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SOME FRIEDMANN- ROBERTSON-WALKER (FRW) MODELS

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***Abstract* :** We have attempted to explore some cosmological scenarios arising from the field equations with variable Λ (cosmological constant) taken in a way which conserves the matter tensor. From the field equations and the conservation equation, an equation is obtained in ρ, R, Λ, which points out two trivial solutions: $2\ddot{\rho}/\rho = 3\dfrac{\dot{\rho}^2}{\rho^2}$ with $\Lambda = \left(\dfrac{1+3\omega}{1+\omega}\right)\dfrac{k}{R^2}$ and $\dot{\rho} = 0$. The solutions of the field equations have been investigated for $\Lambda = \gamma/R^2$ where γ is a fixed pure number of the order of unity.

Introduction : In Einstein's field equations, there are two parameters, the cosmological constant Λ, and the gravitational constant G. The Newtonian constant of gravitation G, plays the role of a coupling constant between geometry any matter in the Einstein field equations. Numerous arguments have been proposed in the past few decades in which G varies with time, such as Bergmann[4], Dirac[7], Dreitlein[8], Gasperini[9], Hoyle[10, 11], Linde[14] and Wesson[16]. The cosmological constant Λ as a function of time has also been considered by several authors in various variable G theories by Bicknell[6] and Lau[13]. Several authors such as Beeshan[3] and Kalligas[12] have proposed to link the variation of G with that of Λ. This approach preverses the conservation of the energy momentum tensor and leaves the form of the Einstein field equations unchanged, Pradhan[15] presented cylindrically symmetric inhomogeneous cosmological models with viscous fluid and varying Λ. Bali and Rareek[2] have investigated Bianchi Type V magnetized string dust cosmological models with Pertov-type degenerate. In this paper our aim is to explore some cosmological scenarios arising from field equations with variable G and Λ taken in a way which conserves the matter tensor.

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The Field Equations : Let us consider an isotropic and homogeneous space time presented by Friedman –Robertson- Walker (FRW) metric together with perfect fluid. The Einstein field equation gives two independent equations with time dependent G and Λ as

$$\frac{\dot{R}^2}{R^2} = \frac{8\pi G(t)}{3}\rho + \frac{\Lambda(t)}{3} - \frac{k}{R^2}, \tag{1}$$

$$\frac{\ddot{R}}{R} = \frac{4\pi}{3}(1+3\omega)\rho G(t) + \frac{\Lambda(t)}{3}, \tag{2}$$

where we have taken an equation of state p = ωρ, ω = constant. In view of equations (1) and (2), we obtain

$$G\dot{\rho} + 3(1+\omega)\rho G\frac{\dot{R}}{R} + \rho\dot{G} + \frac{\dot{\Lambda}}{8\pi} = 0 \tag{3}$$

The law of conservation gives.

$$\dot{\rho} + 3(1+\omega)\rho\frac{\dot{R}}{R} = 0. \tag{4}$$

Hence, we obtain

$$\dot{G} = \frac{\dot{\Lambda}}{8\pi\rho}. \tag{5}$$

Integrating eq. (4), one obtains

$$\rho = CR^{-3(1+\omega)},\ C = \text{Constant} > 0. \tag{6}$$

Thus, we get

$$G(R) = G_0 + \frac{1}{8\pi c}\left(\Lambda_0 R_0^{3(1+\omega)}\right) - \frac{1}{8\pi c}\Lambda(R)R^{3(1+\omega)}$$

$$+\frac{3(1+\omega)}{8\pi c}\int_{R_0}^{R} R' R'^{(2+3\omega)}\, d\dot{R}, \tag{7}$$

where the subscript zero denotes for the value of the quantity at t =0. If we put ω = -1, we get $\dot{\rho} = 0$, implies ρ = constant. If we consider $\dot{\rho} \neq 0$ we obtain

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$$\frac{2\ddot{\rho}}{\rho}-\frac{3\dot{\rho}^2}{\rho^2}=3(1+\omega)^2\left[\frac{(1+3\omega)k}{(1+\omega)R^2}-\Lambda\right],\omega\neq-1. \tag{8}$$

Now R(t) must satisfy the differential equation

$$\frac{2}{1+\omega}\dot{H}+3H^2\frac{(1+3\omega)}{(1+\omega)}\frac{k}{R^2}-\Lambda=0, \tag{9}$$

where H = $\dot{R}/R$ be the Hubble parameter.

A Trivial Solution : A trivial solution of eq. (8) reads

$$\frac{2\ddot{\rho}}{\rho}-3\dot{\rho}^2/\rho \qquad \text{with} \quad \Lambda=\frac{1+3\omega}{1+\omega}\frac{k}{R^2}. \tag{10}$$

Several authors have suggested the from of Λ as Λ = Const./R^2. However in the present case the constant assumes different values in different phases, for example, in radiation-dominated phase ω = 1/3, in matter-dominated phase ω = 0. Again, we obtain

$$2\dot{H}+3(1+\omega)H^2=0, \tag{11}$$

which gives solution

$$H=\frac{2}{3(1+\omega)}\frac{1}{(t+constant),} \tag{12}$$

and hence,

$$R=(mt)^{\frac{2}{3}(1+\omega)}$$

where m be the constant of integration, with R = 0 at t = 0. is obvious that R is independent of k. R → ∞. This is a significant deviation from the standard model. However, Λ >0, required a spatially compact universe. One may evaluate the time t= t_{cau}, when the whole universe becomes causally connected.

