

## A FINITE ELEMENT STUDY OF MHD VISCOUS INCOMPRESSIBLE FLUID FLOW IN A POROUS MEDIUM IN COAXIAL CYLINDERS

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### ABSTRACT:

This paper presents a finite element Galerkin method for simulating the flow of a viscous, incompressible fluid through a porous medium established between two coaxial cylinders. The governing equations are solved numerically, and the temperature and velocity profiles of the flow are obtained. The results show that the Galerkin finite element method improves the accuracy of the temperature and velocity profiles, compared to other numerical methods. The study has implications for the areas of fluid mechanics and applied mathematics. The results can be used to design and optimize devices that use porous media to control the flow of fluids. The study also provides insights into the applied physics of fluid flow through porous medium.

**KEY WORDS:** Galerkin's method, porous medium, viscous flow, incompressible fluid, temperature profile.

**MSC classification 2020 :** 76D55, 76M10

### 1. INTRODUCTION:

The study of viscous incompressible fluid flow through a porous medium between two coaxial cylinders is of interest to many researchers because it has applications in many engineering and technological fields, including geophysics, astrophysics, and boundary layer control in the field of aerodynamics. The applications of magnetohydrodynamics are numerous. Sodium chloride and induction flow measurements can be used to cool nuclear reactors, based on the potential difference in the fluid in the direction perpendicular to the motion. The convection is a significant mechanism that affects a wide range of environments, like the cooling of electronic

circuit boards in computers, large-scale atmospheric circulation, and the climatic effects of lakes and oceans. It is caused by the effect of density gradients in conjunction with gravitational field. Free convection flows in tubes and channels with permeable walls have been the focus of extensive study.

Researchers are increasingly using heat energy in their pursuit for the finest heat transmission mechanism. Problems with various boundary conditions, geometries, nanofluids, magnetohydrodynamic (MHD) applications, hybrid nanofluids, etc. are taken into consideration. Mixed convection problem is formed with the help of natural convection (buoyancy force) and forced convection.

In his research on the magnetohydrodynamic flow between rotating co-axial disks, Stephenson (1969) used a model. Laminar free convective flow through coaxial circular pipes was examined by Gupta et al. in 1979 with and without heat sources. Varshney (1979) convinced fluid to flow in an irregularly shaped conduit through an unstable MHD medium. In their 1979 study, Rath and Jena looked at fluid fluctuations between two coaxial cylinders. A conducting, viscous, incompressible fluid was encouraged to flow through porous media in an equilateral triangular tube by Gupta and Sharma (1981). In their 1981 study, Pathak and Upadhyay looked at the stability of a dusty flow between two revolving coaxial cylinders.

In a partially filled, rapidly rotating cylinder with a differentially rotating end cap, Shadday et al. (1983) investigated the flow of an incompressible fluid. In their study from 1989, Pillai et al. covered the flow of a conducting fluid between two coaxially rotating porous cylinders enclosed in a permeable bed. In a rotating coaxial rheometer system using bingham plastic, Javadpour and Bhattacharya (1991) discussed an axial flow. In porous coaxial circular cylinders, constant flow of an elastico-viscous fluid was studied by Gupta and Gupta in 1996. The inconsistent heat transfer of a monatomic gas between two coaxial circular cylinders has been studied by Abourabia et al. in 2002. In a coaxial cylinder, Ratnam and Malleswari (2004) looked at convection flow through a porous material. Krishna and Rao (2005) investigated the viscous flow through a porous media in a triangular duct using finite element analysis. In coaxial cylinders, Nobre et al. (2006) investigated the effects of surfaces on the propagation of energy through optical modes.

