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A STUDY ON MATHEMATICAL USAGE OF DISPERSIVE PARTIAL DIFFERENTIAL EQUATION

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Abstract

In most cases, the global analysis for linearized equations is used as the foundation for the nonlinear evolution partial differential equations analysis. In order to achieve this goal, one makes an effort to create and investigate global solutions for relevant linear equations. In general, solutions to different types of partial differential equations lead to representations of the solutions in a variety of distinct forms. Partial elliptic differential equations, for instance, result in parametrices that take the form of pseudo-differential operators. On the other hand, it is possible to create propagators for hyperbolic equations in the form of Fourier integral operators. Other equations, such as the Schrödinger equation or the linearized Korteweg-de Vries equation, lead to oscillatory integrals that are more generic in nature. The solutions of Schrödinger equations may be shown in the form of Legendrian oscillatory integrals, but the oscillatory integrals that are generated by more generic evolution partial differential equations are of a more broad nature. Methods of representing solutions, calculus of solution operators and propagators, global weighted L2 and other estimates, spectral qualities, functional analytic properties, and so on are often included as components of the needed analysis.

In this study, we present a brief introduction of numerous techniques to linearizations of nonlinear evolution equations as well as ways to smoothing estimates. In addition, we discuss approaches to smoothing estimates. Here, we demonstrate various different equations together with the relevant difficulties. The smoothing estimates for linear evolution equations are the primary topic of discussion in this article. The evolution equations of both the dispersive and non-dispersive kinds are the primary focus of our attention. We will discuss their normal forms and canonical transforms that can be used for the reduction of general equations to these normal forms, and we will introduce comparison principles that can be used to obtain additional information about equations in their normal form. In order to accomplish this goal, we will discuss their normal forms and canonical

transforms that can be used. In addition, we will provide an overview of related problems such as necessary estimates for Fourier integral and pseudo-differential operators, estimates in weighted L2–spaces for pseudo-differential and Fourier integral operators under minimal conditions, global calculus of these operators, and various other aspects and applications.

Keywords:

Equation, Partial, Dispersive

Introduction

A partial differential equation (PDE) is called dispersive if, when no boundary conditions are imposed, its wave solutions spread out in space as they evolve in time. As an example, consider $iu_t + u_{xx} = 0$. If we try a simple wave of the form $u(x,t) = Ae^{i(kx-\omega t)}$, we see that it satisfies the equation if and only if $\omega = k^2$. This is called the dispersive relation and shows that the frequency is a real valued function of the wave number. If we denote the phase velocity by v = ω k we can write the solution as $u(x,t) = Ae^{ik(x-v(k)t)}$ and notice that the wave travels with velocity k. Thus the wave propagates in such a way that large wave numbers travel faster than smaller ones. (Trying a wave solution of the same form to the heat equation $iu_t + u_{xx} = f(u)$, we obtain that the ω is complexd valued and the wave solution decays exponential in time. On the other hand the transport equation $u_t - u_{xx} = 0$, and the one dimensional wave equation $u_{tt} = u_{xx}$ are traveling waves with constant velocity.) If we add nonlinear effects and study we will see that even the existence of solutions over small times requires delicate techniques.

Going back to the linear equation, consider $u_0(x) = \int_{\mathbb{R}} \hat{u}_0(k)e^{ikx}dk$. For each fixed k the wave solution becomes $u(x,t) = \hat{u}_0(k)e^{ik(x-kt)} = \hat{u}_0(k)e^{ikx}e^{-ik^2t}$. Summing over k (integrating) we obtain the solution to our problem

$$u(x,t) = \int_{\mathbb{R}} \hat{u}_0(k)e^{ikx-ik^2t}dk.$$

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Since $|\hat{u}(k,t)| = |\hat{u}_0(k)|$ we have that $\|u(t)\|_{L^2} = \|u_0\|_{L^2}$. Thus the conservation of the L^2 norm (mass conservation or total probability) and the fact that high frequencies travel faster, leads to the conclusion that not only the solution will disperse into separate waves but that its amplitude will decay over time. This is not anymore the case for solutions over compact domains. The dispersion is limited and for the nonlinear dispersive problems we notice a migration from low to high frequencies. This fact is captured by zooming more closely in the Sobolev norm

$$\|u\|_{H^s} = \sqrt{\int |\hat{u}(k)|^2(1+|k|)^{2s}dk}$$

and observing that it actually grows over time. To analyze further the properties of dispersive PDEs and outline some recent developments we start with a concrete example

