RESOLVABILITY OF THE STARPHENE STRUCTURE AND APPLICATIONS IN ELECTRONICS

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Abstract:

Polycyclic aromatic hydrocarbons, or starphenes, are composed of three distinct acene-arm variants. The fundamental building blocks for the downsizing of various electronic devices, particularly organic ones, are starphenes. It was also a key component of several logical gates. Every electrical circuit, structure, or network in network topology can be represented as a graph with line segments (branches) acting as edges and primary nodes (or simply nodes) alternating to vertices. Resolvability parameters of a graph are a relatively recent specialized field in which the unique location of each primary node is obtained by forming the network as a whole. This article investigates the metric, edge metric dimension, and generalizations as resolvability characteristics of starphene structure. We demonstrated the consistent cardinalities of all the parameters examined for the starphene graph. Transforming the entire structure into a fresh shape provided by resolvability parameters facilitates understanding and handling of structures.

1. Introduction

The graphical representation of electric circuits known as network topology. Complex and complicated electric circuits or networks are relatively not easy to work on and study in their original forms, to make them easy and understandable, network topology is used. Any electric circuit or network can be transformed or shaped into its equivalent graph, in this procedure of terraforming an electric network into graph, open circuits took places of current sources and short circuits are came up in place of the passive elements and voltage sources. Open circuits usually denoted by nodes (or principal nodes) in network topology and vertices in pure mathematical graph theory, whereas short circuits are called as line segments (or branches) in network topology and edges in graph theory conceptualization. The formal definition of graphical representation of an electric circuit or network is defined as:

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Peer review under responsibility of Ain Shams University. Definition 1.1 [25]. "Let @ðVð Þ@ ; Eð Þ@ Þ is an electric circuit (network) with Vð Þ@ is called as set of principal nodes (vertex set) and Eð Þ@ is the set of branches (edge set). The total number of principal nodes in an electric network are jVð Þ@ j and the count of branches usually denoted as jEð Þ@ j, basically these are order and size of a equivalent graph of an electric network."

To elaborate more in depth we took an example to transform an electric circuit into its corresponding graph. In the circuit shown in Fig. 1 (left), the labeling with 1; 2; 3, and 4, we can see are the four principal nodes (or vertices). There are also labeling with 15 X; 2 X; 5 X; 5 X, and 10 X are the resistors having resistances in X-unit, 2 A is a current source, 10 V

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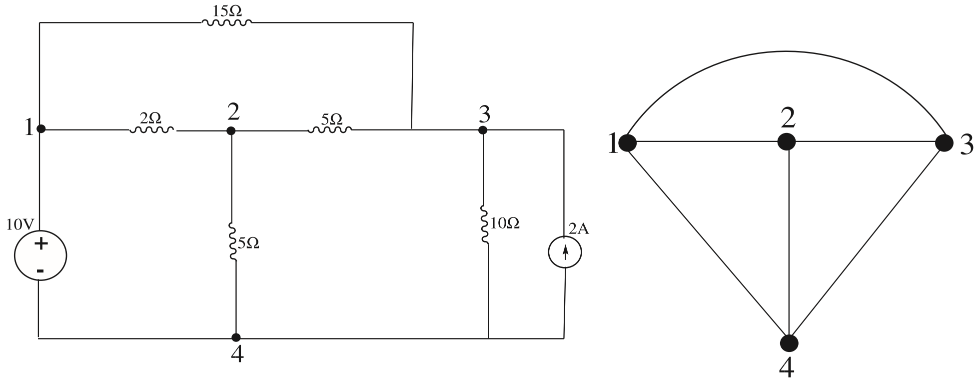
which is a voltage source, these are all seven branches (or edges) in the above circuit. There is another drawing shown in the following Fig. 1 (right), which is an equivalent graph of the electric circuit. For more detail of transformation of a electric circuit to graph see [53].

We will show some technique of graph to demonstrate theoretical parameters in the context of electronics. There are several different approaches to examine and investigate circuits of electric when it comes to graph theory. In 1975, [50,13] presented an effective notion of network visualization; in this concept, a small number of main nodes are chosen so that the whole set of principle nodes can be identified in relating to a distance vector, this is referred to as the metric basis or resolving set. This notion laid the groundwork for a number of graph theoretical parameters that are used in a variety of electrical and chemical engineering,



and in other areas. The fault-tolerant concept of the definition of resolving set, described by [5], is also a unique approach of examining a graph (structure) wherein fault-tolerant of a solemn main vertex from the resolving set may be allow while the full collection of primary vertices still has a unique location. In 2018, [21] developed the edge metric resolving set, which assigns a unique location to the whole set of branches (edges) instead of main nodes. The authors in [28], investigated the edge version of a fault-tolerant resolving set in 2020. In [22], authors presented the combination version of resolving and edge metric resolving set in 2017, which allows whole sets of main nodes and branches to be uniquely identified. Partition resolving set [6] is created when the whole set of primary vertices, is split down into subgroups and the requirement of acquiring distinct position of the set of principal nodes is met. All of the aforementioned ideas are referred to be metric-based resolvability parameters, and they've been investigated for many circuits, networks and graphs. For more versions of graph theoretical aspects, we refer to see [11,12,41].

