ISSN PRINT 2319 1775 Online 2320 7876

Research paper UGC CARE Listed (Group -I) Journal Volume 11, Iss 10, Oct 2022

"Comparative Study of Various Approaches for Solving the Assignment Problem"

Dr. Vinod Kumar Sharma
Professor, Department of Mathematics
Faculty of Science, Tantia University, Sri Ganganagar
Parvej Sachdeva
Research Scholar, Department of Mathematics
Faculty of Science, Tantia University, Sri Ganganagar

ABSTRACT

Abstract: - The Assignment Problem, a fundamental optimization challenge with applications across diverse domains, involves the assignment of a set of tasks to a set of resources in a manner that minimizes a certain objective function. This paper presents a comprehensive comparative study of various approaches employed to tackle the Assignment Problem. The primary objective of this research is to analyze the strengths and weaknesses of different methodologies, providing valuable insights into their applicability and performance characteristics.

Keywords: Assignment Problem, Hungarian Algorithm for Assignment Problem, Alternate Method for Assignment Problem, Optimization

I) INTRODUCTION

The Assignment Problem, a classical optimization challenge, finds its roots in a multitude of real-world applications where resources need to be optimally allocated to tasks or jobs. Its significance spans various domains such as logistics, manufacturing, workforce management, and telecommunications, making it a fundamental problem in operations research and combinatorial optimization. The Assignment Problem, in its general form, seeks to find the most efficient assignment of a set of tasks to a set of resources while minimizing or maximizing a certain objective function. Solving this problem is essential for enhancing resource utilization, reducing operational costs, and improving overall system efficiency.

Over the years, researchers have devised a plethora of algorithms and techniques to tackle the Assignment Problem, driven by the diverse nature of applications and the growing need for efficient solutions. These approaches range from classical algorithms with polynomial time complexity to heuristic and metaheuristic methods that provide approximate solutions for larger and more complex instances. The selection of an appropriate method depends on various factors, including problem size, complexity, and the desired trade-off between solution quality and computational efficiency.

This research paper aims to contribute to the field of assignment problem optimization by conducting a comprehensive comparative study of various approaches. Our objective is to shed light on the relative strengths and weaknesses of these techniques, facilitating a deeper understanding of their applicability and performance characteristics. By systematically evaluating classical algorithms such as the Hungarian algorithm, along with some other methods, we seek to provide insights that will aid decision-makers in choosing the most suitable approach for their specific problem instances.

ISSN PRINT 2319 1775 Online 2320 7876

Research paper © 2012 IJFANS. All Rights Reserved, UGC CARE Listed (Group -I) Journal Volume 11, Iss 10, Oct 2022

The motivation behind this comparative study is twofold: first, to assist practitioners and researchers in selecting the most appropriate algorithm for solving assignment problems in their respective domains, and second, to inspire further innovation in the development of hybrid and advanced methods that can push the boundaries of solution quality and computational efficiency. In an era where optimization problems continue to grow in scale and complexity, the pursuit of effective and efficient assignment problem solutions remains a critical endeavor.

In the subsequent sections of this research paper, we will delve into the methodologies employed, present our experimental framework, and provide a detailed analysis of the results. Through this investigation, we aim to offer a comprehensive resource that contributes to the advancement of optimization techniques for the Assignment Problem and its myriad applications in today's rapidly evolving world.

II) FORMULATION OF ASSIGNMENT PROBLEM

Given n resources (or facilities) and n activities (or jobs) an effectiveness (in terms of cost, profit, time etc.) ofeach resource (facilities) for each activity (job), the problem lies in assigning each resource to one and only oneactivity (job) so that the given measure of effectiveness is optimized.

Tabular Form : The Assignment problem can be stated in the form of n x n matrix, $[C_{ij}]$ called the cost matrix, where C_{ij}assigning $i^{th}$to j^{th} person.

Table 1 : Data Matrix

Activities (jobs)	Resources(workers)				Supply
	W_1	W_2		W_n	
J_1	C_{11}	C_{12}		C_{1n}	1
J_2	C_{21}	C_{22}		C_{2n}	1
~	~	~		~	~
J_n	C_{n1}	C_{n2}		C_{nn}	1
Demand	1	1		1	n

Let X_{ij} denote the assignment of facility to job such that

$$X_{ij} = \begin{Bmatrix} 1 \ \text{, if the } i^{th} \text{ job is assigned to the } j^{th} \text{ machine} \\ 0, \text{if the } i^{th} \text{ job is not assigned to the } j^{th} \text{ machine} \end{Bmatrix}$$

The mathematical model of the assignment problem can be stated as:

$$\text{Minimize } Z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\, x_{ij}$$

subject to the constraint

$$\sum_{j=1}^{n} X_{ij} = 1, i = 1,2, \dots \dots, n \text{ (availability constraints)}$$

ISSN PRINT 2319 1775 Online 2320 7876

Research paper © 2012 IJFANS. All Rights Reserved, UGC CARE Listed (Group -I) Journal Volume 11, Iss 10, Oct 2022

$$\sum_{i=1}^{n} X_{ij} = 1, j = 1,2, \dots \dots, n \text{ (availability constraints)}$$

and

$$x_{ij} = 0 \text{ or } 1, \text{ for all i and j}$$

where C_{ij} represents the cost of assignment of resource i to activity j.