In the presence of heat radiation, Mazumdar and Deka (2007) investigated MHD flow across an impulsively begun infinite vertical plate. Finite element and boundary element contact stress

analysis with remeshing approach was covered by Oysu (2007). The numerical solution to the MHD flow of micropolar fluid between two concentric porous cylinders was studied by Srinivasacharya and Shifera in 2008. The changing free convection flow through heated horizontal circular cylinders was studied by Hossain et al. in 2009. Makinde et al. (2009) investigated MHD viscous flow in a rotating porous medium cylindrical annulus. Loganathan et al. (2010) examined the impact of MHD on a moving semi-infinite vertical cylinder with heat and mass transmission. Ahmed and Sarmah (2011) explored the MHD transient flow across an impulsively begun infinite horizontal porous plate in a rotating system with hall current. Kumar et al. (2011) produced a Crank-Nicholson system to transiently MHD free convective flow through a semi-infinite vertical porous plate with constant suction and a temperature dependent heat source. A reliable MHD steady flow via channels with porous borders was investigated by Kumar et al. in (2010). Kumar et al. (2010) used a porous effect with a changeable permeability value to create MHD free convective fluctuations flow. In their 2014 study, Kaur et al. looked at the analysis of heat transfer in hydromagnetically rotating flows of viscous fluid through non-homogeneous porous media with constant heat source/sink. Dzikowski et al. (2018) investigated depth-averaged lattice Boltzmann and finite element methods for single-phase flows in fractures with obstacles. In their study published in 2020, Agarwal and Kumar looked at the impact of viscous dissipation on MHD unsteady flow over a vertical porous media with constant suction. A mathematical model for both Newtonian and non-Newtonian flow was examined by Kumar and Verma in 2021, along with its applications. Kumar (2022) a computational work with applications that profounded a periodic on convective heat transmission via a semi-infinite vertical porous wall. Onyanha et al. (2023) Optimization of magnetohydrodynamic parameters in two- dimensional incompressible fluid flow on a porous channel.

In the current study, we have examined the free and forced convection of viscous fluid flow through coaxial porous effect cylinders while accounting for viscous dissipation. Using Galerkin's method, the governing equations have been resolved. The fluid flow's velocity and temperature profiles are generated numerically, and graphs are used to illustrate how they behave for various values of the governing parameters.

## 2. Formulation of the problem:

In the current study, we are taking into consideration a fully developed constant laminar free and forced convective flow of a viscous incompressible fluid through a porous material in between two vertical coaxial cylinders. Let  $(r^*, \varphi^*, z^*)$  be the cylindrical coordinate axes such that  $r^* = a$  and  $r^* = b$  are the radii of inner and outer cylinders respectively. Assuming that the pipes are long enough so that all the physical quantities are independent of  $\varphi^*$  and  $z^*$ . Based on the motion's rotational symmetry, the azimuthal velocity is zero. The governing equations are given below in non-dimensional form:

$$\frac{d^2 w}{dr^2} + \frac{1}{r} \frac{dw}{dr} = P + G_r \theta - \frac{w}{D} - Kw - Mw, \quad \dots (1)$$

$$\frac{d^2 \theta}{dr^2} + \frac{1}{r} \frac{d\theta}{dr} + h \left( \frac{dw}{dr} \right)^2 = 0, \quad \dots (2)$$

where  $r = \frac{r^*}{a}$ ,  $w = \frac{aw^*}{v}$ ,  $P^* = P_D + P_s$ ,  $\theta = \frac{T^* - T_0}{T_1 - T_n}$ ,  $P = \frac{a^3}{\rho v^2} \left( \frac{-\partial P^*}{\partial z^*} \right)$ ,  $w$  is the non-dimensional velocity component along  $z$ -axis,  $\rho$  the density of the fluid,  $P$  the fluid pressure,  $T$  the non-dimensional temperature of the fluid,  $\mu$  the coefficient of the viscosity,  $C_p$  the specific heat at constant pressure,  $\nu$  the kinematic viscosity,  $K$  the permeability of porous medium,  $k$  the coefficient of thermal conductivity,  $\rho_0$  the equilibrium density,  $T_0$  the equilibrium temperature,  $P_s$  the equilibrium pressure and  $P_D$  the dynamic pressure.

$$G_r = \frac{g\beta(T_1 - T_0)}{\nu^2} \text{ (Grashoff number)}$$

$$h = \frac{\mu v^2}{k(T_1 - T_n)} \text{ (Eckert number)}, \quad D^{-1} = \frac{a^2}{K} \text{ (Darcy's parameter)}$$

and  $P = -\frac{\partial p}{\partial z}$  (Pressure gradient).

