

Examining the Flow of MHD Casson Nanofluid with Radiative Heat Source over a Nonlinear Inclined Surface, Accounting for Soret and Dufour Effects.

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Abstract

This article delves into the analysis of boundary layer flow of MHD Casson Nanofluid over an inclined extending surface, considering thermal radiation, heat source/sink, Soret and Dufour effects. The study employs the Buongiorno model, incorporating Brownian motion and thermophoresis properties to explore the thermal efficiencies of fluid flows. The nonlinear problem concerning Casson Nanofluid flow over an inclined channel is formulated to gain insights into heat and mass exchange phenomena. Key flow parameters of the intensified boundary layer are considered. The governing nonlinear partial differential equations are transformed into ordinary differential equations, subsequently numerically solved using the homotopy analysis method (HAM). Numerical outcomes, along with graphical representations in tables and graphs, are presented. It's observed that an increase in the inclination parameter leads to reduced surface friction, but conversely affects the Nusselt number and Sherwood number. The concentration field demonstrates a decreasing trend with the inclination parameter, whereas the chemical reaction rate parameter exhibits an opposite increasing trend. These findings align remarkably well with previous studies conducted by other researchers.

Introduction

In recent years, nanofluids have garnered significant attention within the scientific community due to their exceptional thermal performance and remarkable potential for enhancing heat transfer efficiency without inducing pressure drops. A nanofluid is composed of diverse nanoparticles, such as Al_2O_3 , Cu, and CuO, dispersed within a base liquid, which can be substances like oil, water, ethylene glycol, among others. Extensive research has shown that the thermal conductivity of the base fluid often differs from that of the nanofluid [1-3]. This divergence in thermal conductivity is what makes nanofluids an attractive choice as a working fluid, primarily due to their elevated thermal conductivity. [4] explored the pivotal factors contributing to the enhancement of nanofluid's thermal conductivity. He identified the significant roles played by Brownian motion and thermophoresis effects in conventional fluids in augmenting thermal conductivity [5-7]. To the best of our knowledge, minimal research has been undertaken on Casson nanofluid in the context of an inclined stretching surface, while simultaneously considering thermal radiation, heat source, chemical reaction, Brownian motion, and the Soret and Dufour effects [8]. This investigation was prompted by Liao's homotopy analysis method, as applied in that prior study. The model employed here is a recent development inspired [9-11], and the outcomes derived from this current research are novel. Our findings reveal that the Dufour effect leads to a reduction in the Nusselt and Sherwood numbers due to the influence of Soret effect. Additionally, the non-linear radiative heat exchange intensifies fluid temperature. This study holds the potential for future extension to exponentially inclined stretching surfaces [12].

Mathematical formulation

Examined in this context is the steady flow of a two-dimensional boundary layer of Casson nanofluid over a nonlinear surface that extends at an inclination is shown in the below diagram [13].

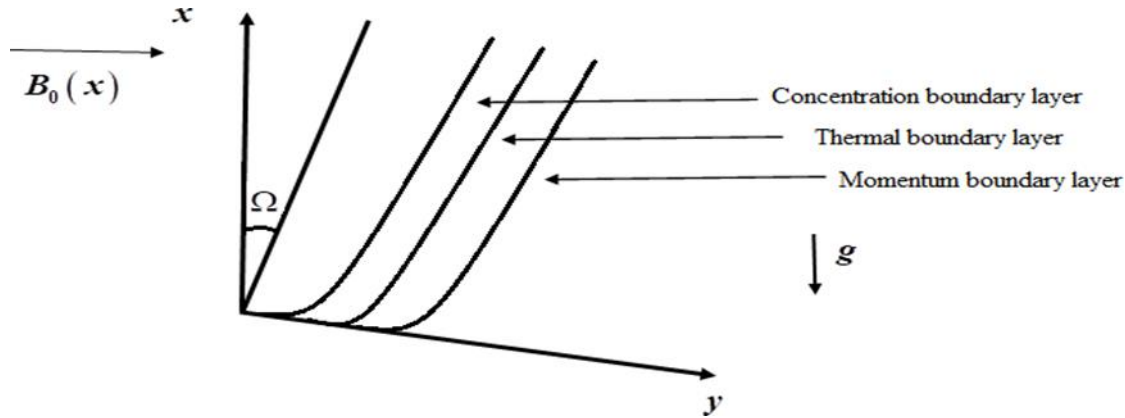


Fig. 1. Geometry of the physical model and the coordinate system

Using these assumptions, the modelled governing equations are defined as follows

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = v \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} + g \left[\beta_t (T - T_\infty) + \beta_c (C - C_\infty) \right] \cos \Omega - \frac{\sigma B_0^2(x) u}{\rho}, \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho c_p)_f} \frac{\partial q_r}{\partial y} + \tau \left[D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] + \frac{D_T K_T}{C_s C_p} \frac{\partial^2 C}{\partial y^2} + \frac{Q_0}{(\rho c_p)_f} (T - T_\infty), \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T K_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} - Kr (C - C_\infty), \quad (4)$$

The Rosseland approximation (for radiation flux) is defined as follows:

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y}. \quad (5)$$

$$T^4 = 4T_\infty^3 T - 3T_\infty^4 \quad (6)$$

Then, with the help of Eq (6) and (7), we obtain

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \left(\alpha + \frac{16\sigma^*}{3k^*(\rho c_p)_f} \right) \frac{\partial^2 T}{\partial y^2} + \tau \left[D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] + \frac{D_T K_T}{C_s C_p} \frac{\partial^2 C}{\partial y^2}, \quad (7)$$

$$\begin{aligned} u = u_w(x) = ax^m, v = 0, T = T_w, C = C_w, at y = 0 \\ u \rightarrow u_\infty(x) = 0, v \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty at y \rightarrow \infty, \end{aligned} \quad (8)$$

