

THE UPPER RESTRAINED DETOUR EDGE MONOPHONIC DOMINATION NUMBER OF A GRAPH

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ABSTRACT- In this paper the concept of minimal restrained detour edge monophonic domination number $M$of a graph G is introduced. For a connected graph $G = (V, E)$ of order at least two, aminimal restrained detour edge monophonic dominating set M of a graph G is a detour edge monophonic dominating set such that either $M = V$ or the sub graph induced by $V - M$ has no isolated vertices. The minimum cardinality a minimal restrained detour edge monophonic dominating set of G is the minimal restrained detour edge monophonic domination number of G and is denoted by ${\gamma_{dem_r}}^+(G)$. We determine bounds for it and characterize graphs which realize these bounds. It is shown that For any three positive integers a, b, c and d, with $2 \leq a \leq b \leq c \leq d$, there is a connected graph G with $dm(G) = a$, $\gamma_{dm}(G) = b$,$\gamma_{dm_r}(G) = c$ and ${\gamma_{dem_r}}^+(G) = d$.

Keywords :Minimal detour edge monophonic dominating set,minimal detour edge monophonic domination number, minimalrestrained edge detour monophonic dominating set and minimalrestrained detour edge monophonic domination number.

I. INTRODUCTION

By a graph $G = (V, E)$ we mean a finite undirected connected graph without loops or multiple edges. The order and size of G are denoted by p and q, respectively. The neighborhood of a vertex v of G is the set $N(v)$ consisting of all vertices u which are adjacent with v. A vertex v of G is an extreme vertex if the sub graph induced by its neighborhood is complete. A vertex with degree one is called an end vertex. A vertex v of a connected graph G is called a support vertex of G if it is adjacent to an end vertex of G. A vertex v in a connected graph G is a cut vertex of G, if $G - v$is disconnected. A chord of a path $u_1, u_2, u_3, \ldots, u_k$ in G is an edge $u_i u_j$ with $j \geq i + 2$. A path P is called a monophonic path if it is a chordless path. A set M of vertices of G is aedge monophonic set of G if each vertex of G lies on a u-v monophonic path for some u and v in M. The minimum cardinality of aedge monophonic set of G is the edge monophonic number of G and is denoted by e(G). For a subset D of vertices, we call D a dominating set for each $x \in V(G) - D$, x is adjacent to at least one vertex of D. The domination number of D is the minimum cardinality of a dominating set of G and is denoted by $\gamma_m(G)$ [4]. A set of vertices M in G is called a monophonic dominating set if M is both edge monophonic set and a dominating set. The minimum cardinality of aedge monophonic dominating set of G is the edge monophonic domination number of G and is denoted by $\gamma_{em}(G)$ [5]. A longest $x - y$ monphonic path is called an $x - y$ detour monophonic path. A set M of a graph G is a detour edge monophonic set of G if each vertex v of G is lies on an $x - y$ detour monophonic path, for some x and y in M. The minimum cardinality of a detour edge monophonic set of G is the detour edge monophonic number of G and is denoted by $dem(G)$ [6]. A minimalrestrained detour edge monophonic dominating set of G is a detour edge monophonic dominating set M such that either $M = V$ or the sub graph induced by $V - M$ has no isolated vertices.

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Definition 2.1 A restrained detour edge monophonic dominating set M in a connected graph G is called a minimal restrained detour edge monophonic dominating set if no proper subset of M is a restrained detour edge monophonic dominating set. The maximum counting number in the midst of all minimal restrained detour edge monophonic dominating set is labeled upper restrained detour edge monophonic domination number of G, and is denoted by $\gamma_{dem_r}{}^{+}(G)$.

Example 2.2 For the graph G given in Figure 2.1, the restrained detour edge monophonic dominating sets are $M_1 = \{v_1, v_2, v_3, v_4, v_5\}, M_2 = \{v_2, v_3, v_4, v_5, v_6\}, M_3 = \{v_1, v_2, v_3, v_6, v_7, v_8\}$, $M_4 = \{v_1, v_2, v_3, v_4, v_7, v_8\}$, and $M_5 = \{v_3, v_4, v_5, v_6, v_7, v_8\}$. In this graph, the upper restrained detour edge monophonic domination number is 6 and the restrained detour edge monophonic domination number is 5.

Theorem 2.3 Each extreme vertex of a connected graph G belongs to every minimal restrained detour edge monophonic dominating set of G.

Proof. Since every minimal restrained detour edge monophonic dominating set of G is a restrained detour monophonic dominating set of G, the theorem follows from Theorem 1.2.

