

## Finite Element Analysis of Unsteady Viscoelastic Nano-Fluid Flow Induced by Radiation, featuring Multiple Slip Effects over a Permeable Stretching Sheet in a Magnetohydrodynamic (MHD) Configuration.

DR.KOTTAPALLI RAJA SEKHARA PRASAD, Dept of chemistry, Koneru Lakshmaiah Education Foundation, India-522302

### Abstract

This study delves into the repercussions of multiple slips within the Jeffrey fluid model, examining its impact on an unsteady magnetohydrodynamic viscoelastic buoyant nanofluid. This investigation is conducted in the context of Soret and radiation effects, encompassing a permeable stretching sheet. By employing suitable transformations, pertinent nonlinear differential equations are derived and subsequently addressed numerically using the finite element method. The analysis encompasses a thorough exploration of the influence of various controlling parameters on dimensionless quantities, including velocity, temperature, concentration, and nanofluid volume fraction profiles. Furthermore, it scrutinizes the effects of these parameters on dimensionless numbers such as local Nusselt, Sherwood, nano-particle Sherwood, and the local friction coefficient. An essential finding of this research is the examination of the impact of multiple slips, revealing that their presence amplifies the boundary layer flow.

### Introduction

Recent advancements in technology have sparked a keen interest among engineers and scientists in the study of non-Newtonian materials. The widespread utilization of liquids across various technological and industrial domains serves as a primary catalyst for scientific curiosity [1]. A myriad of applications, such as paints, colloids, suspensions, cosmetics, polymer solutions, foods, specialized lubricants, paper production, coal-water mixtures, ketchup, adhesives, ink, blood, select oils, fiber technology, and clay coating, underscores the significance of non-Newtonian fluids [2]. Despite the diverse array of non-Newtonian liquids, a unifying law to elucidate their viscous and elastic properties remains elusive [3]. Undeterred by these complexities, researchers have made significant contributions to the existing literature, delving into a wide spectrum of non-Newtonian fluids [4]. Among these, viscoelastic fluids assume a paramount role within contemporary research endeavors, owing to their extensive applications in engineering and industrial processes. Several recent investigations have advanced our understanding in this realm [5]. Kumar et al. explored the influence of non-linear thermal radiation on the flow of mixed viscoelastic nano liquids under double diffusion convection boundary conditions on a stretchable film. Shit et al. delved into convective heat transfer and magnetohydrodynamic (MHD) viscoelastic nanofluid flow driven by a stretchable film [6]. Sheikholeslami et al. conducted a comprehensive study on the interplay of magnetic fields and thermal radiation on the hydrothermal behavior of Fe<sub>3</sub>O<sub>4</sub>-H<sub>2</sub>O nanofluids [7]. The effects of Lorentz forces on the flow of CuO-water nanofluids within a permeable enclosure were examined using computational methods, shedding light on nanofluid fluxes and heat transfer within a cavity, building upon prior work [8].

Guided by a suitable similarity transformation method, the governing partial differential equations are converted into ordinary differential equations (ODEs). The ensuing ODEs are addressed through a

hybrid approach, combining the finite element method [9] for numerical solutions. The outcomes are thoroughly examined, elucidated, and presented through a combination of tabular summaries and graphical illustrations[10].

### Mathematical formulation

Let's contemplate the dynamic characteristics of a two-dimensional magnetohydrodynamic (MHD) boundary layer [11]. This layer is formed by an electrically conductive liquid, which is immersed in a viscoelastic floating nanofluid. The system also accounts for the presence of multiple slips and thermal radiation effects, all taking place over a linearly stretching sheet [12].