Consider the semi-linear Schr¨odinger equation (NLS) in arbitrary dimensions

$$\begin{cases} iu_t + \Delta u + \lambda |u|^{p-1}u = 0, \quad x \in \mathbb{R}^n, \quad t \in \mathbb{R}, \\ u(x,0) = u_0(x) \in H^s(\mathbb{R}^n). \end{cases}$$

for any $1 < p < \infty$. $H^s(\mathbb{R}^n)$ (the s Sobolev space) is a Banach space that contains all functions that along with their distributional s−derivatives belong to $L^2(\mathbb{R}^n)$ This norm is equivalent (through the basic properties of the Fourier transform) to

$$\|f\|_{H^s(\mathbb{R}^n)} = \left(\int_{\mathbb{R}^n} |\widehat{f}(\xi)|^2(1+|\xi|)^{2s}d\xi\right)^{\frac{1}{2}} < \infty.$$

The restriction on the values of p for equation (1) will become apparent shortly. When $\lambda = -1$ the equation is called defocusing and when λ = 1 it is called focusing. NLS is a basic dispersive model that appears in nonlinear optics and water wave theory. We will study problems related to the NLS

- of local-in-time nature (local existence of solutions, uniqueness, regularity),
- of global-in-time nature (existence for large times, finite time blow-up, scattering).

The problem of scattering is the most difficult of them all. Assume that there is one solution that can be used everywhere (which is true for large data in the defocusing case). After then, the issue may be broken down into two more manageable problems: one involving the existence of the wave operator, and the other concerning asymptotic completeness. In a moment, we shall show that the L p norms of linear solutions decrease throughout the course of time. Because of the temporal decay, it appears that for high values of p, the nonlinearity may become insignificant as time progresses towards infinity. As a result, we anticipate that the answer to the linear equation will provide us with an approximation of u. It is important to note that the application of this theory to huge data sets is not at all simple. Even in the case of scattering and global solutions in the focusing issue, we can have global solutions for tiny data sets. 7. A solution will be referred to be a strong solution if it is capable of satisfying (at least locally) all of these qualities. In the following notes, you will find a definition that is more in-depth later on. Let us just note the fact that it is really important for our solutions to be equipped with all of these additional qualities. For instance, the fact that local H1 solutions satisfy the energy conservation law is a byproduct not only of the existence of the local-in-time solution but also of the regularity and continuity with respect to the properties of the initial data. This is because the existence of the local solution in time is a prerequisite for the solution. The evidence is sufficient to demonstrate that, the vast majority of the time (local-in-time existence does, after all, stem from Banach's fixed point theorems), the map from the data to the solution is uniformly continuous, and in certain instances, it is even analytic.

When searching for local solutions, constructing the aforementioned Banach space X is the more difficult problem to solve than any of the others. This method is delicate (the development of smooth solutions that is done traditionally being the obvious exception) and is built on specific estimations that are satisfied by the linear solution. Recalling what we learned in our PDE lectures during our undergraduate (or graduate) studies, we may solve the linear issue by using the Fourier transform. Then, for smooth starting data, the solution to the linear equation is provided as the convolution of the data with the tempered distribution. This would be the case if the data were in the Schwartz class S(R n).

$$K_t(x) = \frac{1}{(4\pi i t)^{\frac{n}{2}}} e^{i\frac{|x|^2}{4t}}.$$

Thus we write for the solution

$$u(x,t) = U(t)u_0(x) = e^{it\Delta}u_0(x) = K_t \star u_0(x) = \frac{1}{(4\pi i t)^{\frac{n}{2}}} \int_{\mathbb{R}^n} e^{i\frac{|x-y|^2}{4t}} u_0(y)dy.$$