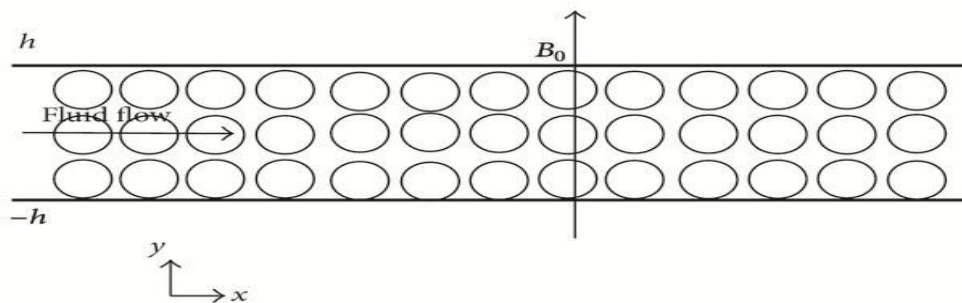


Figure 1. Physical model

The hydrostatic balance equation as given below

$$-\frac{\partial p_s}{\partial z} - \rho_0 g = 0, \quad \dots (3)$$

The appropriate corresponding boundary conditions are:

$$w(1) = w(s) = 0, \theta(1) = 0, \theta(s) = 1, \text{ where } s = \frac{b}{a}. \quad \dots (4)$$

### 3. Computational Method:

The software Mathematica (version-11) has been used to solve the dimensionless partial differential equations employing Galerkin's technique. The finite element method of Galerkin's approach has been utilized to resolve equations (1) and (2) under boundary conditions (4). The solution of second-order differential equations is proposed employing an efficient iterative technique. This method steps a solution to a single second-order differential equation using the finite element technique as a numerical tool. By reducing the need to solve two second-order differential equations simultaneously or a nonlinear second-order differential equation, the method is more efficient in comparison to those that do. The weight function and approximation function are equal in Galerkin's technique. Suppose domain  $(r_a, r_b)$  of the radial direction is divided into  $n$  subintervals and length of each interval is  $h_s = (r_b - r_a)/n$ , where  $s = r_b/r_a$  (the gap duct between two coaxial cylinders).

The Galerkin's method is standard form as defined by

$$\int_{\Omega} \varepsilon \varphi_i d\Omega = 0, \quad \dots (5)$$

where  $\varepsilon$  is residual function and  $\varphi_i$  weighting function.

From equation (1), reduce as a residual function form

$$\varepsilon = -\frac{d^2 w}{dr^2} - \frac{1}{r} \frac{dw}{dr} - \left(\frac{1}{D} + K^2 - M\right) w + P + G_r \theta.$$

We use the Galerkin's integral in the standard form

$$-\int_{\Omega} \left[ \frac{d^2 w}{dr^2} + \frac{1}{r} \frac{dw}{dr} + \left(\frac{1}{D} + K^2 - M\right) w - P - G_r \theta \right] \varphi_i d\Omega = 0, \quad \dots (6)$$

where  $\Omega$  is the volume of the annulus bounded by coaxial cylinders of unit length. The weak form of the equation (6) is given below:

$$- \int_0^1 \int_0^{2\pi} \int_{r_a}^{r_b} \left[ r \frac{dw}{dr} + \left( \frac{1}{D} + K^2 - M \right) w - P - G_r \theta \right] \varphi_i r dr d\theta dz = 0. \quad \dots$$

(7)