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0, \quad (1)$$

$$\begin{aligned} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= \frac{v}{1 + \lambda^*} \left[ \frac{\partial^2 u}{\partial y^2} + \lambda_t \left( u \frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} \right. \right. \\ &\quad \left. \left. - \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} + v \frac{\partial^3 u}{\partial y^3} \right) \right] \\ &\quad - \frac{\sigma B^2(x)u}{\rho} + g\beta_T(T - T_\infty) + g\beta_C(C - C_\infty) \\ &\quad + g\beta_\phi(\phi - \phi_\infty) \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \alpha \left( 1 + \frac{16T_\infty^3 \sigma^*}{3k^* \kappa} \right) \frac{\partial^2 T}{\partial y^2} \\ &\quad + \tau \left[ D_B \frac{\partial \psi}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \right] - \frac{\sigma B^2(x)u}{\rho} \end{aligned} \quad (3)$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_s \frac{\partial^2 C}{\partial y^2} + D_{CT} \frac{\partial^2 T}{\partial y^2} \quad (4)$$

$$\frac{\partial \phi}{\partial t} + u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} = D_B \frac{\partial^2 \phi}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} \quad (5)$$

with respect to the boundary conditions

$$\begin{aligned} u &= U(x, t) + U_{slip}, v = v_w, T = T_w(x, t) + T_{slip}, \\ C &= C_w(x, t) + C_{slip}, \end{aligned} \quad (6)$$

$$\phi = \phi_w(x, t) + \phi_{slip} \quad \text{at } y \rightarrow 0$$

$$u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty, \phi \rightarrow \phi_\infty, \quad \text{as } y \rightarrow \infty, \quad (7)$$

$$\eta = \sqrt{\frac{a}{\nu(1-\lambda t)}}y, \quad \psi = \sqrt{\frac{av}{(1-\lambda t)}}xf(\eta),$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad S(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad \gamma(\eta) = \frac{\phi - \phi_\infty}{\phi_w - \phi_\infty} \quad (8)$$

$$f''' + \beta(2f'f''' - f''^2 - ff^{iv})$$

$$+ (1 + \lambda)(ff'' - f'^2 - A\left[\frac{\eta}{2}f'' + f'\right])$$

$$- Mf' + \lambda_1\theta + \lambda_2S + \lambda_3\gamma = 0, \quad (9)$$

$$(1 + R)\frac{1}{Pr}\theta'' - f'\theta + f\theta' - A\left(\frac{\eta}{2}\theta' + 2\theta\right)$$

$$+ Nb\gamma'\theta' + Nt\theta'^2 - Mf' = 0 \quad (10)$$

$$\frac{1}{Sc}S'' - f'S + fS' - A\left(\frac{\eta}{2}S' + 2S\right) + S_r\theta'' = 0 \quad (11)$$

$$\gamma'' - Le\left[f'\gamma - f\gamma' + A\left(\frac{\eta}{2}\gamma' + 2\gamma\right)\right] + \frac{Nt}{Nb}\theta'' = 0 \quad (12)$$

and the transformed boundary conditions Eqs. (6) and (7) are:

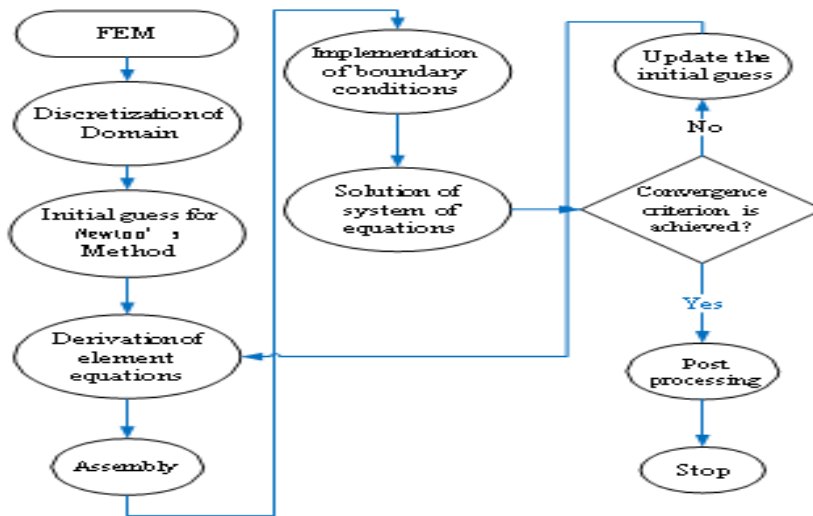
$$f(0) = f_w, f'(0) = 1 + S_f f''(0), \theta(0) = 1 + S_t \theta'(0),$$

$$S(0) = 1 + S_p S'(0), \quad (13)$$

$$\gamma(0) = 1 + S_g \gamma'(0),$$

$$f'(\infty) \rightarrow 0, \theta(\infty) \rightarrow 0, S(\infty) \rightarrow 0, \gamma(\infty) \rightarrow 0, \quad (14)$$