Another fact from our undergraduate (or graduate) machinery is Duhamel's principle: Let I be any time interval and suppose that $u \in C_t^1\mathcal{S}(I \times \mathbb{R}^n)$; and that $F \in C_t^0\mathcal{S}(I \times \mathbb{R}^n)$. Then u solves

$$\begin{cases} iu_t + \Delta u = F, & x \in \mathbb{R}^n, \quad t \in \mathbb{R}, \\ u(x,t_0) = u(t_0) \in \mathcal{S}(\mathbb{R}^n) \end{cases}$$

if and only if

$$u(x,t) = e^{i(t-t_0)\Delta}u(t_0) - i\int_0^t e^{i(t-s)\Delta}F(s)ds.$$

For less regular Hs data, a time interval I which contains zero, and

$$F \in C(H^s(\mathbb{R}^n); H^{s-2}(\mathbb{R}^n)).$$

We are fortunate in that we are able to acquire the well-known Strichartz estimates by extending these fundamental dispersive estimates through duality (by utilising a T T? argument). The names Strichartz, Ginibre, and Velo, as well as Keel and Tao, are important in this context.

REVIEW LITERATURE

M. Burak Erdoğan (2012) The discipline of nonlinear dispersive partial differential equations (PDEs) is one that is rapidly expanding and has gotten exceptionally sophisticated over the course of the past few years. The authors of this book want to give graduate students as well as advanced undergraduate students in the fields of mathematics, engineering, and the physical sciences with

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an introduction that is self-contained and easy to understand. Students are given the opportunity to acquire knowledge of the topic in a condensed amount of time by focusing on model cases that simplify the presentation without compromising the in-depth technical aspects of the theory. Both traditional and contemporary approaches that are currently being utilised in the field are described in great detail. This book is suitable for both independent study by students who already have some experience with analysis and the instruction of a graduate-level introductory course on nonlinear dispersive PDEs that lasts for an entire semester. Applications of the theory are also taught, together with a wealth of tasks, in order to connect dispersive PDEs with the more broad fields of dynamical systems and mathematical physics.

Abdul-Majid Wazwaz (2013) In this article, we provide a novel method for solving the third-order dispersive partial differential equations in spaces of one dimension and higher. The Adomian decomposition method is the primary foundation for our approach. When dealing with non-homogeneous issues, we will employ the modified decomposition approach in conjunction with the noise terms phenomena. Testing numerical examples that were created to capture the qualities of the suggested algorithms is one of the ways that we illustrate our work.

Dr. K.A. Patel (2020) In this paper, I propose a New Integral Transform, which is currently neither generally recognised nor utilised to any significant degree. The objective of this work is to examine the application of the Elzaki Transform with combination of the basic iteration Method for the purpose of solving the Fokker-Planck equation as well as some other equations that are quite similar to it. The approach is capable of reducing the amount of computing effort required and may be used without difficulty to a wide variety of linear and non-linear partial differential equations. In this way of approaching the problem, the answer is discovered in the form of a convergent series that has components that are simple to compute. I have included various examples in both one-dimensional and two-dimensional scenarios in order to provide an overview of the methodology.

Using the online live broadcast and on-demand platform that is built on the B/S architecture as the application side, this article proposes a video image forensic system that is capable of meeting the needs of many types of police and multiple application scenarios. The system solves the problems of quick reaction and concealment by using mobile phones as the video image capture terminal, and it solves the problems of device mobility and connection maintenance by using 5G communication technology as the transmission medium. The problem of diversification of the use

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and application modes of multiple types of police has been solved; the problem of managing video image evidence in a centralised storage, audit, and export method has been solved; and the problem of ensuring the authenticity and security of the evidence has been solved. The system is capable of realising a number of functions, including the collection, transmission, storage, and application of video image evidence. In addition to this, the system is capable of realising the application-side video image live broadcast function in accordance with actual work needs, and it solves the problem of large-scale case command and decision-making that has been plaguing public safety organisations. The purpose of this research is to offer a denoising method that is based on the two-way coupling diffusion equation in order to eliminate the noise that is present in the public security forensic photographs and to smooth the noise while maintaining the features of the image. It is possible to design a new diffusion function that has a better diffusion impact than the model that was originally used by making improvements to the partial differential equation of the second order. We established a novel model for the denoising algorithm by combining the adaptive edge threshold and the stop criterion in order to produce a model that is capable of producing superior denoising outcomes.