Integrating equation (7) by parts, we get

$$\int_{r_a}^{r_b} \left[ r^2 \frac{dw}{dr} \frac{d\varphi_i}{dr} + \left\{ \frac{dw}{dr} + \left( \frac{1}{D} + K^2 - M \right) w \right\} \varphi_i r \right] dr = \int_{r_a}^{r_b} \{P + G_r \theta\} \varphi_i r dr + \left[ \varphi_i \left( r^2 \frac{dw}{dr} \right) \right]_{r_a}^{r_b}$$

or

$$\int_{r_a}^{r_b} \left[ r^2 \frac{dw}{dr} \frac{d\varphi_i}{dr} + \left\{ \frac{dw}{dr} + \left( \frac{1}{D} + K^2 - M \right) w \right\} \varphi_i r \right] dr = \int_{r_a}^{r_b} \{P + G_r \theta\} \varphi_i r dr - \varphi_i(r_a) Q_1^e + \varphi_i(r_b) Q_2^e$$

$$\text{where } Q_1^e = r_a^2 \left( \frac{dw}{dr} \right)_{r=r_a} \text{ and } Q_2^e = r_b^2 \left( \frac{dw}{dr} \right)_{r=r_b}.$$

Similarly, weak form of equation (2) can be expressed as

$$\int_{r_a}^{r_b} \left[ r^2 \frac{d\theta}{dr} \frac{d\varphi_i}{dr} \right] dr = - \int_{r_a}^{r_b} \left\{ h \left( \frac{dw}{dr} \right)^2 \right\} \varphi_i r dr - \varphi_i(r_a) T_1^e + \varphi_i(r_b) T_2^e,$$

$$\text{where } T_1^e = r_a^2 \left( \frac{d\theta}{dr} \right)_{r=r_a} \text{ and } T_2^e = r_b^2 \left( \frac{d\theta}{dr} \right)_{r=r_b}.$$

Let us assume that  $w(r) = \sum_{j=1}^n w_j^e \psi_j^e(r)$  and  $\theta(r) = \sum_{j=1}^n \theta_j^e \psi_j^e(r)$  are the approximate solutions of the equations (1) and (2). Assuming  $\varphi_i = \psi_1^e, \psi_2^e, \psi_3^e, \dots, \psi_n^e$  (weight function is equal to the approximation function). Therefore, the finite element method for velocity and temperature profiles is as given below:

$$[K^e][w^e] = [f^e][Q^e], \quad \dots (8)$$

$$\text{and } [S^e][\theta^e] = [g^e][T^e], \quad \dots (9)$$

$$\text{where } K_{ij}^e = \int_{r_a}^{r_b} \left[ r^2 \frac{dw_j^e}{dr} \frac{d\psi_j^e}{dr} + \left\{ \frac{dw_j^e}{dr} + \left( \frac{1}{D} + K^2 - M \right) w_j^e \right\} \psi_j^e r \right] dr, \quad f_i^e = \int_{r_a}^{r_b} \{P + G_r \theta\} \psi_j^e r dr$$

$$K_{ij}^e = \int_{r_a}^{r_b} \left[ r^2 \frac{d\theta_j^e}{dr} \frac{d\psi_j^e}{dr} \right] dr, \quad g^e = - \int_{r_a}^{r_b} \left\{ h \left( \frac{dw_j^e}{dr} \right)^2 \right\} \psi_j^e r dr.$$

Taking the approximate function as a linear interpolation function of the form ( $h_e = r_b - r_a$ ),

$$\psi_1^e(r) = \frac{r_b - r}{h_e} \text{ and } \psi_2^e(r) = \frac{r - r_a}{h_e}.$$

The standard finite element form of Galerkin is represented by equations (8) and (9). By using the predictor-corrector method or a different technique similar to it, the results can be determined.

#### 4. RESULTS AND DISCUSSION:

In the present model, the velocity profile indicates in addition to the quantity of the flow; it also shows its direction, changes carried on by the structure of a particular domain, and fluctuations in the magnitude of the flow with respect to parameters. When examining the free and forced convection flow of viscous incompressible fluid via coaxial cylinders, the effects of viscous dissipation are taken into consideration. The numerical solution is obtained with the finite element method. Here  $G_r > 0$  indicates that the temperature of the outer boundary is higher than the inner in a coaxial cylinder duct or vice versa ( $G_r < 0$ ).

