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Research Paper

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Tulgeity of Restricted Super line Graph of Path graph

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Abstract: Tulgeity $\tau(G)$ is the maximum number of disjoint, point induced, non-acyclic subgraphs contained in *G*. In this paper we find the formula for tulgeity of the restricted super line graps of path graph is derived.
Key words: Tulgeity, Super line graph.

1. ***Introduction***:

Point partition number [4](Gray Chartrand,1971) of a graph *G* is the minimum number of subsets into which the point- set of *G* can be partitioned so that the sub graph induced by each subset has the property *P*. Dual to this concept of point partition number of a graph is the maximum number of subsets into which the point set of *G* can be partitioned such that the subgraph induced by each subset does not have the property *P*. Define the property *P* such that a graph *G* has the property *P* if *G* contains no subgraph that is homeomorphic from the complete graph K_3.This point partition number, and dual point partition number for the property *P* is referred as point arboricity and tulgeity of *G* respectively. Equivalently the tulgeity is the maximum number of vertex disjoint cycles in *G* so that each subgraph is not acyclic..The formula for tulgeity of complete bipartite graph was given in Gray Chartrand.,1968. Akbar Ali.et.al and Paniyappan [3,5] given the tulgeity of line, middle, total graphs of some class of graphs. It is observed that in any graph G, a $K_{1,2}$ [K_2 in *G* induces a C_3 in $L_2(G)$ that leads to
maximum number of cycles. This made us to work on Tulgeity of restricted superline graphs.
In this paper we find the tulgeity of $L_2(P_n)$. For the terminology not given here refer [2]
All graphs considered in this paper are simple graphs. The vertices of $L_r(G)$ are the *r*-element subsets of $E(G)$ and two vertices *S* and *T* are adjacent if there exists atleast one pair of edges, one from each of the sets *S* and *T*, which are adjacent in *G*.

2. ***Main Theorem***:

To avoid the complexity in listing the vertices of super line graph , in this chapter we represent the vertex induced by the edges e_i,e_j in *G* as v_{ij} instead of $\{e_ie_j\}$ in $L_2(G)$.

Outline of the proof : Here we derive the formula for tulgeity of superline graph of index 2 in six cases. We covered all the vertices of $RL_2(G)$ with C_3s whenever $\left|\binom{E(G)}{2}\right|$ is a multiple of 3. If $\left|\binom{E(G)}{2}\right|$ is not a multiple of 3, then we cover $\left|\binom{E(G)}{2}-4\right|$ vertices with C_3s and the remaining 4 vertices with C_4 .
Thus we obtain maximum number of induced cyclic subgraphs .

Theorem 2.1: For $n \geq 6$, the tulgeity of Super line graph of index 2 of the path graph is $\tau(L_2(P_n)) = \left\lfloor \frac{|V(L_2(P_n)|}{3} \right\rfloor = \left\lfloor \frac{(n-1)(n-2)}{6} \right\rfloor$

Proof: Let $E(P_n) = \{1,2,3....n-1\}$. By definition of $L_2(G)$, $V(L_2(P_n)) = \{v_{i,j} / i \neq j \,\&1 \leq i < j \leq n-1, i, j \in E(P_n)\}$.

Thus there are $\frac{(n-1)(n-2)}{2}$ vertices. By division algorithm we express *n* as *n*=6*q*+*t*, $0 \leq t \leq 5$

Since Tulgeity is the maximum number of disjoint cycles and it is possible with a cycle of length 3, here we partition all vertices into C3s when ever $n \equiv 1,2,4,5 (\text{mod} 6)$. In other 2 cases, that is When $n \equiv 0,3 (\text{mod} 6), |V(l_2(P_{n)})|$ is not divisible by 3. So, it is not possible to partition vertex set of RL2(Pn) into only c3s. Instead the vertex set is partitioned into one c4 and rest to c3s.

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$$\tau(L_2(P_n)) \le \left\lfloor \frac{|V(RL_2(P_n))|}{3} \right\rfloor = \left\lfloor \frac{(n-1)(n-2)}{6} \right\rfloor \text{-------------} 1$$

Case (i): $n \equiv o(\bmod 6)$.

In this case $\binom{|E(Pn)|}{2}$ is not a multiple of 3. So we partitioned all the vertices with one C_4 and the remaining vertices with C_3s.