Hence, we get

$$G=G_0+\frac{k}{4\pi c(1+\omega)}(mt)^{2(1+3\omega)/3(1+\omega)} \tag{14}$$

here we obtain

$$m = (1+\omega)\sqrt{6\pi c G_0}, \qquad (15)$$

for self consistency of the system. It is obvious that G has finite non-zero value G0 initially. It goes on increasing as t in RD phase and then $t^{2/3}$ in MD phase with $\dot{G} \to 0$ as t $\to\infty$. It is interesting to note that the deceleration parameter q = - $\ddot{R}/RH^2$ is constant in this model. i.e.

$$q = \frac{1+3\omega}{2}, \qquad (16)$$

showing that q = 1 and ½ in RD and MD phases respectively.

Solution for $\Lambda = \gamma/R^2$:For this given value of Λ, one obtains

$$G = G_0 + \frac{\gamma}{4\pi c(1+\omega)}\left[R^{(1+3\omega)} - R_0^{(1+3\omega)}\right] \qquad (17)$$

$$H = \left[\left\{\left(\frac{1+\omega}{1+3\omega}\right)\gamma - k\right\}\frac{1}{R^2} + \frac{A}{R^{3(1+\omega)}}\right]^{\frac{1}{2}} \qquad (18)$$

where A be constant of integration

$$A = \frac{8\pi c}{3}\left[G_0 - \frac{\gamma}{4\pi c(1+3\omega)}R_0^{(1+3\omega)}\right] \qquad (19)$$

Now, we obtain

$$\ddot{R} = -\frac{(1+3\omega)A}{2}R^{-(2+3\omega)} \qquad (20)$$

which shows that $\ddot{R} \gtrless 0$ accordingly as $\ddot{A} \gtrless 0$ It is obvious that for non-zero γ and R_0, one may select A < 0 to avoid $\ddot{R} \leq 0$ and hence the initial singularity. For A = 0, which gives a linearly expanding universe.

$$R = \left[\left(\frac{2}{3}\gamma - k\right)(t+\beta)^2 - A\left(\frac{2}{3}\gamma - k\right)^{-1}\right]^{\frac{1}{2}}, \qquad (22)$$

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where β as the constant of integration. Case (1) For $R_0 \neq$ and $H_0 = 0$, we get $c = \rho_0 R_0^4$, $A = -\left(\frac{2}{3}\gamma - k\right)R_0^2$, indicating that $\gamma > \frac{2}{3}k$ for expansion. For γ = 3, we recover the model by Abdel – Rahman[1], Case (ii) $R_0 = 0$, we get $A = \left(\frac{2}{3}\gamma - k\right)^2 \beta^2$. For A > 0, we get

$$R = \left[\frac{3\sqrt{A}}{2}t + B_3\right]^{2/3} \tag{23}$$

where B_3 be the constant of integration. Hence, in MD phase it is described by equation (23) depending upon the values of k and γ.

***Solutions for Constant Density* :** For ρ = constant = ρ_c, we get

$$(1+\omega)\dot{R} = 0, \tag{24}$$

showing three possibilities (1) ω = = -1 with R = R (t) (ii) R = constant = R_c with ω ≠ -1 (iii) R = R_c with ω = -1. In view of the above, we get

$$\ddot{R} = \frac{8\pi\rho_c D}{3}R \tag{25}$$

where

$$D = G + \frac{\Lambda}{8\pi\rho_c} = G_0 + \frac{\Lambda_0}{8\pi\rho_c} \tag{26}$$

showing a inflation for Λ > 0 and G > 0, and we obtain

$$R = R_0 \exp\left[\sqrt{\frac{8\pi\rho_c D}{3}}t\right] \tag{27}$$

showing classical inflation as obtained by Kalligas[12]. For static universe with R = R_c, we obtain

$$\Lambda = \frac{(1+3\omega)}{(1+\omega 0)R_c^2}, \qquad G = \frac{k}{4\pi(1+\omega)\rho_c R_c^2}, \tag{28}$$

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Concluding Remarks : We have presented two trivial solutions: $2\ddot{\rho}/\rho = 3\dot{\rho}/\rho^2$ with $\Lambda = \left(\frac{1+3\omega)}{(1+\omega}\right)\frac{k}{R^2}$ and $\dot{\rho} = 0$. The case with $\Lambda = \left(\frac{1+3\omega)}{(1+\omega}\right)\frac{k}{R^2}$ yields a model starting from by bang with finite G_0 and has a phase wise constant deceleration parameter q. The model reduces to standard model for k = 0. We recover the Berman[5] result for q = 1/2 in the present phase of evolution. We have also investigated for $\Lambda = \lambda/R^2$. In special case, a linearly expanding model is obtained which is free from horizon problem. We recover the model of Abdel- Rahman[1] with ω = -1.

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