Figures (2) and (6) depicts that the axial velocity of the fluid be maximum at the mid region while velocity profile upwards for  $G_r > 0$  and downwards for  $G_r < 0$ . From these figures we observe that the enhancement in the velocity profile depends upon the nature of the gap duct between two coaxial cylinders.

Figures (3) and (7) depicts that the velocity profile rises with increasing Darcy's parameter values  $D^{-1}$  in both narrow and broader gap ducts, while the larger gap duct exhibits a greater increase than the narrower gap duct.

Figure (4) depicts that the temperature profiles for various values of ( $G_r$ ) in a narrow gap duct. It shows that temperature profile increases with increase in Grashoff number ( $G_r$ ). For  $G_r > 0$ , the temperature profile is positive or negative according as the actual temperature is higher or lower than the temperature of the inner boundary.

Figure (5) depicts that the temperature profiles for various values of Darcy's parameter  $D^{-1}$ . The temperature profile increases with the decrease in the permeability of the porous medium and effect of magnetic field. Figures (8) and (9) depicts that the temperature profiles for various values of Grashoff number ( $G_r$ ) and  $D^{-1}$  respectively in wide gap duct of the coaxial cylinders. The temperature assumes positive values faster as we move from the inner boundary to outer boundary than in the case of narrow gap duct. The nature of temperature profile in wider gap duct of coaxial cylinders for positive or negative values of  $G_r$ ,  $D^{-1}$  is similar to that of the corresponding case in a narrow gap duct of the coaxial cylinders. The present algorithm is economic and efficient having a sharp convergence.