For $n = 6$, partition of vertices of $L_2(P_6)$ is given by

$V(RL_2(G)) = \{v_{3,4}, v_{1,4}, v_{1,3};\ v_{2,3}, v_{3,5}, v_{2,5}\} \cup \{v_{1,2}, v_{1,5}, v_{4,5}, v_{2,4}, v_{1,2}\}$

Thus $\tau(L_2(P_6)) = \left\lfloor \frac{(6-1)(6-2)}{6} \right\rfloor = 3$

For n=12, partition of vertices of $L_2(P_{12})$ is given by

$$V(L_2(P_{12})) = \left\{ \begin{matrix} v_{1,3}, v_{1,4}, v_{3,4};\ v_{1,7}, v_{1,8}, v_{2,10};\ v_{1,9}, v_{1,10}, v_{2,4};\ v_{2,5}, v_{2,6}, v_{1,11};\ v_{2,7}, v_{2,8}, v_{3,11}; \\ v_{3,5}, v_{3,6}, v_{2,9}; v_{3,7}, v_{3,8}, v_{2,3};\ v_{3,9}, v_{3,10}, v_{1,2};\ v_{4,6}, v_{4,7}, v_{5,11};\ v_{5,8}, v_{5,7}, v_{4,10}; \\ v_{8,9}, v_{8,11}, v_{9,11}; v_{4,8}, v_{4,9}, v_{4,5}; v_{6,7}, v_{6,10}, v_{7,10};\ v_{8,10}, v_{6,11}, v_{7,8};\ v_{6,8}, v_{6,9}, v_{1,5}; \\ v_{5,6}, v_{5,9}, v_{4,11};\ v_{5,10}, v_{7,9}, v_{9,10} \end{matrix} \right\} \cup \{v_{1,6}, v_{2,11}, v_{10,11}, v_{7,11}, v_{1,6}\}$$

$$\text{Thus } \tau(L_2(P_{12})) = \left\lfloor \frac{(12-1)(12-2)}{6} \right\rfloor = 18$$

For $n \ge 18$: The set of C_3s that contained in $L_2(P_n)$ are partitioned as S_1, S_k, S_l are given as follows.

$$S_1 = \{v_{1,3}, v_{1,4}, v_{4,3}\} \cup \left\{ \begin{matrix} v_{i,i+2}, v_{i,i+3}, v_{i-1,n-1} \\ 2 \le i \le n-6 \\ i \ne 2,8,14,\dots,n-11 \end{matrix} \right\} \cup \left\{ \begin{matrix} v_{i,i+2}, v_{i,i+3}, v_{i-1,4r+2j-1} \\ i = 2,8,14,\dots,n-11 \\ 1 \le j \le \frac{n-6}{6} \end{matrix} \right\} \Rightarrow |S_1| = n-6$$

For each k,2is lessthan or equal to k leq eq 2q-2

$$S_k = \left\{ \begin{matrix} v_{i,i+2k}, v_{i,i+2k+1}, v_{i+1,n-k} / i \text{ isodd}, \\ v_{i,i+2k}, v_{i,i+2k+1}, v_{i-1,n-k} / i \text{ iseven} \end{matrix} \right\}, 1 \le i \le n-3k-3, \quad \Rightarrow |S_k| = n-3k-3$$

$$\sum_{k=2}^{2q-2} |S_k| = \sum_{k=2}^{2q-2} (n-3k-3)$$

$$= (n-9) + (n-12) + (n-15) + \dots\dots 6 + 3$$

$$= 3 + 6 + 9 + \dots\dots + (n-9)$$

$$= \frac{(n-9)(n-6)}{6}$$

Remaining vertices are partitioned as

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$$S_l = \left\{ \begin{array}{l} v_{1,4q-1}, v_{1,4q}, v_{1,2}; v_{2,4q}, v_{2,4q+1}, v_{1,n-1}; v_{3,4q+1}, v_{3,4q+2}, v_{2,3}; v_{4,4q}, v_{4,4q+1}, v_{4,5}; v_{5,4q+1}, v_{5,4q+2}, v_{6,7}; \\ \qquad v_{6,4q+2}, v_{5,6}, v_{7,8}; \\ v_{7,4q+1}, v_{7,4q+2}, v_{8,9},; v_{8,4q+2}, v_{8,4q+3}, v_{7,n-1}; v_{9,4q+3}, v_{9,4q+4}, v_{9,10}; v_{10,4q+2}, v_{10,4q+3}, v_{10,11}; v_{11,4r+3}, v_{11,4q+4}, v_{12,13}; \\ \qquad v_{12,4q+4}, v_{11,12}, v_{13,14}; \\ v_{13,4q+3}, v_{13,4q+4}, v_{14,15},; v_{14,4q+4}, v_{14,4q+5}, v_{13,n-1}; v_{15,4q+5}, v_{15,4q+6}, v_{14,15}; v_{16,4q+4}, v_{16,4q+5}, v_{16,17}; v_{17,4q+5}, v_{17,4q+6}, v_{18,19}; \\ \qquad v_{12,4q+4}, v_{11,12}, v_{13,14}; \\ \qquad \ldots\ldots\ldots\ldots \qquad \ldots\ldots\ldots\ldots \\ \qquad v_{n-16,n-6}, v_{n-16,n-5}, v_{n-15,n-1}; \\ v_{n-15,n-5}, v_{n-15,n-4}, v_{n-15,n-14}; v_{n-14,n-6}, v_{n-14,n-5}, v_{n-14,n-13}; v_{n-13,n-5}, v_{n-13,n-4}, v_{n-12,n-11}; v_{n-12,n-4}, v_{n-13,n-12}, v_{n-11,n-10}; \\ v_{n-11,n-5}, v_{n-11,n-4}, v_{n-10,n-9}; \\ \qquad v_{n-10,n-4}, v_{n-10,n-3}, v_{n-11,n-10}; \\ v_{n-9,n-3}, v_{n-9,n-2}, v_{n-9,n-8}; v_{n-8,n-4}, v_{n-8,n-3}, v_{n-8,n-7}; v_{n-7,n-3}, v_{n-7,n-2}, v_{n-7,n-6}; v_{n-6,n-2}, v_{n-6,n-1}, v_{n-6,n-5} \end{array} \right\}$$