The researchers of [14,27,7,6], examined complexity or computational cost and demonstrated that all of the parameters from resolvability family, investigated in this work relate to the nondeterministic polynomial time-hardness. The investigators are inspired by the proven results of the metric dimension which has a wide range of practical applications in our everyday lives and is well-studied. Metric dimension is used in a variety of disciplines, including the weighing of coins [50], robot navigation [23], pharmaceutical chemistry [7], computer networks [30,46] also linked to this idea, coastguard loran, sonar, and facility locating difficulties connected to this concept in the foundational article by [49], for a more comprehensive examination and the uses of this pointer [35,36]. In several real-world applications, the partition dimension is also described, as an illustration, by [4], the process of identifying a network and also its verification is connected to this idea, the piloting or

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guidance of a robot also linked to this concept [23], the popular relationship Djokovic-Winkler linked to this concept [5], for the coding of games, their decoding and other strategies of games and especially mastermind games brief in [10]. See [7,13,18,19,32] for additional information on how to use and use these resolvability factors.

As previously stated, the chemical field is blessed with the applications of this parameter. A huge number of publications are done and published whether in the graph prospectives or particularly related to metric dimension. The VC5C7 and HNaphtalenic nano-tubes are detailed in [16] with the pointer of metric, cellulose network is studied in [47], in which they computed some sharps boundaries on this parameter, silicate star is another chemical rich structure and [48] made a point of discussion this structure for metric dimension, two types of structure in which one is alpha-boron nanotubes and other one is twodimensional lattice, are detailed in [15], relating to the metric pointer and also linked with its applications. The partition dimension parameter is detailed in [3] with inconstant cardinality, a chemical structure which is a fullerene with ð4; 6Þ type, is also examined in [31] by using the notion of partition resolving set. We ask you to look at the papers [34,9,29] for some recent publications on the resolving partition set.

Moving on to the edge metric dimension, [26] examined the barycentric subdivision of the Cayley graph, [55,1] presented a few works on the convex polytopes structure, and [51] addressed the chemical structures of wheel graphs. In addition, the foundational work on the edge metric dimension is published in the reference [52], which includes a quantitative comparison between metric and its various variants. Some current work can be gained by the references [39,45,39,2,42], for the fault-tolerant idea mentioned in [17] for basic graphs and [37] for diverse connectivity networks along with the deployment of their applications.

The following are some very fundamental ideas and early mathematical definitions that are very helpful in comprehending the study work conducted in this research.

Definition 1.2 ([34,25]). "Suppose @ðVð Þ@ ; Eð Þ@ Þ is an undirected graph of an electric circuit (network) with Vð Þ@ is called as set of principal nodes (vertex set) and Eð Þ@ is the set of branches (edge set). The distance between two principal nodes f1;f2 2 Vð Þ@ , denoted as dðf1;f2Þ is the minimum count of branches between f1 f2 path."

Definition 1.3 ([50,25]). "Suppose R Vð Þ@ is the subset of principal nodes set and defined as R ¼ ff1;f2;...;fsg, and let a principal node f 2 Vð Þ@ . The identification rðfjRÞ of a principal node f with respect to R is actually a s-ordered distances ðdðf;f1Þ; dðf;f2Þ;...; dðf;fsÞÞ. If each principal node from Vð Þ@ have unique identification according to the ordered subset R, then this subset renamed as a resolving set of network @. The minimum numbers of the

V St lðð ;m;nÞÞ ¼ faf;: 1 6f 2ðl þ n 1Þg [fbf;: 1 6f 2ðl þ m 1Þg [fcf;: 1 6f
2ðm þ n 1Þg;

E St lðð ;m;nÞÞ ¼ fafafþ1;: 1 6f 2ðl þ nÞ 3g [fbfbfþ1;: 1 6f 2ðl þ mÞ 3g[fcfcfþ1;: 1 6f 2ðm þ nÞ
3g [fafbf;: 1 6f 2l 1; f ¼ oddg[fafcj;: 2l 6f 2ðl þ n 1Þ; f ¼ even; j ¼ 2ðl þ nÞ
1 fg[fbfcj;: 2l 6f 2ðl þ m 1Þ; f;j ¼ even; 2n 6 j 2ðm þ n 1Þg:

elements in the subset R is actually the metric dimension of @ and it is denoted by the term dimð Þ@ ."

Definition 1.4 ([5,25]). "A particular chosen ordered subset which were actually resolving set symbolize by R of a network @ is considered to be a fault-tolerant denoted by ðRf Þ, now if for each member of f 2 R, with the condition R n f is also remain a resolving set for the network @. The fault-tolerant metric dimension will be the least possible elements in the fault-tolerant resolving set and labeled as dimf ð Þ@ ."

Definition 1.5 ([21,24]). "A principal node f 2 Vð Þ@ and a branch e ¼ f1f2 2 Eð Þ@ , the distance between f and e is defined as dðe;fÞ ¼ minfdðf1;fÞ; dðf2;fÞg. Suppose Re Vð Þ@ is the subset of principal nodes set and defined as Re ¼ ff1;f2;...;fsg, and a branch e 2 Eð Þ@ . The identification rðejReÞ of a branch e with respect to Re is actually a s-tuple distances ðdðe;f1Þ; dðe;f2Þ;...; dðe;fsÞÞ. If each branch from Eð Þ@ have unique identification according to Re, then Re is called an edge metric resolving set of network @. The least possible elements in the set Re is labeled as the edge metric dimension of @ and it is represented by dimeð Þ@ ."