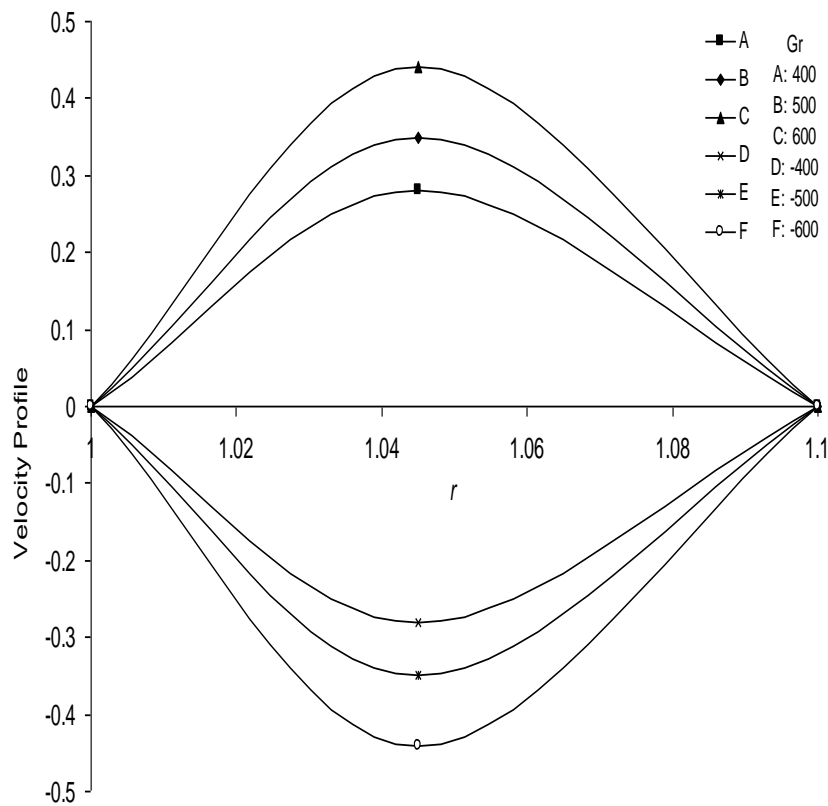


Figure 2. The Velocity Profiles for Various Number of Grashoff Number  $G_r$ ,

$$s = 1.1, P = 1, D^{-1} = 10000, h = 0.001, M = 0.732$$



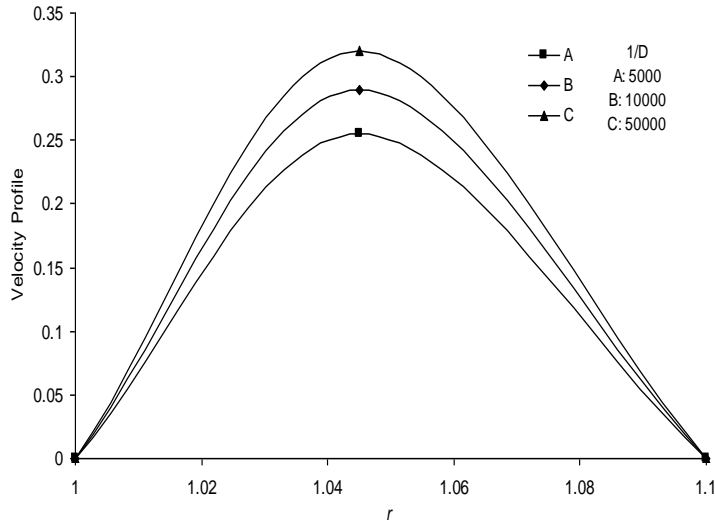


Figure 3. The Velocity Profiles for Various Values  $D^{-1}$

$$s = 1.1, P = 1, G_r = 400, h = 0.001, K = 1$$

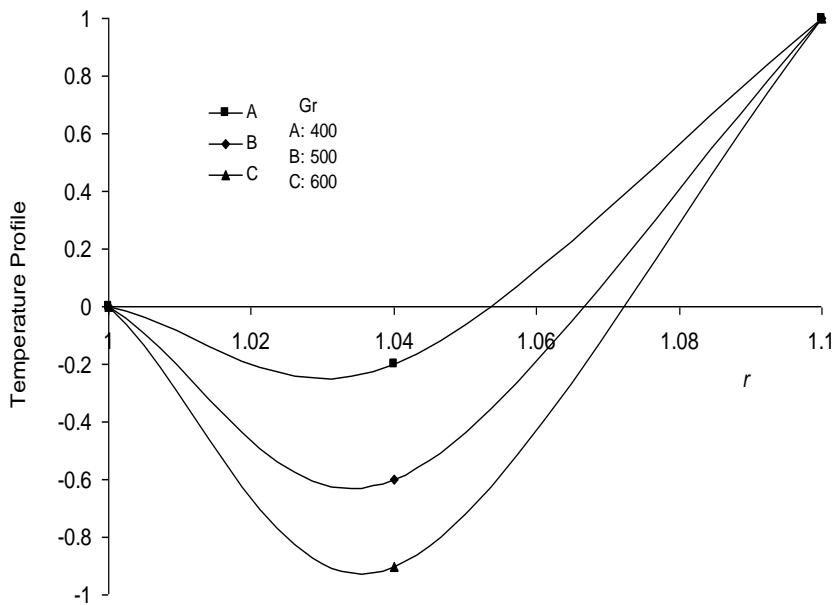


Figure 4. The temperature profiles for various values of Grashoff number  $G_r$ ,