$$\cup \{v_{n-5,n-3}, v_{n-5,n-4}, v_{n-3,n-2}; v_{n-4,n-2}, v_{n-1,n-4}, v_{n-1,n-2}\} \cup \{v_{n-5,n-2}, v_{n-5,n-1}, v_{n-5,n-2}, v_{n-3,n-1}, v_{n-5,n-2}\}$$

$$|S_l| = (n-6)+2+1 = n-3$$

Clearly all cycles in the above sets are distinct and hence the total number of disjoint cyclic subgraphs =

$$|S_1| + \sum_{k=2}^{2r-2} |S_k| + |S_l| = (n-6) + \frac{(n-9)(n-6)}{6} + (n-3) = \frac{n(n-3)}{6}$$

Thus the vertex set is partitioned into $\left(\frac{n(n-3)}{6} - 1\right) C_3 s$ and a C_4

So, $\tau(L_2(P_n)) \geq \frac{n(n-3)}{6} \geq \left\lfloor \frac{(n-1)(n-2)}{6} \right\rfloor$

Case (ii): $n \equiv 1 (\bmod 6), n = 6q+1$.

Here $\left| \begin{pmatrix} |E(Pn)| \\ 2 \end{pmatrix} \right|$ is a multiple of 3. So, we partition all the vertices into C_3s.

For n=7, partition of vertices of $RL_2(P_7)$ is given by

$V(RL_2(P_7)) = \{v_{1,2}, v_{2,4}, v_{2,5}; v_{1,3}, v_{3,4}, v_{46}; v_{1,6}, v_{2,6}, v_{4,5}; v_{1,5}, v_{1,4}, v_{5,6}; v_{2,3}, v_{3,5}, v_{3,6}\}$

Thus $\tau(L_2(P_7)) = \left\lfloor \frac{(7-1)(7-2)}{6} \right\rfloor = 5$

For n=13,the vertex disjoint $C_3 s$ are given as

$$V(L_2(P_{13})) = \left\{ \begin{array}{l} v_{1,3}, v_{1,4}, v_{3,4}; v_{2,4}, v_{2,5}, v_{1,12}; v_{3,5}, v_{3,6}, v_{2,12}; v_{4,6}, v_{4,7}, v_{3,12}; v_{5,7}, v_{5,8}, v_{4,10}; v_{6,8}, v_{6,9}, v_{5,12}; v_{7,9}, v_{7,10}, v_{9,10}; \\ v_{1,5}, v_{1,6}, v_{2,11}; v_{2,7}, v_{1,11}, v_{2,6}; v_{12.}, v_{89}, v_{1\ 10}; v_{4,8}, v_{4,9}, v_{3,11}; v_{1,7}, v_{1,8}, v_{2,10}; v_{37}, v_{38}, v_{4\ 11}; v_{2,8}, v_{2,9}, v_{2,3}; \\ v_{3,9}, v_{3,10}, v_{4,12}; v_{5,9}, v_{5,10}, v_{5,6}; v_{4,5}, v_{5,11}, v_{6,7}; v_{6,10}, v_{6,11}, v_{7,8}; v_{7,11}, v_{9,12}, v_{10,12}; v_{8,12}, v_{8,9}, v_{7,12}; v_{8,10}, v_{8,11}, v_{10,11}; \\ v_{9,11}, v_{6,12}, v_{11,12} \end{array} \right\}$$

Thus $\tau(L_2(P_{13})) = 22$.

For $n \geq 19$, The cyclic decomposition of $L_2(P_n)$ is given as below.

