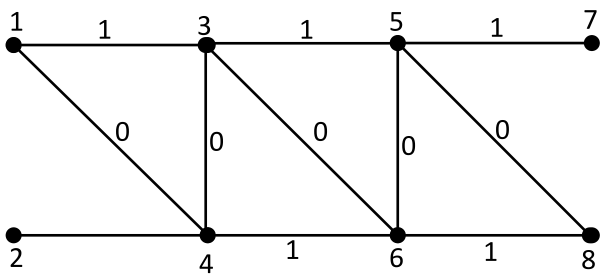

Figure 3.

Theorem 2.7. The Double Ladder $P_n \times P_3$ is a sum of power n divisor cordial graph.

Proof: Let $V(P_n \times P_3) = \{\alpha_i, \beta_i, \gamma_i : 1 \leq i \leq n\}$ and $E(P_n \times P_3) = \{\alpha_i\alpha_{i+1}, \beta_i\beta_{i+1}, \gamma_i\gamma_{i+1} : 1 \leq i \leq n-1\} \cup \{\alpha_i\beta_i, \beta_i\gamma_i : 1 \leq i \leq n\}$. Then the graph $P_n \times P_3$ has $3n$ vertices and $5n-3$ edges. Define a function $f: V(P_n \times P_3) \to \{1, 2, \dots, |V(P_n \times P_3)|\}$ by

$f(\alpha_i) = 2i - 1, 1 \le i \le n$, $f(\beta_i) = 2i, 1 \le i \le n$, $f(\gamma_i) = f(\beta_i) + i, 1 \le i \le n$. Then the induced edge labels are $f^*(\alpha_i\alpha_{i+1}) = 1, 1 \le i \le n - 1$, $f^*(\beta_i\beta_{i+1}) = 1, 1 \le i \le n - 1$, $f^*(\alpha_i\beta_i) = 0, 1 \le i \le n$, $f^*(\gamma_i\gamma_{i+1}) = 0, 1 \le i \le n - 1$, $f^*(\beta_{2i-1}\gamma_{2i-1}) = 0, 1 \le i \le \frac{n}{2}$, $f^*(\beta_{2i}\gamma_{2i}) = 1, 1 \le i \le \frac{n}{2}$. Hence $|e_f(0) - e_f(1)| \le 1$. Thus, the graph G is a sum of power n divisor cordial graph.

Example 2.8. The sum of power n divisor cordial labeling of double ladder graph $P_4 \times P_3$ as shown in the figure.

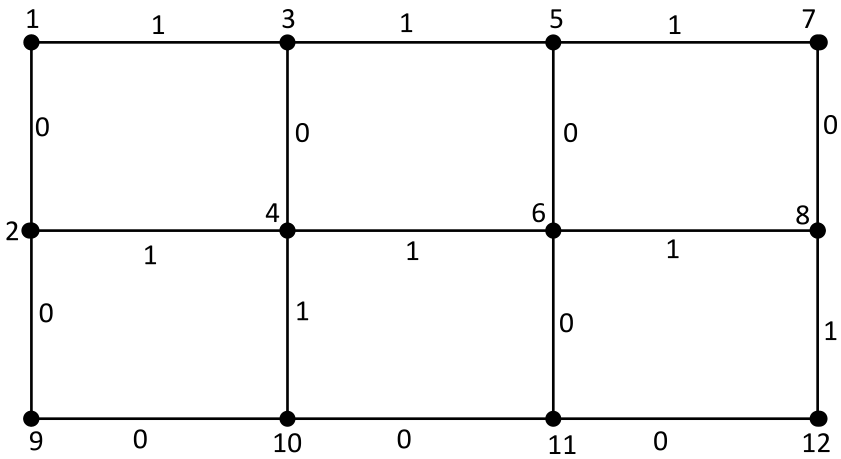

Figure 4.

Theorem 2.9. The Circular ladder graph CL_n is not a sum of power n divisor cordial graph.

Proof: Let $G = CL_n$ be a circular ladder graph. Let $\alpha_1, \alpha_2, \dots, \alpha_n$ be the vertices of inner cycle and $\beta_1, \beta_2, \dots, \beta_n$ be the vertices of outer cycle of circular ladder graph. Then $|V(G)| = 2n$ and $|E(G)| = 3n$. Define a function $f: V(G) \to \{1, 2, \dots, |V(G)|\}$ by $f(\alpha_i) = 2i - 1, 1 \le i \le n$, $f(\beta_i) = 2i, 1 \le i \le n$. Then the induced edge labels are $f^*(\alpha_i\alpha_{i+1}) = 1, 1 \le i \le n - 1$, $f^*(\alpha_n\alpha_1) = 1$, $f^*(\beta_i\beta_{i+1}) = 1, 1 \le i \le n - 1$, $f^*(\beta_n\beta_1) = 1$,$f^*(\alpha_i\beta_i) = 0, 1 \le i \le n$.

The sum of power n divisor cordial labeling of double ladder graph CL_5 as shown in the figure.

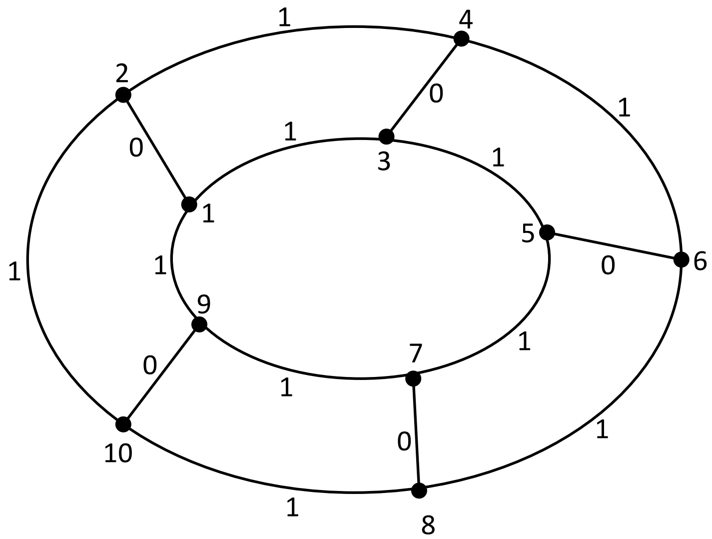

Figure 5.

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Here $|e_f(0) - e_f(1)| \geq 1$. Thus, Circular ladder graph is not a sum of power n divisor cordial graph.

Conclusion:

It is very interesting and challenging as well as to investigate graph families which admit sum of power n divisor cordial labeling. Here we have proved as Hurdle graph Hd_n, graph obtained by attaching P_4 at each vertex of P_n, Double Ladder $P_n \times P_3$, Open Triangular Ladder TL_n, Circular Ladder CL_n,

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