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}, \quad (9)$$

$$\begin{aligned} \zeta = y \sqrt{\frac{(m+1)ax^{m-1}}{2\nu}}, \psi = \sqrt{\frac{2\nu ax^{m+1}}{m+1}} f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \\ \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \end{aligned} \quad (10)$$

$$\left(1 + \frac{1}{\beta} \right) f''' + ff'' - \frac{2m}{m+1} f'^2 + \frac{2}{m+1} (\lambda\theta - \delta\phi) \cos\Omega - \left(\frac{2M}{m+1} \right) f' = 0 \quad (11)$$

$$\left(1 + \frac{4}{3}R \right) \theta'' + Pr f\theta' + Pr Nb\phi'\theta' + Pr Nt\theta'^2 + D_f\phi + \frac{2}{m+1} QPr\theta = 0, \quad (12)$$

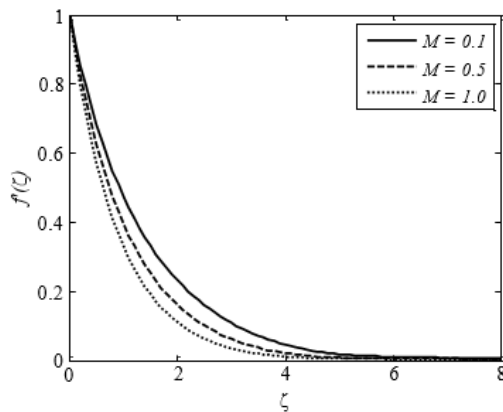
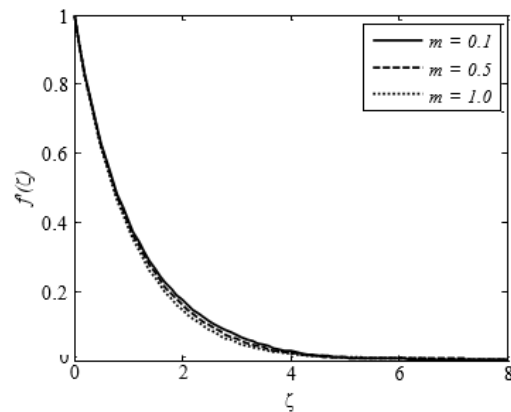
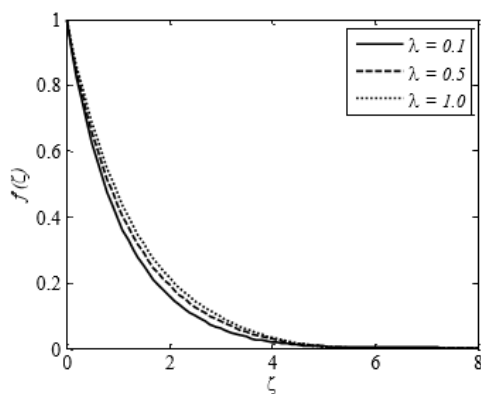
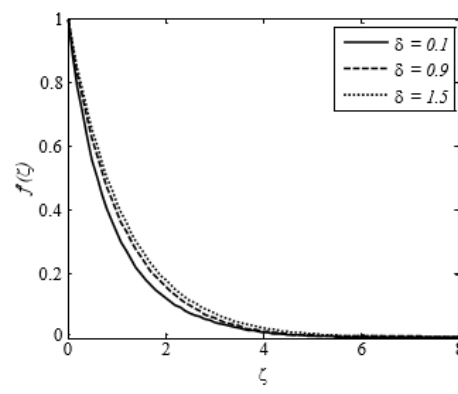
$$\phi'' + Pr Lef\phi' + \frac{Nt}{Nb}\theta'' - \frac{2}{m+1} Pr Le\gamma\phi = 0 \quad (13)$$

The corresponding boundary conditions are transformed to:

$$\begin{aligned} f(\zeta) = 0, f'(\zeta) = 1, \phi(\zeta) = 1, \theta(\zeta) = 1 \text{ at } \zeta = 0, \\ f'(\zeta) \rightarrow 0, \theta(\zeta) \rightarrow 0, \phi(\zeta) \rightarrow 0 \text{ as } \zeta \rightarrow \infty, \end{aligned} \quad (14)$$

Results

In this context, we present graphical and tabular representations illustrating the characteristics of physical variables, including velocity, temperature, concentration, skin friction, Nusselt number, and Sherwood number. Several of these figures are generated by systematically varying a parameter within a predefined range.

Fig. 2. Impact of M on $f'(\zeta)$ Fig. 3. Impact of m on $f'(\zeta)$ Fig. 4. Impact of λ on $f'(\zeta)$ Fig. 5. Impact of δ on $f'(\zeta)$

Conclusions

The following notable conclusions are drawn from the present investigation:

- I. Elevated values of the magnetic field parameter lead to a decline in velocity.
- II. As the Brownian motion parameter increases, the temperature experiences a corresponding increase.
- III. Increasing Dufour constraint values are associated with higher temperature fields.
- IV. A higher Casson fluid factor contributes to a more uniform velocity distribution.
- V. An increase in the inclination parameter results in a reduction of the skin friction coefficient, while exerting the opposite effect on the Nusselt and Sherwood numbers.

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