Definition 1.6 ([28,24]). "An fault-tolerant edge metric resolving set ðRe;f Þ of a network @ is considered, if for each f 2 Re; Re n f is remains an edge metric resolving set for @. The fault-tolerant edge metric dimension will be the minimum amount of members in the fault-tolerant edge metric resolving set and it is described as the entire set of principal nodes have unique identifications, then Rp is named as the partition resolving set of the principal node of a network @. The least possible count of the subsets in that set of Vð Þ@ is labeled as the partition dimension ðpdð ÞÞ@ of @."

Given below are some useful observations and are very necessary in the finding of our main results of the resolvability parameters of our graph @.

Theorem 1.9 [8]. Let dimð Þ@ , the metric and dimf ð Þ@ is the faulttolerant metric dimension of graph @. Then dimf ð Þ@ P dimð Þ þ@ 1: dime;f ð Þ@ ."

Definition 1.7 [22]. If the identifications of entire set of principal nodes and branches are unique with respect to a chosen resolving set Rm ð Þ@ , then Rm is called as mixed metric resolving set, and the minimum count of elements in Rm is called as mixed metric dimension and denoted as dimmð Þ@ .

Definition 1.8 ([6,24]). "Let Rp # Vð Þ@ is the s-elements proper set and rfjRp ¼ fdðf; Rp1Þ; dðf; Rp2Þ;...; dðf; RpsÞg, is the s-tuple disp tance identification of a principal node f in association with R . If Theorem 1.12 [22]. Let dimmð Þ@ be a mixed metric dimension of a graph @. Then dimmð Þ@ P maxfdimð Þ@ ;dimeð Þg@ :

2. Construction of Starphene St lð ;m;nÞ

Starphenes are two-dimensional polycyclic aromatic hydrocarbons (PAH) which are build by three acene arms [33] connected systematically on a centered benzene ring [40]. Starphenes are widely used in many electronic devices, and played a key role in the revolution of miniaturization of electronic devices. The structure used in the single molecule electronics as NOR [44] and as well as NAND [43]. The starphenes are very attractive materials in different electronic applications, especially organic electronics, the starphenes behaved as a component which is organic light emitting diodes in the field effect transistors [20].

Starphenes are belongs to the family of PAH, it has three acene arms we denoted as ðl; m; nÞ arms on a centered benzene ring. We denote the starphene structure or network throughout the work as St lð ; m; nÞ. The total number of vertices in starphene with different l; m and n variation 4ðl þ m þ nÞ 6, and 5ðl þ m þ nÞ 9 are the total nodes or line segments. Given below are the vertex (or principal nodes) and edge set (line segment or branches) of corresponding graph of St lð ; m; nÞ. Theorem 1.10 [28]. Let dimeð Þ@ , the edge metric and dime;f ð Þ@ is the fault-tolerant edge metric dimension of graph @. Then dime;f ð Þ@ P dimeð Þ þ@ 1:

Theorem 1.11 ([50,52]). Let dimð Þ@ ; dimeð Þ@ ; dimmð Þ@ ; pdð Þ@ , are the metric, edge metric dimension, mixed metric dimension and partition dimension of a graph @ respectively. Then

dimð Þ ¼@ dimeð Þ ¼@ 1; iff @ is a path graph;

dimmð Þ ¼@ pdð Þ ¼@ 2; iff @ is a path graph:

Furthermore, by merging the vertex and edge sets of St lð ; m; nÞ created above, the vertices marking utilized in the discoveries reported in Fig. 2, and the generalize St lð ; m; nÞ can be constructed.

3. Results on the resolvability of starphene St lð ;m;nÞ

This section is started by the core of this work, in which the resolving set with cardinality two is chosen from the possible combinations, later it's generalizations in which the faulttolerant version of resolving set, edge metric dimension and its generalized version which same as the fault-tolerant of given above, mixed metric dimension and at the end final version of all resolvability parameters named as the partition dimension are elaborated.

Lemma 3.1. Let the graph of starphene is St lð ; m; nÞ with l; m; n P 2. Then the cardinality of resolving set of St lð ; m; nÞ is 2.

Proof. Let R ¼ fa1; a2ðlþn1Þg, from the vertex set of graph of starphne St lð ; m; nÞ, with cardinality two. Consider R is one of the potential candidate for the role of resolving set. The identifications of the complete set of nodes in St lð ; m; nÞ with regard to the nodes in R are provided below.