$$s = 1.1, P = 1, D^{-1} = 10000, h = 0.001, K = 1$$

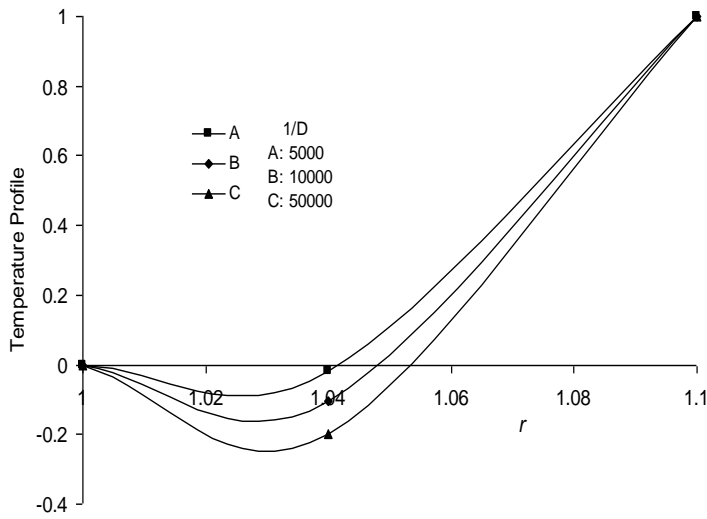


Figure 5. The temperature Profiles for various values  $D^{-1}$ ,  
 $s = 1.1, P = 1, G_r = 400, h = 0.001, K = 1$

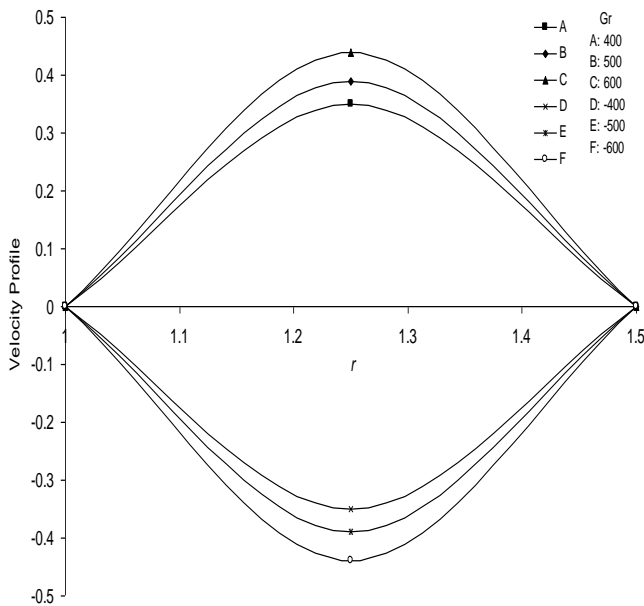


Figure 6. The velocity profiles for various values of Grashoff number  $G_r$ ,  
 $s = 1.5, P = 1, D^{-1} = 10000, h = 0.001, K = 1$

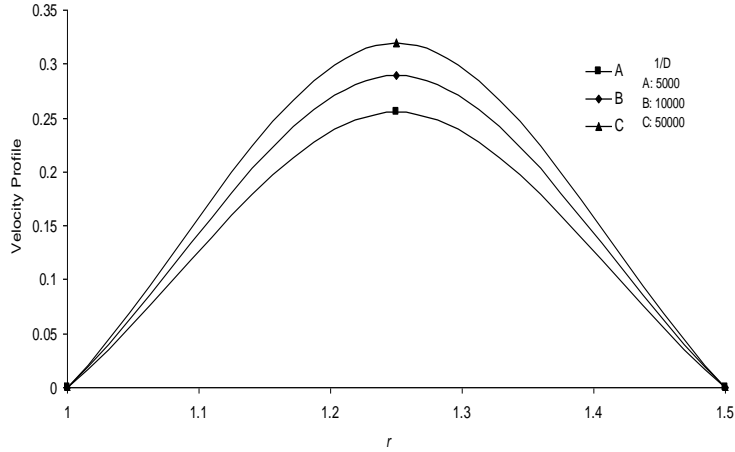


Figure 7. The velocity profiles for various values  $D^{-1}$ ,  
 $s = 1.5, P = 1, G_r = 400, h = 0.001, K = 1$

