Explicit finite difference analysis of Casson fluid flow in parallel plate channel with moderate values of Reynolds number

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ABSTRACT

Transport phenomenon within a fluid, namely momentum is plays vital role in nature, industries and all branches of engineering and sciences. The fluids of most interest to engineers are complex in terms of their rheological and transport properties.This study is focused on solution of the boundary layer flow of Casson fluid in a parallel plate channel with different values of Reynolds number. we note that a positive linear relationship between fully developed length and Reynolds number. Horizontal velocity profile of the flow has a parabolic shape when the flow is fully developed.

1. INTRODUCTION

Fluid flows within the channels are of interesting phenomenon in science and Engineering even in everyday life. Fluid flow within the channels like air ducts, pipes are exclusively in contact with rigid boundaries. Szeri et al. [1] studied the occurrence of fluid flow within a finite parallel plate duct, with numerical approach theoretically predicted the velocity profile for laminar and turbulent modes and found a good agreement between predicted and experimental values. Sarojamma et al. [2] analyzed the flow, heat and mass transport phenomenon on MHD Casson fluid within a parallel plated duct in presence of Stretching walls surrounded by transverse Magnetic field. With RK fourth order shooting technique they discussed the impact of governing parameters on fluid flow variables with their study it was revealed that the magnetic field strength and fluid velocity diminishes with the rise of temperature.

Das et al. [3] investigated the blood flow phenomenon within a constricted blood vessel by means of Casson fluid flow model. In their investigation with the implementation of IBM_MAC methods it is revealed that the absorption parameter and fluid concentration are inversely associated in both tissue and lumen. Amlimohamadi et al. [4] investigated the flow of Cassson fluid within a 2D porous duct consisting of a local constriction by means of numerical approach of SIMPLE algorithm and shown that enhancement of porosity of the channel reduces the shear stress on the constriction. Avramenko et al. Santos et al. [7] presented simulations on laminar fluid flow in a channel consisting of baffles prepared with solid and porous materials. By SIMPLE algorithm approach results compared numerically for friction factor and Nusselt number and compared with available data.

2. MATHEMATICAL FORMULATION

Let us consider a viscous incompressible Casson fluid flow in a parallel plate channel.The constitutive equation for a Casson fluid i.e. the stress-strain relation takes the form:

$$\tau^{\frac{1}{2}} = \tau_0^{\frac{1}{2}} + \mu \dot{\gamma}^{\frac{1}{2}}$$

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + \dfrac{P_y}{\sqrt{2\pi}}\right)e_{ij}, & \pi > \pi_c \\ 2\left(\mu_B + \dfrac{P_y}{\sqrt{2\pi_c}}\right)e_{ij}, & \pi_c > \pi \end{cases} \tag{1}$$

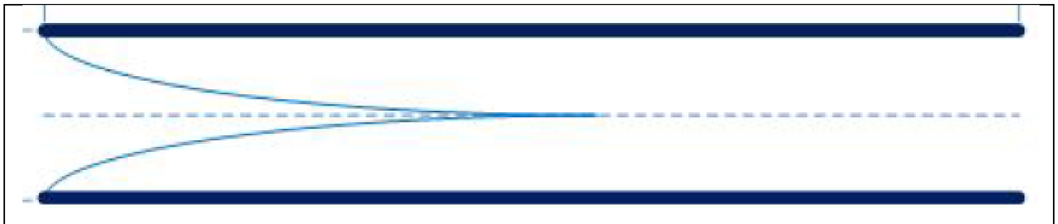


Figure 1: Schematic diagram of the problem

where μ_B is the plastic dynamics viscosity, $\pi = e_{ij}\, e_{ij}$ where e_{ij} is the (i, j)th element of the deformation rate and π_c is the critical value of the deformation rate. The yield stress of non-Newtonian Casson fluid P_y is written as

$$P_y = \frac{\mu_B \sqrt{2\pi}}{\beta} \tag{2}$$

In a case of Casson fluid flow, where $\pi > \pi_c$, then it is possible to say that

$$\mu = \mu_B + \frac{P_y}{\sqrt{2\pi_c}} \tag{3}$$

From the equation (2) and (3), we have

$$\mu = \mu_B \left(1 + \frac{1}{\beta}\right) \tag{4}$$

With the above assumptions, the governing fluid flow equations of Casson fluid in a parallel plate channel as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{5}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{1}{\rho}\frac{\partial P}{\partial X}+\left(1+\frac{1}{\beta}\right)\nu\frac{\partial^2 U}{\partial Y^2} \qquad (6)$$

The channel Reynolds number is based on the channel height, mean velocity and the kinematic viscosity of the fluid, as is shown in Equation (7):

$$\mathrm{Re}=\frac{\rho v H}{\mu} \qquad (7)$$

Velocity of the flow at the inlet is $\frac{Re\mu}{\rho H}$.