To recover the heat source outlined by Poisson's equation, the weighted gradient reproducing kernel collocation technique is shown here. Because it is a well-known fact that the inverse heat source problem does not have a single answer that applies to all situations, this article will focus on the less robust approach that is founded on a priori assumptions. The high-order gradient reproducing kernel approximation is introduced in light of the fourth-order partial differential equation (PDE) in the mathematical model in order to efficiently untangle the problem without calculating the high-order derivatives of reproducing kernel shape functions. This is done in light of the fact that the PDE is a part of the mathematical model. The first step in performing a high-order inverse analysis is to figure out the weights for the weighted collocation approach. The imprecise depiction in the literature is explained in the benchmark analysis, and the accurate interpretation of the numerical data is presented in detail. Two mathematical formulations and four examples are offered to show the practicability of the technique. These examples cover a range of scenarios, including those with a restricted amount of accessible boundary space.

Judy P. Yang (2022) To recover the heat source outlined by Poisson's equation, the weighted gradient reproducing kernel collocation technique is shown here. Because it is a well-known fact

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Nehad Ali Shah (2022) In this article, a novel approach to obtaining closed-form solutions of a nonlinear fractional partial differential equation known as the simplified modified Camassa-Holm (MCH) problem is proposed. The wave transformation and the modified Riemann-Liouville fractional derivative are the two methods that are utilised in the process of transforming the fractional order partial differential equation into an ordinary differential equation of integer order. Several precise solutions in the form of hyperbolic, trigonometric, and rational function solutions may be obtained by applying the innovative (G'/G2)-expansion approach. These solutions each have their own extra free parameters. Forms of soliton solutions, known as singular kink wave soliton solutions, are produced when suitable values are assigned to parameters and distinct values of fractional order travelling wave solutions, such as singular periodic waves. The value of the arbitrary parameters H has an effect on the answers that may be produced using the approach that has been given. When different values are entered for the parameters, the findings from before are rederivated. In addition, the popular commercial programme Mathematica 10 is utilised in order to generate graphical representations of numerical data in both two and three dimensions. The depiction of the approach is spot on, and it enables the generation of further general precise answers.

Discussion

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Since many decades ago, people have been aware of the procedure that is employed to factorise the longitudinal wave equation. In light of this information, the traditional Cauchy-established 2nd-order partial differential equation (PDE) has been broken up into two 1st-order PDEs, in accordance with D'Alemberts' theory, in order to get solutions for forward- and backward-traveling waves. Because of this, the Cauchy equation has to be interpreted as a two-way wave equation. The inherent directional ambiguity of this equation causes irregular phantom effects in numerical computations using finite elements (FE) and finite differences (FD). In the field of seismic applications, an enormous number of different approaches to minimising these disturbances have been devised; nevertheless, to this day, none of these efforts has been successful. The above-mentioned ambiguity is removed, however, by a priori factorization of the longitudinal wave equation for inhomogeneous media. The resulting one-way equations offer a specification of the wave propagation direction based on the geometric position of the transmitter and receiver.

The range of applications for image recognition technology is expanding rapidly in tandem with the fast development of image processing technology, which has allowed for this expansion. The primary approaches to gather image data and fix image data in complex environments include processing, analysing, and repairing graphics and pictures through the use of computer and big data technologies. This work makes a proposal to get rid of the noisy data and repair the picture based on the partial differential equation system that is used in image recognition technology. This is in response to the bad quality of image information that is generated during the process of sports. First, image recognition technology is used to track and obtain the image information in the process of sports, and then the fourth-order partial differential equation is used to optimise and process the image. Image recognition technology is used to track and obtain the image information in the process of sports. In conclusion, in order to address the issue of low image quality and blur during the transmission process, denoising is performed, and image restoration is studied through the application of the adaptive diffusion function in partial differential equation. Both of these are done in an effort to solve the problem. The findings of the study presented in this article reveal that the content of the research makes a significant improvement to the issues of blurred picture and poor quality that arise during the process of sports, and it also makes it possible to automatically follow the aim of sports images. In the picture restoration link, it has the ability to accomplish the normal repair effect while simultaneously reducing the amount of time needed for the repair. The information presented in this research study is useful and applicable to the fields of image processing and restoration.