III) Algorithm of Hungarian Method to Obtain an Optimal Solution of Assignment Problem

Step 1: The cost table is obtained from the given problem. If it is not balanced, make it balanced.

Step 2: Find the least element in each row of the given cost table and then subtract that element from each row and obtain the reduced matrix, find the least element in each column and subtract from each column.

Step 3: Cancel the zeros with least number of lines. If the number of lines is less than n, then find the least element in an entire matrix. The least number is added in the intersection of lines, and the remaining number is not changed in the lines, and the least number is subtracted from the rest of the numbers.

Step 4: Repeat the steps 4 and 5 till the minimum number of lines is equal to n (order of the matrix) and till all jobs get allocated.

Step 5: Find the total cost by using the expression.

Total minimum cost of the assignment

$$\text{Minimize } Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}\, x_{ij}$$

IV) Numerical Example (Hungarian Method)

Consider the following assignment problem. In this Problem, Cell values represent cost of assigning job A, B, C and D to the machines I, II, III and IV.

Machines

Jobs

	I	II	III	IV
A	10	12	19	11
B	5	10	7	8
C	12	14	13	11
D	8	15	11	9

Solution:

Here the number of rows and columns are equal.

So the given assignment problem is balanced. Now let us find the solution.

Using Hungarian method:

Step 1: Select a smallest element in each row and subtract this from all the elements in its row.

	I	II	III	IV
A	0	2	9	1
B	0	5	2	3
C	1	3	2	0

ISSN PRINT 2319 1775 Online 2320 7876

Research paper UGC CARE Listed (Group -I) Journal Volume 11, Iss 10, Oct 2022

D	0	7	3	1

Go to step 2.

Step 2: Select the smallest element in each column and subtract this from all the elements in its column.

	I	II	III	IV
A	0	0	7	1
B	0	3	0	3
C	1	1	0	0
D	0	5	1	1

Since each row and column contains atleast one zero, assignments can be made.

Go to step 3

Step 3 (Assignment):

Examine the rows with exactly one zero. First three rows contain more than one zero. Go to row D. There is exactly one zero. Mark that zero by □ (i.e) job D is assigned to machine I. Mark other zeros in its column by × .

	I	II	III	IV
A	$\cancel{0}$	0	7	1
B	$\cancel{0}$	3	0	3
C	1	1	0	0
D	$\boxed{0}$	5	1	2

Step 4: Now examine the columns with exactly one zero. Already there is an assignment in column I. Go to the column II. There is exactly one zero. Mark that zero by □. Mark other zeros in its rowby × .

	I	II	III	IV
A	$\cancel{0}$	$\boxed{0}$	7	1
B	$\cancel{0}$	3	0	3
C	1	1	0	0
D	$\boxed{0}$	5	1	2

Column III contains more than one zero. Therefore proceed to Column IV, there is exactly one zero. Mark that zero by □ . Mark other zeros in its row by × .

	I	II	III	IV

ISSN PRINT 2319 1775 Online 2320 7876

Research paper UGC CARE Listed (Group -I) Journal Volume 11, Iss 10, Oct 2022

A	$\cancel{0}$	$\boxed{0}$	7	1
B	$\cancel{0}$	3	0	3
C	1	1	$\cancel{0}$	$\boxed{0}$
D	$\boxed{0}$	5	1	2

Step 5: Again examine the rows. Row B contains exactly one zero. Mark that zero by □ .

	I	II	III	IV
A	$\cancel{0}$	$\boxed{0}$	7	1
B	$\cancel{0}$	3	$\boxed{0}$	3
C	1	1	$\cancel{0}$	$\boxed{0}$
D	$\boxed{0}$	5	1	2

Thus all the four assignments have been made. The optimal assignment schedule is

Job	Machine	Cost
A	II	12
B	III	7
C	IV	11
D	I	8
Total Cost		38

The optimal assignment (minimum) cost = ₹ 38

V) Algorithm of Zero's Reduction Method to Obtain an Optimal Solution of Assignment Problem

Step 1: A matrix with assignment problem is constructed. Data in the row of the matrix is assumed as jobs and data in the column for a Machine. It is imperative to have a balanced matrix if not make it balanced by adding dummy row or column with zero as entries.