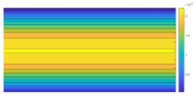
3. NUMERICAL SCHEME

The routine based finite difference method is to solve the governing equation (6) subject to the inlet boundary conditions. The MATLAB code is developed an explicit form of the marching algorithm, which is shown as follows:

$$u_{i,j}\frac{u_{i+1,j}-u_{i,j}}{\Delta x}+v_{i,j}\frac{u_{i,j+1}-u_{i,j-1}}{2\Delta y}=-\frac{1}{\rho}\left(\frac{dP}{dx}\right)_{i+1}+\frac{\mu}{\rho}\left(\frac{u_{i,j+1}-2u_{i,j}+u_{i,j-1}}{\Delta y^2}\right)$$

where i and j are grid numbers in x and y direction.

a.



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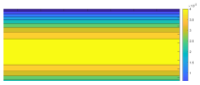

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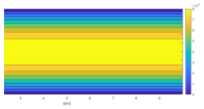
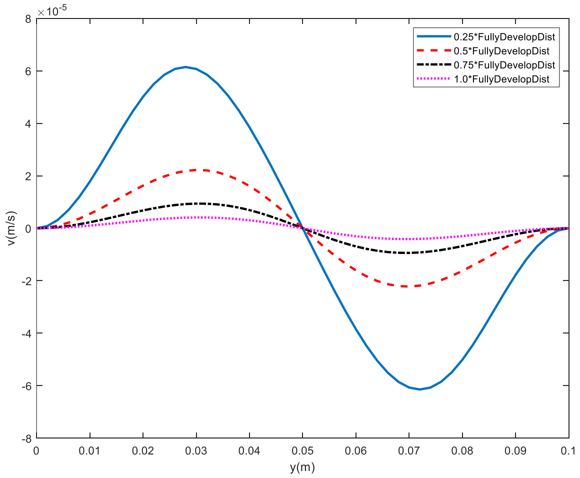


Figure 2: Streamlines Contours of u velocity in the parallel plate channel for different values of Re: a. Re=50, b. Re=100,c. Re=200



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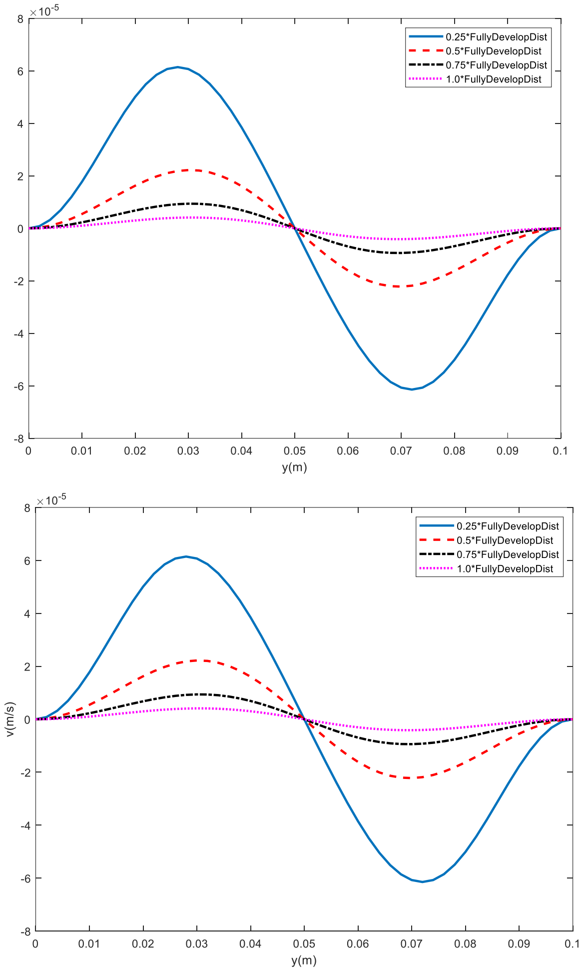

b.

c.

Figure 3: Vertical velocity v in the parallel plate channel for various values of Re at four corresponding locations. a. Re=50, b. Re=100,c. Re=200.

4. RESULTS AND DISCUSSION

This section illustrates the results of some interesting parameters for velocity profiles of Casson fluid flow with β=0.5. we consider a parallel plate channel with a height H of 0.1m and a length of 9m,as is shown in Figure 1 (schematic diagram). The fully-developed length usually can beapproximated as $0.1\times Re \times H$. The dynamic viscosity is set to 10^{-3} kg/m and density ρ is set to 10^{3} kg/m^{3}.Fig.2 represents the horizontal velocity profile of fluid flow within the parallel plates for distinct Reynolds numbers at distinct positions i.e. 0.25,0.50, 0.75 and 1.0 of the respective fully developed length. When the fluid enters into the parallel channel the horizontal velocity profile is observed flat around the channels central line and a parabolic velocity profile is observed when the flow is completely developed.Fig.3 represents the vertical velocity profile of fluid flow within the parallel plate duct for distinct Reynolds numbers at distinct positions i.e 0.25, 0.50, 0.75 and 1.0 of the respective fully developed length. The shape of vertical velocity profile is similar to Sine function as the flow develops it becomes flattened ultimately vertical velocity turn into zero.

5. CONCLUSION

In this numerical simulation, we apply the finite difference method to study the Casson fluid flow in Parallel Plate Channel. By this powerful and newly developed technique, the convergence series solutions are obtained. Graphs are depicted to study the variation of Reynolds number. From the present analysis, we note that a positive linear relationship between fully developed length and Reynolds number. Horizontal velocity profile of the flow has a parabolic shape when the flow is fully developed.

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