Asymptotic homotopy perturbation technique (AHPM) is the name of a novel algorithm-based approach that was developed by the authors of this article for the aim of simulating non-linear and linear differential equations of non-integer and integer order respectively. AHPM has been expanded such that it may be used for numerical treatment to approximate the solution to one of the most significant fractional-order two-dimensional Helmholtz equations, as well as some of the equation's instances. We have compared the solutions generated by the AHPM to the solutions generated by another existing approach as well as the actual solutions generated by the issues that were taken into consideration for the purposes of validation and illustration. In addition, it has been noticed that the symmetry or asymmetry of the solutions to the issues that are being studied does not change based on the definition of homotopy. Estimates of the amount of error are also supplied for the solutions. The fact that AHPM's approximative answers are tabulated and displayed demonstrates that it is both efficient and clear in its presentation of information.

In the past three decades, there has been a substantial amount of research conducted on the topic of partial differential equation (PDE)-based geometric modelling and computer animation. The PDE surface-represented facial blendshapes, on the other hand, have not been examined. In this research, we suggest a new approach of facial blendshapes by making use of PDE surfaces that are defined by curves and represented by Fourier series. In order to construct this new approach, we first design a curve template and then make use of it to extract curves from polygon face models. Afterwards, we utilise this curve template to create the new method. After that, we create a mathematical model of curve-defined PDE surfaces by first proposing a second-order partial differential equation and then combining that equation with the restrictions imposed by the extracted curves, which serve as boundary curves. Next, we solve the second-order partial differential equation that is constrained by the extracted boundary curves using an extended Fourier series representation. This allows us to get an analytical mathematical expression of curve-defined and Fourier series-represented PDE surfaces. The mathematical expression is used to develop a new PDE surface-based interpolation method of creating new facial models from one source facial model and one target facial model as well as a new PDE surface-based blending method of creating even more new facial models from one source facial model and many target facial models. Both of these new methods are used to create new facial models from one source facial model and many target facial models. In order to show the efficacy of the suggested technology and its applicability in creating 3D facial blendshapes, a few examples have been provided.

It is possible to acquire reasonable prediction accuracy when forecasting photovoltaic (PV) energy output based just on the most recent weather and power data. This is true for short-term horizons. NWP systems, which stand for numerical weather prediction, often generate free predictions of the local cloud quantity every six hours. These are significantly delayed by a number of hours and do not give adequate quality. A Differential Polynomial Neural Network, often known as a D-PNN, is an unorthodox method of soft computing that was developed recently and can simulate intricate weather patterns. The n-variable partial differential equation (PDE) of the kth order is expanded by the use of D-PNN into chosen two-variable node PDEs of the first or second order. Their derivatives are simple to turn into Laplace transforms and may be substituted with the use of operator calculus (OC). The D-PNN algorithm validates two-input nodes so that PDE components can be included into its gradually expanding sum model. Because it is represented by PDEs, it may take into account the unpredictability and uncertainty of certain patterns in the surface layer. We compare and evaluate the results of the proposed all-day single-model and intra-day several-step PV prediction methods using differential and stochastic machine learning. In order to estimate the PV production in entire day sequences or particular hours, statistical models are being developed that take into account the individual data time delay. Models are able to calculate accurate forecasts each day thanks to the spatial data from a bigger region and the initially identified daily periods. These factors also allow models to correct for unforeseen pattern alterations and various beginning circumstances. The ideal data samples are trained to forecast the Clear Sky Index inside the chosen horizon. These data samples are selected by the precise temporal shifts that occur between the model inputs and outputs.