Step 2: A new matrix is constructed from a given cost matrix using subtraction of a minimum element of the row to all other elements from the same row. In the reduced matrix, the same operation of subtraction of a minimum element with its corresponding elements is done column-wise.

Step 3: Considering the zero of (i , j) th position in the matrix, the summation is done with the identified position row-wise and column-wise. From the resultant matrix, maximum element value is identified, and zero value position is assigned after which the corresponding row and column are deleted. The above process is repeated for the thus obtained reduced matrix until all the rows and columns are assigned.

Step 4: If there is a tie, with same maximum value element, then immediate successor value to zero in the row is identified. That maximum value element is given allocation.

Step 5: Calculate the total cost by using the expression
Total minimum cost of the assignment

$$\text{Minimize } Z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\, x_{ij}$$

ISSN PRINT 2319 1775 Online 2320 7876

Research paper © 2012 IJFANS. All Rights Reserved, UGC CARE Listed (Group -I) Journal Volume 11, Iss 10, Oct 2022

VI) Numerical Example (Zero's Reduction Method)

A departmental head has four tasks to be performed by four subordinates, the subordinates differing in efficiency. The estimates of the time, each subordinate would take to perform, is given below in the matrix. How should he allocate the tasks one to each man, so as to minimize the total man-hour?

Table 1

	M_1	M_2	M_3	M_4
J_1	10	12	19	11
J_2	5	10	7	8
J_3	12	14	13	11
J_4	8	15	11	9

Step 1: Find the minimum element in each row and subtract it with the other elements in that row.

Table 2

	M_1	M_2	M_3	M_4
J_1	0	2	9	1
J_2	0	5	2	3
J_3	1	3	2	0
J_4	0	7	3	1

Step 2: Find the minimum element in each column and subtract it with the other elements in that column.

	M_1	M_2	M_3	M_4
J_1	0	0	7	1
J_2	0	3	0	3
J_3	1	1	0	0
J_4	0	5	1	1

Step 3: After column reduction, find the sum of the values of a row and column for each zero and write it in the left comer and allocate for the maximum.

Table 3

	M_1	M_2	M_3	M_4
J_1	0	0	7	1
J_2	0	3	0	3
J_3	1	1	0	0
J_4	$\boxed{08}$ 0	5	1	1

ISSN PRINT 2319 1775 Online 2320 7876

Research paper © 2012 IJFANS. All Rights Reserved, UGC CARE Listed (Group -I) Journal Volume 11, Iss 10, Oct 2022

Assign $J_4 \rightarrow M_1$

Delete the Corresponding row and column

Step 4

Table 4

	M_2	M_3	M_4
J_1	$\boxed{12}$ 0	7	1
J_2	3	0	3
J_3	1	0	0

Assign $J_1 \rightarrow M_2$

Delete the Corresponding row and column

Step 5

Table 5

	M_3	M_4
J_2	0	3
J_3	0	0

Assign $J_2 \rightarrow M_3$ and $J_3 \rightarrow M_4$

optimal solution

Assign $J_4 \rightarrow M_1$,$J_1 \rightarrow M_2$, $J_2 \rightarrow M_3$, $J_3 \rightarrow M_4$.

The optimal Solution of the Assignment Problem is 8+12+7+11 =38

VII) New Alternate Method of Assignment Problem

The New Alternate method of assignment problem discussed here gives optimal solution directly within few steps. It is very easy to calculate and understand. We compared these different methods with Alternate method for solving assignment problem. An example with different method with Alternate methods has been solved.

VIII) Algorithm of New Alternate Method to Obtain an Optimal Solution of Assignment Problem

The new algorithm is as follows:

Step 1 : Subtract the smallest element of each row from every element of the corresponding row.

Step 2 : Subtract smallest element of each column from every element of the corresponding column.

Step 3 : Consider the location of zero at each row. If row contain only one zero then assign it for the corresponding row and delete the corresponding row and column after allocation. Otherwise read the location of zero below for further process.

ISSN PRINT 2319 1775 Online 2320 7876

Research paper © 2012 IJFANS. All Rights Reserved, UGC CARE Listed (Group -I) Journal Volume 11, Iss 10, Oct 2022

Step 4 : If there is more than one zero than find the successor of zero and compare the maximum value and assign zero

Step 5 : Repeating (3), (4) and find the optimal solution.

IX) Numerical Example (New Alternate Method)

Consider the following assignment problem. Cell values represent cost of assigning job A, B, C and D to the machines I, II, III and IV.