Corollary 2.4 For the complete graph K_p, $\gamma_{dem_r}{}^{+}(K_p) = p$.

Theorem 2.5 Let G be a connected graph with cut vertices and let M be a minimal restrained detour edge monophonic dominating set of G. If v is a cutvertex of G, then every component of $G - v$ contains an element of M.

Proof. Suppose that there is a component B of $G - v$ such that B contains no vertex of M. Let w be a vertex in B. Since M is a minimal restrained detour edge monophonic dominating set of G, there exist vertices $x, y \in M$ such that w lies on some $x - y$ detour edge monophonic path $P : x = u_0, u_1, \dots, w, \dots, u_l = y$ in G. Let P_1 be the $x - w$ subpath of P and P_2 be the $w - y$ subpath of P. Since v is a cutvertex of G, both P_1 and P_2contains v so that P is not a path, which is a contradiction. Thus every component of $G - v$ contains an element of M.

Corollary 2.6 Let G be a connected graph with cut vertices and let M be a minimal restrained detour edge monophonic dominating set of G. Then every branch of G contains an element of M.

Theorem 2.7 No cut vertex of a connected graph G belongs to any minimal restrained detour edge monophonic dominating set of G.

Proof. Let M be any minimal restrained detour edge monophonic dominating set of G and let $v \in M$ be any vertex. We claim that, v is not a cutvertex of G. Suppose that v is a cutvertex of G. Let $G_1, G_2, \dots, G_r (r \geq 2)$ be the components of $G - v$. Then v is adjacent to at least one vertex of $G_j (1 \leq j \leq r)$. Let $M' = M - \{v\}$. Let u be a vertex of G which lies on a detour monophonic path P joining a pair of vertices, say x and v of M. Assume without loss of generality that $x \in G_1$. Since v is adjacent to at least one vertex of each $G_j (1 \leq j \leq r)$, assume that v is adjacent to a vertex y in $G_k (k = 1)$.

Corollary 2.8 For any tree T with k-end vertices, $\gamma_{dem_r}(T) = \gamma_{dem_r}{}^{+}(T) = k$.

Proof. This follows from Theorems 2.3 and 52.7. Since every end-block B is a branch of G at some cutvertex, it follows by Theorems 2.5 and 2.7 that every minimal restrained detour edge monophonic dominating set of G contains at least one vertex from B that is not a cut vertex. Thus the following corollaries are consequences of Theorems 2.5 and 2.7.

Theorem 2.9 For any connected graph G, $2 \leq \gamma_{dem_r}(G) \leq \gamma_{dem_r}{}^{+}(G) \leq p$.

Proof. It is clear from the definition of minimum restrained detour edge mo

nophonic dominating set that $\gamma_{dm_r}(G) \geq 2$. Since every minimal restrained detour monophonic dominating set is a restrained detour edge monophonic dominating set of $G, \gamma_{dem_r}(G) \leq \gamma_{dem_r}{}^{+}(G)$. Also, since $V(G)$ is a restrained detour edge monophonic dominating set of G, it is clear that $\gamma_{dem_r}{}^{+}(G) \leq p$. Thus $2 \leq \gamma_{dem_r}(G) \leq \gamma_{dem_r}{}^{+}(G) \leq p$.

Theorem 2.10 For a connected graph $G, \gamma_{dem_r}(G) = p$ if and only if $\gamma_{dem_r}{}^{+}(G) = p$.

Proof. Let $\gamma_{dem_r}{}^{+}(G) = p$. Then $M = V(G)$ is the unique minimal restrained detour edge monophonic dominating set of G. Since no proper subset of M is a restrained detour edge monophonic dominating set, it is clear that M is the unique minimum restrained detour edge monophonic dominating set of G and so$\gamma_{dem_r}(G) = p$. The converse follows from Theorem 2.9.

Theorem 2.11 If G is a connected graph of order p with $\gamma_{dem_r}(G) = p - 1$, then $\gamma_{dem_r}{}^{+}(G) = p - 1$.

Proof. Since $\gamma_{dem_r}(G) = p - 1$, it follows from Theorem 2.9 that $\gamma_{dem_r}{}^{+}(G) = p$ or $p - 1$. If $\gamma_{dem_r}{}^{+}(G) = p$, then by Theorem 2.4,$\gamma_{dem_r}(G) = p$, which is a contradiction. Hence $\gamma_{dem_r}{}^{+}(G) = p - 1$

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