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A Note on Existence of Ψ-BoundedSolutionsfor Semi-linearDifference Equations

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***Abstract*—**The main object ofthis paper is develop the if and only if conditions for existence of unique ψ – bounded solution for non-linear semi difference equation y(n+1)=A(n)y(n)+B(n), by using the concept of Banachcontractionprinciple on continuous function.

Keywords-ψ-bounded;existenceanduniqueness;Banachcontractionprinciple

1. Introduction

In this paper we develop the only if condition for existence of unique ψ- bounded solution for semi-linear difference equation

$$Y(n+1)= Y(n)+a(n)+g(n,t) \tag{1.1}$$

where $A \in R^{n\times n}$,g $\in R \times R^n$and g(n, 0)=0,and a(n) isanybounded,continuousfunction.HeregisacontinuousmatrixfunctiononR.The existence and unique of Ψ-bounded solutions for system of differential equations has been discussed by many authors [1-10] . From last four decades onwards the existence of Ψ-bounded solutions for system linear and nonlinear difference equations is studied by different authors [11-17]. But, existence of Ψ-bounded solutions for system of semi linear difference was not yet studied. So, for that reason here we studied the develop of Ψ-bounded solution for semi-non linear difference system.

The concept of semi non-linear difference system plays very important rule in many areas like mathematical modeling, control system, Numerical system, data science analysis, probability and stochastic process.

2. PRELIMINARIES

Throught this paper, we Consider.

i) R^n as the Eucludian n-space For y= ($y = (y_1, y_2 - - - -) \in R^n$

ii) Consider, $\|y\| = max\{y_1,\ y_{2,-----}\}$, corresponding to $y = (y_1,\ y_2, y_{3,}, - - - - -) \in R^n$

iii) Consider the norm of a square matrix M =(m_{ij}) as $\|M\| = \sup_{\|y\|\leq 1}\|My\|$

iv) Consider $\Psi_i: R \to (0, \infty), for i = 1,2,3,4$ be a function and take $\Psi = diag(\Psi_1, \Psi_2, - - - -)$, then Ψ(t) is nonsingular matrix.Clearly, we got Ψ(n) is nonsingular matrix function on R.

Definition (2.1): [10]: A mapping $\varphi: R \to R^n is$called Ψ-bounded on R if Ψ**φ** is bounded on R.

We definethenorm$||A||= \sup_{||x||\leq 1}||Ax||$.Itiswell-knownthat |A|= $\max_{1\leq i\leq n}\sum_{j=i}^{n} |aij|$.

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Definition (2.2.)[10]The fractional order sum operator of order η is defined as

$$\nabla^{-\vartheta} v(t) = \sum_{i=0}^{t-1} \left(\binom{i+\beta-1}{i} \right) v(t-i)\, v(t-i) = \sum_{i=1}^{t} \binom{t-i+\beta-1}{t-i} v(i)$$

Similarly, the fractional order difference operator of order ϑ is defined as

$$\nabla^{\vartheta} v(x) = \sum_{i=0}^{t-1} \binom{i-\beta}{i} \nabla v(t-i) = \sum_{0}^{t} \binom{t-i-\beta-1}{t-i} u(i) - \binom{t-\beta-1}{t-1} v(0)$$

Throughout this paper we assume that, $\beta \in R$, $0<\beta<1$

Definition (2.3)[2]: Let g(t, n)be the function defined for t in positive integers, $0<n<\infty$ Then $\nabla^{\vartheta} v(t+1) = g(t, v(t)), with v(0) = 0$, is taken as nonlinear difference equation with order ϑ

Definition (2.4)[3]: For the following fractional difference system

$$\nabla^{\vartheta} v(t+1) = g(t, v(t)), with v(0) = 0$$

Lemma2.1 Consider X(n) is an nonsingular matrix on R and let Q be a projection If $\varphi: R \to (0,\infty) and consider nonnegative constant N saties fies \sum_0^\infty \varphi(n)\Psi(n)Y(n)QY^{-1}(n)\Psi^{-1}$(n), then a constant M satisfies $|\Psi(n)X(n)Q| \leq Me^{-\sum_0^\infty n}$, further $\lim_{n\to\infty}|\Psi(n)X(n)Q|<1$

3. Main Result

Theorem3.1:

Assume the supplementary images Q_{-1} and Q_1 and a nonnegative constant L satisfies $\sum_{-}^{t}|\psi(n)X(n)Q_{-1}X^{-1}(t)\psi^{-1}(t)| + \sum_0^\infty|\psi(n)X(n)Q_1X^{-1}(t)|\,\psi^{-1}(t) \leq M$. and assume that g(x, n) be a function satiesfies $\|\psi(n)[g(n,t) - g(n,x)]\| \leq \beta\|\psi(n0(y-x)\|\; for -\infty < t < \infty$, $\|\psi(t)\| \leq \mu, \psi(t) \leq \mu.$,

Now, if c(n) is any bounded mapping satisfies $|c| \leq \frac{\mu(1-cm)}{m}$, then x(t) is a unique bounded solution under the condition $\|\psi(y)\| \leq \mu$

Proof: By using above lemma, the condition $|\Psi(n)X(n)Q_{-1}\tau|$ is unbounded for n>0 and bounded for n<0 Hence the solution of (11) is unbounded on R.

Define $B_\Psi = \{y: R \to R: y saties fies \Psi - unbounded and continuous on R saties fies \|\Psi x\| \leq \tau\} and \|\Psi\| = sup\|\Psi(n)y(n)\|$

Consider T be a function satisfies

$$Ty(n) = \sum_{-\infty}^{t} x(n)Q_{-1}\, x^{-1}(t)\{a(t) + g(t, y(t))\} - \sum_{t}^{\infty} X(n)Q_1\, X^{-1}(t)\{a(t) + g(t, y(t))\}$$

Take

$\|\Psi Ty(n)\| =$

$|\sum_0^\infty \Psi(n)X(n)Q_{-1}\, X^{-1}(t)\Psi^{-1}(t)\Psi(t)\{a(t) + g(t, y(t))|\} -$

$\sum_0^{t-1} \Psi(n)X(n)Q_{-1}\, X^{-1}(t)\Psi^{-1}(t)\Psi(t)\{a(t) + g(t, y(t))\} \leq \sigma(1-\beta L) + L\beta\|\Psi(n)Y(n)\| \leq \sigma$

It gives $TY(n) \in d_\Psi$ $and\ got\ T: d_\Psi \to d_{\Psi^i}, it\ gives\ \ T\ is\ a\ contraction\ function\ , which\ stiesfies$

$$\|\Psi(Ty - Tx)\| = \left\| \sum_{0}^{t-1} \Psi(n)X(n)Q_{-1}X^{-1}(t)\Psi^{-1}(t)\Psi(t)\{a(t) + g(t, x(t))\} - \sum_{t-1}^{\infty} \Psi(n)X(n)Q_1\, X^{-1}(n)\Psi^{-1}(n)\Psi(n)\{a(t) + g(t, y(t))\} \right\|$$

$TY(n) = \sum_{-\infty}^{n} X(n)Q_{-1}\, X^{-1}(t)\{a(t) + g(p, t(p)\}$-$\sum_n^\infty X(n)Q_1X^{-1}(t)\{a(t) + g(t, x(t)\})$

$Now\ by\ taking\ \|\Psi TY(n)\| =$

$\left|\sum_{-\infty}^{n} \Psi(n)X(n)Q_{-1}X^{-1}(t)\Psi^{-1}(t)\Psi(t)\{a(t) + g(t, y(t))\} - \sum_n^\infty \Psi(n)X(n)Q_1\Psi^{-1}(t)\Psi(t)\{a(t) + g(t, Y(t)\}\right| \leq$

$\left[\sum^n |\Psi(t)X(n)Q_{-1}X^{-1}(t)\Psi^{-1}(t)| + \sum\ |\Psi(n)X(n)Q_1X^{-1}(t)\Psi^{-1}(t)|\right]\|\Psi(t)\{a(t) + g(t, y(t))\}\| \leq$

$\tau(1-\beta L) + L\beta\|\Psi(t)Y(t)\| \leq \tau(1-\beta L) + L\beta\|n\| \leq \tau$

This gives $TY(n) \in C_\Psi$ and T: $T: C_\Psi \to C_\Psi$

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Now our next aim about Contraction mapping T on B_{Ψ}

Assume

$\|\Psi(Ty - Tx\| =$

$\left\|\sum_{-\infty}^{n} \Psi(t)X(n)Q_{-1}X^{-1}(t)\Psi^{-1}(t)\{a(t) + g(t, y(t))\}\right\| -$