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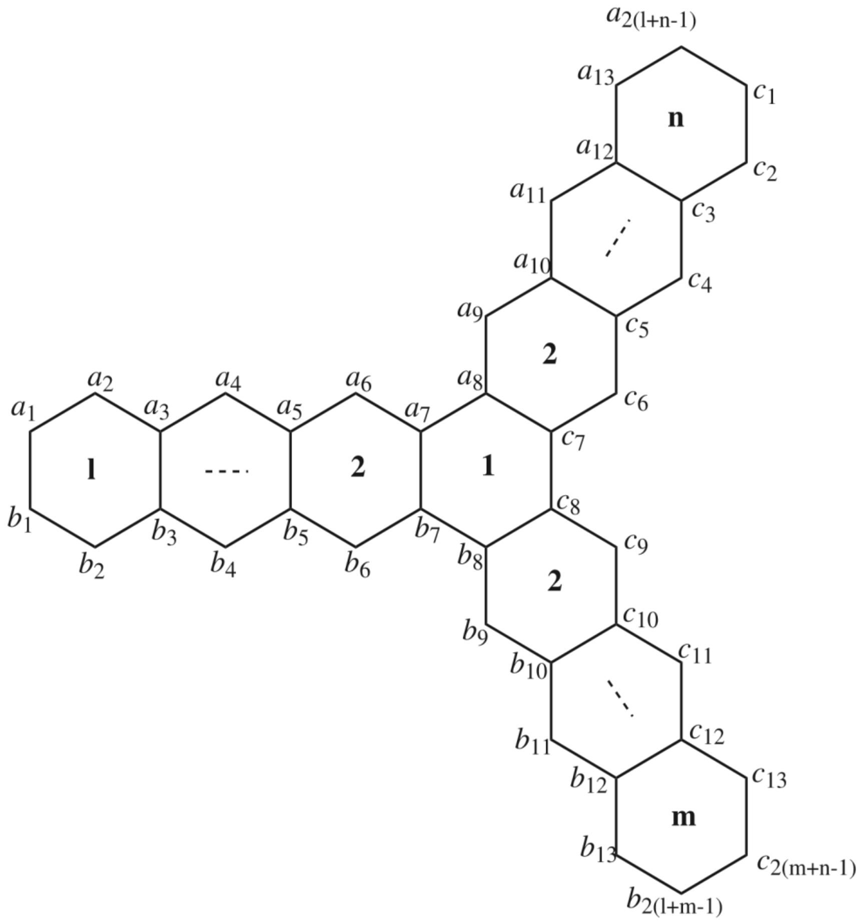

Fig. 2. The labeling of starphene St lð ; m; nÞ.

For f ¼ 1; 2;...; 2ðl þ n 1Þ, the r að fjRÞ, are following; r að fjRÞ ¼ ðf 1; 2ðl þ n 1Þ fÞ:

For f ¼ 1; 2;...; 2ðl þ m 1Þ, the r bð fjRÞ, are following;

r bð fjRÞ ¼ ððff;; 22ððlnþnlÞ þÞ 11 þ ffÞÞ;; ifif ff ¼¼ 211;;22;l...þ ;12;l...;12;ðl þ m 1Þ:

For f ¼ 1; 2;...; 2ðm þ n 1Þ, the r cð fjRÞ, are following;

ð2ðl þ nÞ 1 f; fÞ; if f ¼ 1;2;...;2m 1; Proof. By using the basic technique of double inequality, to prove that the graph of starphene St lð ; m; nÞ has two metric dimension. We refer to see the Lemma 3.1, in that proved we already showed that the potential candidate for the resolving set R ¼ fa1; a2ðlþn1Þg, with 2 cardinality.

Now we will prove that dim St lð ð ; m; nÞÞ P 2. On contrary we can see that the starphene is not a path graph and using Theorem 1.11, it indicated that one metric dimension of St lð ; m; nÞ is not possible. Hence; dim St lð ð ; m; nÞÞ P 2.

Hence,

r c R

Fig. 2. The labeling of starphene St lð ; m; nÞ.

For f ¼ 1; 2;...; 2ðl þ n 1Þ, the r að fjRÞ, are following; r að fjRÞ ¼ ðf 1; 2ðl þ n 1Þ fÞ:

For f ¼ 1; 2;...; 2ðl þ m 1Þ, the r bð fjRÞ, are following;

r bð fjRÞ ¼ ððff;; 22ððlnþnlÞ þÞ 11 þ ffÞÞ;; ifif ff ¼¼ 211;;22;l...þ ;12;l...;12;ðl þ m 1Þ:

For f ¼ 1; 2;...; 2ðm þ n 1Þ, the r cð fjRÞ, are following;

ð2ðl þ nÞ 1 f; fÞ; if f ¼ 1;2;...;2m 1; Proof. By using the basic technique of double inequality, to prove that the graph of starphene St lð ; m; nÞ has two metric dimension. We refer to see the Lemma 3.1, in that proved we already showed that the potential candidate for the resolving set R ¼ fa1; a2ðlþn1Þg, with 2 cardinality.

Now we will prove that dim St lð ð ; m; nÞÞ P 2. On contrary we can see that the starphene is not a path graph and using Theorem 1.11, it indicated that one metric dimension of St lð ; m; nÞ is not possible. Hence; dim St lð ð ; m; nÞÞ P 2.

Hence,

r c R

ð fj Þ ¼ ð2ðn mÞ þ 1 þ f; fÞ; if f ¼ 2m;2m þ 1;...;2ðm þ n 1Þ:dim St lð ð ;m;nÞÞ ¼ 2:

We can see that all the primary nodes held unique identifications and met the idea of a resolving set by concluding that j jR ¼ 2, by looking at the identifications of the whole group of nodes of St lð ; m; nÞ. h

Theorem 3.2. Let the graph of starphene is St lð ; m; nÞ with l; m; n P 2. Then

dim St lð ð ;m;nÞÞ ¼ 2:

Lemma 3.3. Let the graph of starphene is St lð ; m; nÞ with l; m; n P 2. Then the cardinality of fault-tolerant resolving set of St lð ; m; nÞ is 4.

Proof. Let Rf ¼ fa2l1; a2l; b2ðlþm1Þ; c2ðmþn1Þg, from the vertex set of graph of starphne St lð ; m; nÞ, with cardinality four. Consider Rf is one of the potential candidate for the role of fault-tolerant resolving set. The identifications of the complete set of nodes in St lð ; m; nÞ with regard to the nodes in Rf are provided below.