Conclusion

At this juncture, it is important to point out that the method that was just described shares some parallels with a methodology that is referred to as the "method of free parameter analysis." In this approach, the boundary conditions play a crucial part in determining the extent to which the similarities are diminished. In a few distinct classes of hypothetical situations. technique has been applied to the problem of solving fractional-order multi-dimensional dispersive partial differential equations. The findings of the Elzaki decomposition method are accomplished in series form, which provides a greater convergence rate to the recommended strategy. These results are reached for both integer and fractional orders. In order to validate the methodology that is currently being used, illustrative issues are formulated. It has also been discovered via study that the findings of the fractional-order are convergent to the results of the integer-order. Additionally, the results of the suggested technique are compared with the issues' precise solutions, which has demonstrated that approximate solutions are convergent to the problems' exact solutions as the number of terms in the series increases. In this part of the analysis, the correctness of the approach is evaluated with the assistance of several instances. It has been demonstrated that the proposed technique is found to be trustworthy, effective, and straightforward in its application to a variety of connected issues in applied science.

REFERENCES

1) O. E. Ige, R. A. Oderinu, & T. M. Elzaki, "Adomian Polynomial and Elzaki Transform Method for Solving Sine-Gordon Equations.", International Journal of Applied Mathematics (IJAM), Vol. 49, 2019. [2]
2) M. Tatari, M. Dehghan, & M. Razzaghi, "Application of the Adomian decomposition method for the Fokker-Planck equation.", Mathematical and Computer Modelling, Vol. 45, pp. 639–650, 2007. [3]
3) G. A. Rathva, K. S. Tailor & P. H. Bhathawala, "Numerical Solution of one-dimensional Ground water Recharge problem using Variational Iteration Method.", International Journal of Advanced Engineering Research and Studies (IJAERS), Vol. 1, No. 3, pp. 217-219, 2012.
4) [4] K. Shah, & T. Singh, "A Solution of the Burger's Equation Arising in the Longitudinal Dispersion Phenomenon in Fluid Flow through Porous Media by Mixture of New Integral Transform and Homotopy Perturbation Method.", Journal of Geoscience and Environment Protection, Vol. 3, pp. 24–30, 2015. [5]
5) J. M. Khudhir, "Homotopy Perturbation Method for solving Fokker-Planck Equation.", Journal of THI-QAR Science, Vol. 3, No.2, pp. 149-162, 2012. [6]

6) K. Shah, & T. Singh, "Solution of Burger's Equation in a One-Dimensional Groundwater Recharge by Spreading Using q-Homotopy Analysis Method.", European Journal of Pure and Applied Mathematics, Vol. 9, No. 1, pp. 114–124, 2016. [7]
7) K. Shah, "Application of the Homotopy Analysis Method to the Fokker-Planck equation." https://www.researchgate.net/publication/283655722, 2014. [8]
8) K. Wang & S. Liu, "Application of new iterative transform method and modified fractional homotopy analysis transform method for fractional Fornberg-Whitham equation.", Journal of Nonlinear Sciences and Applications, 2015. [9]
9) A. Hemeda & E. E. Eladdad, "New Iterative Methods for Solving Fokker-Planck Equation." Mathematical Problems in Engineering, Vol. 2018, Article ID 6462174, 9 pages, 2018. [10]
10) T. M. Elzaki, & M. A. Hilal, "Solution of Linear and Nonlinear Partial Differential Equations Using Mixture of Elzaki Transform and the Projected Differential Transform Method." Mathematical Theory and Modeling, Vol. 2, No. 4, pp. 50-59, 2012. [11]
11) S. Hesam, A. R. Nazemi, & A. Haghbin, "Analytical solution for the Fokker-Planck equation by differential transform method." Scientia Iranica, Transactions B: Mechanical Engineering, Vol. 19, No. 4, pp. 1140–1145, 2012. [12]
12) Yan, L. "Numerical solutions of fractional Fokker-Planck equations using Iterative Laplace transform method." Abstract and Applied Analysis, Hindawi publishing corporation, Vol. 2013, Article ID 465160, 7 pages, 2013. [13]