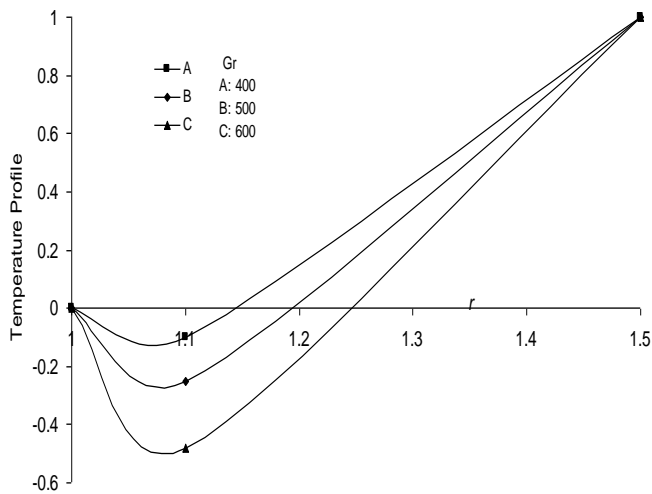


Figure 8. The temperature Profiles for various values of Grashoff number  $G_r$ ,  
 $s = 1.5, P = 1, D^{-1} = 10000, h = 0.001, K = 1$

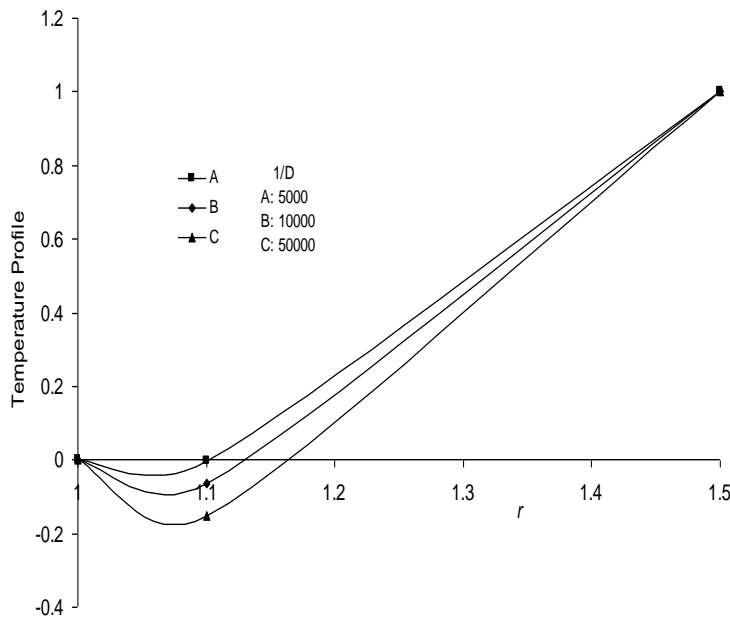


Figure 9. The temperature Profiles for various values  $D^{-1}$ ,

$$s = 1.5, P = 1, G_r = 400, h = 0.001, K = 1$$

## 5. Conclusion:

In addition, the graphical representation of the results demonstrates how the appropriate physical parameters influence the profiles of velocity and temperature. In the present investigation, we examined viscous fluid flow via coaxial porous effect cylinders under both free and forced convection while taking viscous dissipation into account. The finite element scheme, a numerical approach, is used to obtain the solutions. The governing equations have been solved using Galerkin's Method. The graphs are used to show how the velocity and temperature profiles of the fluid flow evolve with various combinations of the parameters.

## NOMENCLATURE:

- $w$ : Velocity component along z-axis
- $\rho$ : Density of the fluid
- $P$ : Fluid pressure
- $T$ : Non-dimensional temperature of the fluid
- $\mu$ : Coefficient of the viscosity
- $C_p$ : Specific heat at constant pressure
- $k$ : Permeability of Porous medium

$k_1$ : Coefficient of thermal conductivity  
 $\rho_0$ : Equilibrium density  
 $T_0$ : Temperature  
 $P_s$ : Equilibrium pressure  
 $P_D$ : Dynamic pressure  
 K- Permeability Parameter

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