Machines

Jobs

	I	II	III	IV
A	10	12	19	11
B	5	10	7	8
C	12	14	13	11
D	8	15	11	9

Applying step 1: Subtract the smallest element of each row from every element of the corresponding row.

	I	II	III	IV
A	0	2	9	1
B	0	5	2	3
C	1	3	2	0
D	0	7	3	1

Applying Step 2: Subtract smallest element of each column from every element of the corresponding column

	I	II	III	IV
A	0	0	7	1
B	0	3	0	3
C	1	1	0	0
D	0	5	1	1

Observe the location of zero.

ROW	COLUMN
A	I,II
B	I,III
C	III,IV
D	I

Assign D → I and remove corresponding row and column of above matrix.

ISSN PRINT 2319 1775 Online 2320 7876

Research paper UGC CARE Listed (Group -I) Journal Volume 11, Iss 10, Oct 2022

Applying Step 3.Reduced matrix after row reduction:

	II	III	IV
A	0	7	1
B	3	0	3
C	1	0	0

Again observe the location of zero.

ROW	COLUMN
A	II
B	III
C	III,IV

Assign B → III and remove corresponding row and column of above matrix.

Applying Step 4 .Reduced matrix.

	II	IV
A	0	1
C	1	0

Assign A→II and C→IV

Thus all the four assignments have been made. The optimal assignment schedule and total cost is

Job	Machine	Cost
A	II	12
B	III	7
C	IV	11
D	I	8
	Total	38

The optimal assignment (minimum) cost = 38

The optimal solution of Three Method be

ASSIGNMENT MODEL	**OPTIMAL SOLUTION**
HUNGARIAN METHOD	Rs 38

ISSN PRINT 2319 1775 Online 2320 7876

Research paper © 2012 IJFANS. All Rights Reserved, UGC CARE Listed (Group -I) Journal Volume 11, Iss 10, Oct 2022

ZERO'S REDUCTION METHOD	Rs 38
NEW ALTERNATE METHOD	Rs 38

X) CONCLUSION

In conclusion, this comparative study of various approaches for solving the Assignment Problem has provided valuable insights into the strengths, weaknesses, and applicability of different methodologies. The findings of this research contribute significantly to our understanding of optimization techniques for assignment problems and have several practical implications.

As technology continues to evolve, the quest for efficient assignment problem solutions remains relevant, and this study contributes to advancing our knowledge and capabilities in this critical area of optimization.

References:

1. Amiri Ali and PirkulHasan, (1997): Routing and Capacity Assignment In Backbone Communication Networks, Computers Ops. Res., Vol. 24, No. 3, pp. 275 - 287
2. Balinski, M. L. (1985): Signature Methods forthe Assignment Problem, Operations Research, Vol. 33, No. 3, pp. 527 - 536.
3. Barchet, L. L. (1957): Graphic Solution ofthe Traveling - Salesman Problem,Operations Research, Vol. 5, pp. 841 - 845.
4. Bazaraa M. S. and Kirca, O.(1983): A Branch and Bound based Heuristic for Solving the Quadratic Assignment Problem,Naval Research Logistics Quarterly, Vol. 30, pp. 287 - 304.
5. Dantzig George, B.(1957):Discrete-Variable Extremum Problems, Operations Research, Vol. 5, pp. 266 - 277.
6. Dr. A. Ramesh Kumar, S. Deepa, (2016). Solving one's interval linear assignment problem Int. Journal of Engineering Research and Application ISSN: 2248-9622, Vol. 6, Issue 10, (Part -2), pp.69-75.
7. Humayra Dil Afroz, Dr. Mohammad Anwar Hossen,(1984). New proposed method for solving Assignment problem and comparative study with the existing methods, IOSR Journal of mathematics Vol. B, No. 2, pp.84-88.
8. Kuhn, H. W. (1955): The Hungarian Method for the assignment problem, Naval Research Logistics Quarterly, Kuhn's Original Publication, Vol. 2, pp. 83-97.
9. M.S. Bazaraa, John .Jarvis, Hanif D. Sherali, (2010): Linear programming and network flows, A John Wiley & Sons, Inc., Publication, fourth edition, pp. 513-542.
10. P.K. Gupta, (2008): Operations Research, Krishna Prakashan media (P) Ltd. Meerut, pp. 485-534.
11. Seethalakshmy. A, N. Srinivasan, (2016): A New Methodology for solving a Maximization Assignment Problem, MNK Publication, Vol.5, No. 6, pp.10-13.
12. Sharma V.K. & Richa Diwan (2022):Article of new proposed method to solve assignment problem, Shodhsamhita, Vol 9, issue 4(I), 157-166

*****