$\sum_{n}^{\infty} \Psi(n)X(n)Q_1X^{-1}(t)\Psi^{-1}(t)\Psi(t)\{a(t), g(t, y(t))\} - \sum \ \Psi(n)X(n)Q_1X^{-1}(t)\Psi^{-1}(t)\Psi(t)\{a(t) + g(t, x(t))\}$

$$\leq \left[\sum_{0}^{t-1} \Psi(n)X(N)Q_{-1}X^{-1}(t)\Psi^{-1}(n) + \sum_{t-1}^{\infty} \Psi(n)X(n)Q_1X^{-1}(t)\Psi^{-1}(t)\right] \|\Psi(t)[g(t, y(t)) - g(t, x(t))]\|$$

$$\leq L\beta\|y - x\|_{\Psi}$$

It follows that $\|T_y - T_x\| \leq \|y - x\|$

Hence T satisfies contraction definition on Banach contraction principle , T gives one and only one fixed point on R.

So, the semi linear difference system (1) has one and only one unique fixed point for $\|\Psi y\| \leq \tau$

In other hand, if (1.1) is satisfied by y(n) under the condition $\|\Psi(y)\| \leq \tau$, x is Ψ-bounded solution for the system 1.1.

Theorem3.2: If

There are supplementary projections Q-1 and Q1 ,non negative constant L satisfies

$$|\Psi(n)X(n)Q_{-1}X^{-1}(t)\Psi^{-1}(t)| \leq L_1 e^{-\beta(n-t)} for t \leq n$$
$$|\Psi(n)X(n)Q_1X^{-1}(t)\Psi^{-1}(t)| \leq L_2 e^{-\alpha(t-n)} under n \leq t$$

Here L1, L2 , α and β are nonnegative constants. The function g(n, y(n)) satisfies $\|\Psi(n)g(n, y(n))\| \leq \vartheta\|\Psi(n)Y(n)\|$. Then the nonlinear semi difference equation (1.1) has atleast one Ψ-bounded solution on set of real numbers

Proof: By using Tychnoff fixed point theorem on continuous function, we prove this theorem

Aim (1): Take $\|\Psi(Ty_n(n) - TY(n))\| =$

$$\left\|\sum_{0}^{n-1} \Psi(n)X(n)Q_{-1}X^{-1}(t)\Psi^{-1}(t)\Psi(t)\{a(t) + g(t, y_n(t)\}\right\| - \sum_{n-1}^{\infty} \Psi(n)X(n)Q_{-1}X^{-1}(t)\Psi(t)\{a(t) + g(t, y_n(t)\}$$
$$- \sum_{0}^{n-1} \Psi(n)X(n)Q_1X^{-1}(t)\Psi^{-1}(t)\Psi(t)\{a(t) + G(t, y(t))\}$$
$$\leq L_1\rho \sum_{n-1}^{\infty} e^{-\beta(t-n)} \|\Psi(n)[y_n(t) - y(t)]\| + L_2\sigma \sum_{n-1}^{\infty} e^{-\alpha(t-n)} \|\Psi(n)[y_n(t) - y(t)]\| But y_n(t)$$
$$\to y(t), it gives that T y_n(t) \to y(t)$$

Aim(2): Assume TG is bounded uniformly

Take $\|\Psi Ty(n)\| \leq L_1 \sum \ e^{-\beta(n-t)} \|\Psi(t)\|\Psi(n)\{a(t) + g(t, y(t)\} + L_2 \sum \ e^{-\alpha(n-t)} \|\Psi(n)\{a(t) + g(t, y(t))\}\| \leq \rho\sigma\left(\frac{L_1}{\alpha} + \frac{L_2}{\beta}\right) < \rho$

From above discussion it follows that Ty(n) is a solution for nonlinear semi difference equation and it is bounded So non-linear semi difference equation has Ψ-bounded solution which is unique.

.Now for system (1.1) fixed point exists

Reversely, assume that ,ifu(t) satiesfies(1.1)under the condition|| f x||≤ ρthen$y = x - Tx$isa f −satiesfies(1.2),therefore$y$=0.