For f ¼ 1; 2;...; 2ðl þ n 1Þ, the r a fjRf , are following;

8>ðjf 2l þ 1j; jf 2lj; 2ðl þ mÞ 1 f; 2ðl þ mÞ fÞ; if f ¼ 1;2;...;2l 1; r afjRf ¼ <ðjf 2l þ 1j; jf 2lj; 1 þ f; 2ðm lÞ þ fÞ; if f ¼ 2l;2l þ 1;...;

>: 2ðl þ n 1Þ:

>>>8ð2l f; 2l f þ 1; 2ðl þ m 1Þ f; 2ðl þ mÞ 1 fÞ;

r b fjRf ¼ <ðf 2l þ 2; f 2l þ 3; 2ðl þ m 1Þ f; 2ðl þ mÞ 1 fÞ;

>>>: if f ¼ 1;2;...; 2l 1; if f ¼ 2l;2l þ 1;

...;2ðl þ m 1Þ:

For f ¼ 1; 2;...; 2ðl þ m 1Þ, the r b fjRf , are following;

For f ¼ 1; 2;...; 2ðm þ n 1Þ, the r c fjRf , are following;

8>ð2n f þ 1; 2n f; f; 2ðm þ n 1Þ fÞ; if f ¼ 1;2;...;2n 1; r cfjRf ¼ <ðf 2m þ 3; f 2m þ 2; f; 2ðm þ n 1Þ fÞ; if f ¼ 2n;2n þ 1;...;

>: 2ðm þ n 1Þ:

We can see that all the primary nodes held unique identifications and met the idea of a fault-tolerant resolving set by concluding that Rf ¼ 4, by looking at the identifications of the whole group of nodes of St lð ; m; nÞ. h

Theorem 3.4. If the graph of starphene is St lð ; m; nÞ with l; m; n P 2, then dimf ðSt lð ;m;nÞÞ ¼ 4:

Proof. By using the basic technique of double inequality, to prove that the graph of starphene St lð ; m; nÞ has four fault-tolerant metric dimension. We refer to see the Lemma 3.3, in that proved we already showed that the potential candidate for the fault-tolerant resolving set Rf ¼ fa2l1; a2l; b2ðlþm1Þ; c2ðmþn1Þg, with four cardinality. one time for each loop, resulted in the same identifications are; rvfjR0f ¼ rvjjR0f , where vf;vj 2 fl acenearmverticesg.

Case 3: Let R0f fcf : f ¼ 1; 2;...; 2ðm þ n 1Þg, and containing three members one time for each loop, resulted in the same identifications are; rvfjR0f ¼ rvjjR0f , where vf 2 fl acenearmverticesg, and vj 2 fm acenearmverticesg.

Case 4: Let R0f faf; bj : f ¼ 1; 2;...; 2ðl þ n 1Þ; and j ¼ 1; 2;...; 2ðl þ m 1Þg, and containing three members one time for each loop, resulted in the same identifications are; rvfjR0f ¼ rvjjR0f , where vf;vj 2 fn acenearmverticesg.

Case 5: Let R0f faf; cj : f ¼ 1; 2;...; 2ðl þ n 1Þ; and j ¼ 1; 2;...; 2ðm þ n 1Þg, and containing three members one time for each loop, resulted in the same identifications are; rvfjR0f ¼ rvjjR0f , where vf;vj 2 fn acenearmverticesg.

Case 6: Let R0f fbf; cj : f ¼ 1; 2;...; 2ðl þ m 1Þ; and j ¼ 1; 2;...; 2ðm þ n 1Þg, and containing three members one time for each loop, resulted in the same identifications are; rvfjR0f ¼ rvjjRf0 , where vf 2 fl acenearmverticesg, and vj 2 fm acenearmverticesg.

All the above chosen cases are resulted in that there is no candidate for fault-tolerant resolving set with cardinality three and implied that the graph does not have dimf ðSt lð ; m; nÞÞ ¼ 3. Hence; dimf ðSt lð ; m; nÞÞ P 4.

Now by relating both inequalities, end up with conclusion that dimf ðSt lð ;m;nÞÞ ¼ 4:

Lemma 3.5. Let the graph of starphene is St lð ; m; nÞ with l; m; n P 2. Then the cardinality of edge metric resolving set of St lð ; m; nÞ is 3.

8><ðf 1; 2ðl þ nÞ 3 f; 2ðl þ m 1Þ fÞ;
r að fafþ1jReÞ ¼ ðf 1; 2ðl þ nÞ 3 f; 2ðm lÞ þ 1 þ fÞ;
>: if f ¼ 1;2;...;2l 2; if f ¼ 2l 1;2l;...; 2ðl þ nÞ 3:

Proof. Let Re ¼ fa1; a2ðlþn1Þ; b2ðlþm1Þg, from the vertex set of graph of starphne St lð ; m; nÞ, with cardinality three. Consider Re is one of the potential candidate for the role of edge resolving set. The identifications of the complete set of edges in St lð ; m; nÞ with regard to the nodes in Re are provided below.