### Finite element method



## Results

The velocity profile experiences a decline as the magnetic parameter  $M$  increases. This trend holds true for scenarios involving suction, injection, and the absence of suction [13]. Figure 2 illustrates these variations in the presence or absence of hydrodynamic slip ( $S_f$ ). Turning attention to Figure 3, the impact of the viscoelastic parameter and the presence or absence of the unsteadiness parameter  $A$  is explored [14]. In both instances of hydrodynamic and non-hydrodynamic slip, the analysis delves into how these factors influence the augmentation of the velocity profile [15].

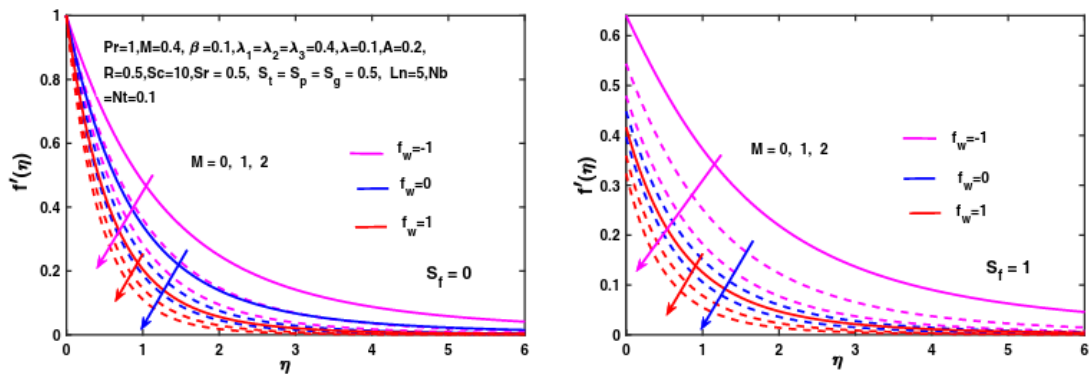


Fig. 2 Effect of  $M$  and  $f_w$  on velocity distribution with slip and no slip condition  $S_f$

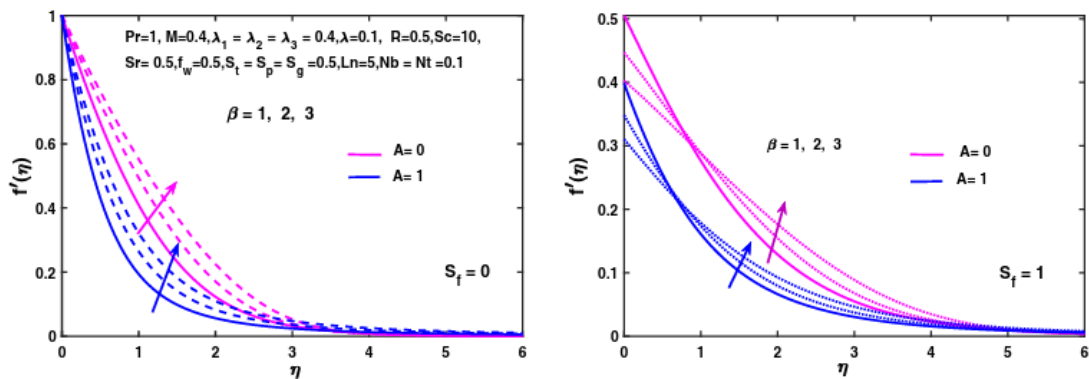


Fig. 3 Effect of  $\beta$  and  $A$  on velocity distribution with slip and no slip condition  $S_f$

### Conclusions

Key findings are summarized as follows:

- The presence of multiple slips leads to an increase in boundary layers.
- The velocity distribution is amplified by the viscoelastic parameter ( $\beta$ ) and buoyancy parameters ( $\beta_1, \beta_2, \beta_3$ ), while it diminishes with an elevated magnetic parameter ( $M$ ).
- Temperature experiences decay influenced by the unsteady parameter ( $A$ ) and buoyancy parameter ( $\beta_1$ ), but it is heightened by radiation ( $R$ ) and the thermophoresis parameter ( $Nt$ ).
- Concentration profiles are enhanced by higher Soret numbers ( $Sr$ ) and reduced by larger Schmidt numbers ( $Sc$ ).
- An increase in the Brownian motion parameter ( $Nb$ ) results in an enhanced nano-fluid volume fraction profile, while a higher Lewis number ( $Le$ ) leads to reduction.

### References

- [1] Khan SU, Ali N, Sajid M, Hayat T (2018) Heat transfer characteristics in oscillatory hydromagnetic channel flow of Maxwell fluid using Cattaneo–Christov model. *Proc Natl Acad Sci India Sect A Phys Sci* 89:1–9
- [2] Basha HT, Makinde OD, Arora A, Singh A, Sivaraj R (2018) Unsteady flow of chemically reacting nanofluid over a cone and plate with heat source/sink. In: *Defect and diffusion forum*, vol 387. Trans Tech Publications, pp 615–624
- [3] Dash G, Ojha KL (2018) Viscoelastic hydromagnetic flow between two porous parallel plates in the presence of sinusoidal pressure gradient. *Alex Eng J* 57(4):3463–3471
- [4] Sheikholeslami M, Ganji D (2017) Transportation of mhd nanofluid free convection in a porous semi annulus using numerical approach. *Chem Phys Lett* 669:202–210
- [5] Akbar NS, Tripathi D, Khan ZH, Bég OA (2016) A numerical study of magnetohydrodynamic transport of nanofluids over a vertical stretching sheet with exponential temperature-dependent viscosity and buoyancy effects. *Chem Phys Lett* 661:20–30
- [6] Ahmed SE, Aly AM, Raizah Z (2019) Heat transfer enhancement from an inclined plate through a heat generating and variable porosity porous medium using nanofluids due to solar radiation. *SN Appl Sci* 1(7):661
- [7] Hassan AR, Fenuga OJ (2019) The effects of thermal radiation on the flow of a reactive hydromagnetic heat generating couple stress fluid through a porous channel. *SN Appl Sci* 1(10):1278
- [8] Shit G, Haldar R, Ghosh S (2016) Convective heat transfer and MHD viscoelastic nanofluid flow induced by a stretching sheet. *Int J Appl Comput Math* 2(4):593–608
- [9] Sheikholeslami M, Shamlooei M (2017) Fe<sub>3</sub>O<sub>4</sub>–H<sub>2</sub>O nanofluid natural convection in presence of thermal radiation. *Int J Hydrog Energy* 42(9):5708–5718
- [10] Sheikholeslami M, Zeeshan A (2017) Analysis of flow and heat transfer in water based nanofluid due to magnetic field in a porous enclosure with constant heat flux using cvfem. *Comput Methods Appl Mech Eng* 320:68–81
- [11] Sheikholeslami M, Vajravelu K (2017) Nanofluid flow and heat transfer in a cavity with variable magnetic field. *Appl Math Comput* 298:272–282
- [12] Hussanan A, Salleh MZ, Khan I, Shafie S (2018) Analytical solution for suction and injection flow of a viscoplastic casson fluid past a stretching surface in the presence of viscous dissipation. *Neural Comput Appl* 29(12):1507–1515
- [13] Majeed A, Zeeshan A, Alamri SZ, Ellahi R (2018) Heat transfer analysis in ferromagnetic viscoelastic fluid flow over a stretching sheet with suction. *Neural Comput Appl* 30(6):1947–1955
- [14] Ellahi R (2013) The effects of mhd and temperature dependent viscosity on the flow of non-newtonian nanofluid in a pipe: analytical solutions. *Appl Math Model* 37(3):1451–1467
- [15] Choi SU, Eastman JA (1995) Enhancing thermal conductivity of fluids with nanoparticles. Technical report, Argonne National Lab, IL (United States)