Theorem 3.3: Let B be a banach space and P be bounded closed and convex subset of B , then the operator., satisfies following hypothesis

L1: The mapping s to g(n, y, x) satisfies measurable property on I for y, x in banach space and the mappings y to g(n, y,x) and x to g(n,y,x) are continuous on banach space for almost everywhere

L2: There exists mappings q: Z to Positive real numbers

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satiesfies$|g(n,y,x| \leq \frac{q(n)}{1+|x+y|}$ $\boldsymbol{for\ almost\ everywhere}$, then solution of (1) is attractive locally

Proof: For any y in Banach space B, derive the operator

$\tau\ satiesfies\ (\tau y)n = \frac{d_1}{\Gamma(\beta)}\left(\frac{n^\rho - t^\rho}{\rho}\right) + \sum_a^n p^{\rho-1}\left(\frac{n^\rho - t^\rho}{\rho}\right)\frac{f(t)}{\Gamma(\beta)}$

Clearly, the function $\Delta(y)$ is continuous on set of integers for any y in Banach space satisfies

$$\left|\left(\frac{n^\rho - b^\rho}{\rho}\right)^{1-\beta}(\Delta y)(n)\right| \leq \frac{|d_1|}{\Gamma(\beta)} + \left(\frac{n^\rho - t^\rho}{\rho}\right)^{1-\beta}\sum_n^t t^{\rho-1}\left(\frac{n^\rho - t^\rho}{\rho}\right)^{\beta-1}\frac{|f(t)|}{\Gamma(\beta)}$$

$$\leq \frac{|d_1|}{\Gamma(\beta)} + \left(\frac{n^\rho - t^\rho}{\rho}\right)^{1-\gamma}\left(\rho J_\beta^\beta\right)(n)$$

Proof:

Let $\{y_n\}$ be a sequence such that $y_n \to y$ in banach space. Then, for every n in I, we got $\left|\left(\frac{s^\rho - a^\rho}{\rho}\right)^{1-\beta}(\Delta y_n)(n) - \left(\frac{s^\rho - y^\rho}{\rho}\right)^{\rho-1}\right| \leq \frac{\left(\frac{\frac{n^\rho}{\rho}}{\Gamma(\beta)}\right)^{1-\beta}}{\rho} \leq \frac{(\alpha-\beta)^\rho}{\rho}\sum_n^\infty t^{n-1}\left(\frac{n^\rho - t^\rho}{\rho}\right)^{\rho-1} q(t)$

$$\left|\left(\frac{n^\rho - t^\rho}{\rho}\right)^{\rho-1}(\Delta y)(t) - \left(\frac{(n-1)^\rho - t^\rho}{\rho}\right)^{\rho}\right| \leq \frac{1}{\Gamma(\beta)}\sum_\beta^n y^\rho\left(\frac{n^\rho - t^\rho}{\rho}\right)^{n-1}\rho(t)$$

$$we\ got\ \|\Delta(x) - y_0\| \leq 2p$$

Moreover, if y is solution of IVP

$$|y(n) - y_0(n)| \leq \frac{\left(\frac{s^\rho - t^\rho}{\rho}\right)^{1-\beta}}{\left(\frac{\rho}{\alpha}\right)^{1-\beta}} Consequently, all solutions of (1) are locally compact$$

Theorem 3.4: The homogeneous linear non autonomous difference equation with order β is obtained by $\nabla^\beta v(n+1) = c(mn)v(n) + a(n) with\ initial\ condition\ v(0) = v_0$ with the corresponding nonhomogeneous equation is given by $\nabla^\beta v(n+1) = b(t)v(n) + a(n), with\ initial\ condition\ v(0) = v_0$

Proof: Now here we find the solutions for the above system -1

$$v(n) = v(0)\sum_0^{n-1}[1 + bA(n-1,b;i)] + a\sum_0^{n-1}A(t-1,\beta;i)\sum_0^{n}[1 + A(n-1,\beta;l)b]$$

$$= v(0)\sum_0^{n-1}[1 + b\beta(n-1,\beta,j)] + \frac{a}{b}\left[\sum_0^{n-1}[1 + bA(t-1,\beta;i)]\right]$$

$$= \left[v(0) + \frac{a}{b}\right]\sum_0^{n-1}[1 + aA(n-1,\beta,i) - 1] - \frac{a}{b}$$

Hence the proof.

Results: In this paper, we developed the conditions for existence of solutions for semi-linear difference equations under certain conditions..these results plays important role for developing the existence and uniqueness of ψ-bounded solutions for non-linear semi difference equations.

Conclusions: The semi-Linear difference equations plays very important role in more fieds like

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Data structures, probability and stochastic process and dynamical systems. So, for that reason here we developed the conditions for existence the solution of nonlinear difference system. This work will be helpful for developing the existence of ψ-bounded solution for semi-linear difference system.

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