For f ¼ 1; 2;...; 2ðl þ nÞ 3, the r a efafþ1jRe, are following;For Case 4: Let R0e faf; bj : f ¼ 1; 2;...; 2ðl þ n 1Þ; and j ¼ f ¼ 1; 2;...; 2ðl þ mÞ 3, the r b fbfþ1jR , are following; 1; 2;...; 2ðl þ m 1Þg, and containing three members one time

>8ðf; 2ðl þ m 1Þ f; 2ðl þ mÞ 3 fÞ; if f ¼ 1;2;...;2l 2;
r bð fbfþ1jReÞ ¼<ðf; 2ðn lÞ þ 1 þ f; 2ðl þ mÞ 3 fÞ; if f ¼ 2l 1;2l;...;

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>: 2ðl þ mÞ 3:

For f ¼ 1; 2;...; 2ðm þ nÞ 3, the r cð fcfþ1jReÞ, are following;

8><ð2ðl þ n 1Þ f; f; 2ðm þ n 1Þ fÞ;

r cð fcfþ1jReÞ ¼ ðf 2ðl nÞ þ 1; f; 2ðm þ n 1Þ fÞ;

>: if f ¼ 1;2;...;2n 2; if f ¼ 2n 1;2n;...; 2ðm þ nÞ 3:

For f ¼ 1; 3; 5;...; 2l 1, the r að fbfjReÞ, are following; r að fbfjReÞ ¼ ðf 1; 2ðl þ n 1Þ f; 2ðl þ m 1Þ fÞ:

For f ¼ 2l; 2l þ 2; 2l þ 4;...; 2ðl þ n 1Þ, and j ¼ 2ðl þ nÞ 1 f, the r a fcjjRe, are following; r a fcjjRe ¼ ðf 1; 2ðl þ n 1Þ f; 2ðm lÞ þ fÞ:

For f ¼ 2l; 2l þ 2; 2l þ 4;...; 2ðl þ m 1Þ, and j ¼ 2n; 2n þ 2;...; 2ðm þ n 1Þ, the r b fcjjRe, are following; r b fcjjRe ¼ ðf; f þ 1; 2ðl þ m 1Þ fÞ:

We can see that all the primary edges held unique identifications and met the idea of a edge resolving set by concluding that jRej ¼ 3, by looking at the identifications of the whole group of edges of St lð ; m; nÞ. h

Theorem 3.6. If the graph of starphene is St lð ; m; nÞ with l; m; n P 2, then dimeðSt lð ;m;nÞÞ ¼ 3:

Proof. To prove that the edge metric dimension of St lð ; m; nÞ is 3, we choose the double inequality method for dimeðSt lð ; m; nÞÞ 3, we are referring the Lemma 3.5 which is a candidate for the edge metric resolving set with cardinality three, it is taken as

Re ¼ fa1; a2ðlþn1Þ; bðlþm1Þg.

Now for dimeðSt lð ; m; nÞÞ P 3, by contradiction we get dimeðSt lð ; m; nÞÞ ¼ 2, for this, suppose there is a candidate of edge metric resolving set is R0e with cardinality 2. Given below are some discussion in the support of this assumption.

Case 1: Let R0e faf : f ¼ 1; 2;...; 2ðl þ n 1Þg, and containing three members one time for each loop, resulted in the same identifications are; r e fjR0e ¼ r e jjR0e, where ef; ej 2 fcentralbenzeneringedgesg.

Case 2: Let R0e fbf : f ¼ 1; 2;...; 2ðl þ m 1Þg, and containing three members one time for each loop, resulted in the same identifications are; r e fjR0e ¼ r e jjR0e, where ef; ej 2 fcentralbenzeneringedgesg.

Case 3: Let R0e fcf : f ¼ 1; 2;...; 2ðm þ n 1Þg, and containing three members one time for each loop, resulted in the same identifications are; r e fjR0e ¼ r e jjR0e, where ef; ej 2 fcentralbenzeneringedgesg.

for each loop, resulted in the same identifications are; r e fjR0e ¼ r e jjR0e, where ef; ej 2 fm acenearmedgesg.

Case 5: Let R0e faf; cj : f ¼ 1; 2;...; 2ðl þ n 1Þ; and j ¼ 1; 2;...; 2ðm þ n 1Þg, and containing three members one time for each loop, resulted in the same identifications are; r e fjR0e ¼ r e jjR0e, where ef 2 fcentralbenzeneringedgesg, and ej 2 fl acenearmedgesg.

Case 6: Let R0e fbf; cj : f ¼ 1; 2;...; 2ðl þ m 1Þ; and j ¼

1; 2;...; 2ðm þ n 1Þg, and containing three members one time for each loop, resulted in the same identifications are; r e fjR0e ¼ r e jjR0e, where ef 2 fcentralbenzeneringedgesg, and ej 2 fm acenearmedgesg.

Analogously, from the above discussion we can observe that we are unable to get a single candidate from the possible combinations which are jV St lð ð ;m;nÞÞjC2 ¼ 2!ðjjV St lV St lðð ðð;;mm;;nnÞÞÞjj! 2Þ! ¼ 2ð4ðð4lþðlþmmþþnÞnÞ6Þ8!Þ! of the set of all principle nodes of starphene graph St lð ; m; nÞ. This indicate that the edge metric dimension of St lð ; m; nÞ; 2 is not possible. Hence; dimeðSt lð ; m; nÞÞ P 3.

Now by relating both acquired inequalities, end up with conclusion that

dimeðSt lð ;m;nÞÞ ¼ 3:

Lemma 3.7. Let the graph of starphene is St lð ; m; nÞ with l; m; n P 2. Then the cardinality of fault-tolerant edge metric resolving set of St lð ; m; nÞ is 4.

Proof. Let Re;f ¼ fa2l; a2l1; b2ðlþm1Þ; c2ðmþn1Þg, from the vertex set of graph of starphne St lð ; m; nÞ, with cardinality four. Consider Re;f is one of the potential candidate for the role of edge faulttolerant resolving set. The identifications of the complete set of edges in St lð ; m; nÞ with regard to the nodes in Re;f are provided below.

For f ¼ 1; 2;...; 2ðl þ nÞ 3, the r a fafþ1jRe;f , are following;

>8ð2l f 2; 2l 1 f; 2ðl þ m 1Þ f; 2ðl þ mÞ 1 fÞ;

>>>>>>>>>>>if f ¼ 1;2;...;2l 2;

>ðf 2l þ 1; 2l 1 f; 2ðm lÞ þ 1 þ f; 2ðl þ mÞ 1 fÞ;

r a fafþ1jRe;f ¼ ><>>>>>>>>if f ¼ 2l 1;

>ðf 2l þ 1; f 2l; 2ðm lÞ þ 1 þ f; 2ðm lÞ þ fÞ;

:>>>>if f ¼ 2l;2l þ 1;...;2ðl þ nÞ 3:

For f ¼ 1; 2;...; 2ðl þ mÞ 3, the r b fbfþ1jRe;f , are following; Proof. Let Rm ¼ fa1; a2ðlþn1Þ; b2ðlþm1Þg, from the vertex set ofm

f f 8>>>ð2l f 1; 2l f; 2ðl þ mÞ f 3; 2ðl þ m 1Þ fÞ; is one of the potential candidate for the role of mixed resolvinggraph of starphne St lð ; m; nÞ, with cardinality three. Consider R

þ1 e;f <if f ¼ 1;2;...;2l 2; set. The identifications of the complete set of nodes in St lð ; m; nÞ

r b b jR ¼ >ðf þ 2 1ð lÞ; f 2l þ 3; 2ðl þ mÞ f 3; 2ðl þ m 1Þ fÞ; with regard to the nodes in Rm are provided below.

>:>if f ¼ 2l 1;2l;...;2ðl þ mÞ 3: For f ¼ 1; 2;...; 2ðl þ n 1Þ, the r að fjRmÞ, are following;

r að fjRmÞ ¼ ððff 11;; 22ððll þþ nn 11Þ Þ ff;; 21ðþl þfÞm; Þ 1 fÞ; ifif ff ¼¼ 12;l;22;l...þ ;12;l...;12;ðl þ n 1Þ:

For f ¼ 1; 2;...; 2ðm þ nÞ 3, the r c fcfþ1jRe;f , are following;

8ð2n f; 2n f 1; 2ðm þ n 1Þ f; 2ðm þ nÞ f 3Þ;

f f 1 e;f <>>>if f ¼ 1;2;...;2n 2;

r c c þ jR ¼ ðf 2n þ 3; f 2n þ 2; 2ðm þ n 1Þ f; 2ðm þ nÞ f 3Þ;

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>>>:if f ¼ 2n 1;2n;...;2ðm þ nÞ 3:

For f ¼ 1; 3; 5;...; 2l 1, the r a fbfjRe;f , are given below; r a fbfjRe;f ¼ ð2l f 1; 2l f; 2ðl þ m 1Þ f; 2ðl þ mÞ f 1Þ:

For f ¼ 2l; 2l þ 2; 2l þ 4;...; 2ðl þ n 1Þ, and j ¼ 2ðl þ nÞ 1 f, the

r a fcjjRe;f , are given below; r a fcjjRe;f ¼ ðf 2l þ 1; f 2l; 2ðm lÞ þ f; 2ðm lÞ þ f 1Þ:

For f ¼ 2l; 2l þ 2; 2l þ 4;...; 2ðl þ m 1Þ, and j ¼ 2n; 2nþ

2;...; 2ðm þ n 1Þ, the r b fcjjRe;f , are given below; r b fcjjRe;f ¼ ðf þ 2 1ð lÞ; f þ 2 1ð lÞ; 2ðl þ m 1Þ f; 2ðm þ n 1Þ fÞ:

We can see that all the primary edges held unique identifications and met the idea of a edge fault-tolerant resolving set by concluding that Re;f ¼ 4, by looking at the identifications of the whole group of edges of St lð ; m; nÞ. h

Theorem 3.8. Let the graph of starphene is St lð ; m; nÞ with l; m; n P 2. Then dime;f ðSt lð ;m;nÞÞ ¼ 4:

Proof. To show that the graph of starphene St lð ; m; nÞ has faulttolerant edge metric dimension 4, we are implementing the method of double inequality and implied at dime;f ðSt lð ; m; nÞÞ 4, which is already proven by the Lemma 3.7, it proved that there is a candidate for the fault-tolerant edge metric resolving set with cardinality four, it is taken as Re;f ¼ fa2l1; a2l; b2ðlþm1Þ; c2ðmþn1Þg.

Now for dime;f ðSt lð ; m; nÞÞ P 4, by contradiction we get dime;f ðSt lð ; m; nÞÞ ¼ 3, and by referring the Theorem 1.10 and

Theorem 3.6 concluded that 3 fault-tolerant edge metric dimension of St lð ; m; nÞ is not possible. Hence; dime;f ðSt lð ; m; nÞÞ P 4.

Now by relating both acquired inequalities, end up on the conclusion that

dime;f ðSt lð ;m;nÞÞ ¼ 4:

Lemma 3.9. Let the graph of starphene is St lð ; m; nÞ with l; m; n P 2. Then the cardinality of mixed metric resolving set of St lð ; m; nÞ is 3.

For f ¼ 1; 2;...; 2ðl þ m 1Þ, the r bð fjRmÞ, are following;

r bð fjRmÞ ¼ ððff;; 22ððlnþnlÞ Þ þ 11 þ ff;; 22ððll þþ mm 11Þ Þ ffÞÞ;; ifif ff ¼¼ 12;l;22;l...þ ;12;l...;12;ðl þ m 1Þ:

For f ¼ 1; 2;...; 2ðm þ n 1Þ, the r cð fjRmÞ, are following;

r cð fjRmÞ ¼ ðð22ððnl þnmÞ Þ þ11þf;f;f;f;fÞf;Þ; ifif ff ¼¼ 21m;2;;2...m;þ2m1;...1;;2ðm þ n 1Þ:

The above identifications are just the nodes identifications, to fulfill the definition of mixed we need the identifications of entire line segment set as well, as we know that Re ¼ Rm, mean that the cardinalities of both edge metric and mixed metric resolving sets are same, therefore, for the identifications of entire branches set we refer the Lemma 3.5.

We can see that all the primary edges and nodes as well held unique identifications and met the idea of a mixed resolving set by concluding that jRmj ¼ 3, by looking at the identifications of the whole group of edges and nodes of St lð ; m; nÞ. h

Theorem 3.10. Let the graph of starphene is St lð ; m; nÞ with l; m; n P 2. Then dimmðSt lð ;m;nÞÞ ¼ 3:

Proof. To show that the graph of starphene St lð ; m; nÞ has mixed metric dimension 3, by implementing the method of double inequality, and referring the Lemma 3.9 in which one of the candidate of mixed metric resolving set with cardinality 3 is given and it is taken as Rm ¼ fa1; a2ðlþn1Þ; b2ðlþm1Þg.

Now we will prove that dimmðSt lð ; m; nÞÞ P 3. On contrary we can see that the starphene is not a path graph (see Theorem 1.11) and using Theorem 1.12, it indicate that 2 mixed metric dimension of St lð ; m; nÞ is not possible. Hence; dimmðSt lð ; m; nÞÞ P 3.

Hence,

dimmðSt lð ;m;nÞÞ ¼ 3:

Lemma 3.11. Let the graph of starphene is St lð ; m; nÞ with l; m; n P 2. Then the cardinality of partition resolving set of St lð ; m; nÞ is 3.

Proof. Let Rp ¼ fRp1; Rp2; Rp3g, where Rp1 ¼ fa1g; Rp2 ¼ fa2ðlþn1Þg; Rp3 ¼ V St lð ð ; m; nÞÞ n fa1; a2ðlþn1Þg, of the vertex set of graph of starphne St lð ; m; nÞ, with cardinality three. Consider Rp is one of the potential candidate for the role of partition resolving set. The identifications of the complete set of nodes in St lð ; m; nÞ with regard to the nodes in Rp are provided below.

Table 1

Resolvability parameters of starphene St lð ; m; nÞ.

dim St lð ð ; m; nÞÞ ¼ 2 dimeðSt lð ; m; nÞÞ ¼ dimmðSt lð ; m; nÞÞ ¼ pd St lð ð ; m; nÞÞ ¼ 3 dimf ðSt lð ; m; nÞÞ ¼ dime;f ðSt lð ; m; nÞÞ ¼ 4

For f ¼ 1; 2;...; 2ðl þ n 1Þ, the r a fjRp, are following; r a fjRp ¼ ðf 1; 2ðl þ n 1Þ f; zÞ:

Where z ¼ 10;;   ifotherwisef ¼ 1; 2ð:1 þ n 1Þ; .

For f ¼ 1; 2;...; 2ðl þ m 1Þ, the r b fjRp, are following;

r b fjRp ¼ ððff;; 22ððnl þnlÞ þÞ 11 þ ff;; 00ÞÞ;;   ifif ff ¼¼ 211;;22;l...þ ;12;l...;12;ðl þ m 1Þ:

For f ¼ 1; 2;...; 2ðm þ n 1Þ, the r c fjRp, are following;

r c fjRp ¼ ðð22ððnl þnmÞ Þ þ11þf;f;f;f;0Þ0;Þ;   ifif ff ¼¼ 21m;2;;2...m;þ2m1;...1;;2ðm þ n 1Þ:

We can see that all the primary nodes as well held unique identifications and met the idea of a partition resolving set by concluding that Rp ¼ 3, by looking at the identifications of the whole group of nodes of St lð ; m; nÞ. h

Theorem 3.12. Let the graph of starphene is St lð ; m; nÞ with l; m; n P 2. Then pd St lð ð ;m;nÞÞ ¼ 3:

Proof. To show that St lð ; m; nÞ has the partition dimension which is 3. From Lemma 3.11 given above shows that there is a candidate of the partition resolving set with cardinality 3 and it is been taken as, Rp ¼ fRp1; Rp2; Rp3g, where Rp1 ¼ fa1g; Rp2 ¼ fa2ðlþn1Þg, and Rp3 ¼ V St lð ð ; m; nÞÞ n fa1; a2ðlþn1Þg. By using Lemma 3.11 and Theorem 1.11, it is concluded that pd St lð ð ;m;nÞÞ ¼ 3:

4. Conclusion

This article examines the structure of the starphene St lð; m; nÞ in terms of various resolvability parameters, particularly those that depend on a graph's metric. The first of these parameters is referred to as the metric dimension, and numerous generalizations are offered before arriving at the mixed metric dimension. Additionally, the partition dimension is a generalization of the original idea of resolvability parameters. Table 1 presents the conclusion observations based on the research work conducted.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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