The set of C_3s that contained in $RL_2(P_n)$ are partitioned as S_1, S_k, S_l and are given as

$$S_1 = \{v_{1,3}, v_{1,4}, v_{4,3}\} \cup \left\{\begin{matrix} v_{i,i+2}, v_{i,i+3}, v_{i-1,n-1} \\ 2 \le i \le n-6 \\ i \ne 5,17,29...n-8 \end{matrix}\right\} \cup \left\{\begin{matrix} v_{i,i+2}, v_{i,i+3}, v_{i-1,4q+2j} \\ i = 5,17,29...n-8 \\ 1 \le j \le q-1 \end{matrix}\right\} \Rightarrow |S_1| = n-6$$

For each k, $2 \le k \le 2q-1$

$$S_k = \left\{\begin{matrix} v_{i,i+2k}, v_{i,i+2k+1}, v_{i+1,n-k} / i \ isodd, \\ v_{i,i+2k}, v_{i,i+2k+1}, v_{i-1,n-k} / i \ iseven \end{matrix}\right\}, 1 \le i \le n-3k-3, \quad \Rightarrow |S_k| = n-3k-3$$

$$\sum_{k=2}^{2q-1} |S_k| = \sum_{k=2}^{2q-1} (n-3k-3)$$
$$= (n-9) + (n-12) + (n-15) + ...4 + 1$$
$$= 1 + 3 + 6 + ... + (n-9)$$
$$= \frac{(n-7)(n-8)}{6}$$

The remaining vertices can be decomposed into C_3s in the following way.

$$S_l = \left\{\begin{matrix} v_{1,2}, v_{1,4q+1}, v_{1,4q+2}; v_{2,3}, v_{2,4q}, v_{2,4q+1}; \\ v_{3,4q+1}, v_{3,4q+2}, v_{4,n-1}; v_{4,5}, v_{6,7}, v_{5,4q+3}; \\ v_{5,6}, v_{5,4q+1}, v_{5,4q+2}; v_{6,4q+2}, v_{7,8}, v_{6,4q+3}; v_{7,4r+3}, v_{7,4q+4}, v_{8,9}; v_{8,4q+2}, v_{8,4q+3}, v_{9,10}; \\ v_{9,4q+3}, v_{9,4q+4} v_{10,n-1}; v_{10,11}, v_{12,13}, v_{11,4q+5}; \\ v_{11,12}, v_{11,4q+3}, v_{11,4q+4}; v_{12,4q+4}, v_{12,4q+5}, v_{13,14}; v_{13,4q+5}, v_{13,4q+6}, v_{14,15}; v_{14,4q+4}, v_{14,4q+5}, v_{15,16}; \\ v_{15,4q+5}, v_{15,4q+6}, v_{16,n-1}; v_{16,17}, v_{17,4q+7}, v_{18,19}; \\ .... \\ v_{n-9,n-8}, v_{n-7,n-6}, v_{n-2,n-8}; v_{n-6,n-2}, v_{n-8,n-3}, v_{n-8,n-7}; \\ v_{n-7,n-3}, v_{n-7,n-2}, v_{n-6,n-5}; \end{matrix}\right\}$$

$$\cup \{v_{n-8,n-4}, v_{n-4,n-3}, v_{n-3,n-1}; v_{n-5,n-4}, v_{n-5,n-1}, v_{n-6,n-1}; v_{n-4,n-2}, v_{n-4,n-1}, v_{n-1,n-2}; v_{n-3,n-2}, v_{n-5,n-3}, v_{n-5,n-2}\}$$

Clearly allC3s in above sets are distinct

Thus $|S_l| = (n-7) + 4 = n-3$

Total number of disjoint cyclic subgraphs =

$$|S_1| + \sum_{k=2}^{2q-1} |S_k| + |S_l| = (n-6) + \frac{(n-7)(n-8)}{6} + (n-3) = (2n-9) + \frac{(n-7)(n-8)}{6} = \frac{(n-1)(n-2)}{6}$$

Thus the vertex set is partitioned into $\frac{(n-1)(n-2)}{6} C_3s$

So,
$$\tau(RL_2(P_n)) \ge \left\lfloor \frac{(n-1)(n-2)}{6} \right\rfloor$$

***Case* 3:** $n \equiv 2(\mathrm{mod}6)$,

In this case $\left|\binom{E(G)}{2}\right|$ is multiple of 3. So we partitioned all the vertices with C_3s.

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For $n=8$, C_3 decomposition is given by

$$V(L_2(P_8))=\{v_{1,3},v_{1,4},v_{3,4};v_{1,6},v_{1,7},v_{2,3};v_{2,4},v_{2,5},v_{3,7};v_{2,6},v_{2,7},v_{1,2};v_{5,6},v_{3,5},v_{3,6};v_{4,5},v_{5,7},v_{1,5};v_{4,6},v_{4,7},v_{6,7}\}$$

$$\tau(L_2(P_8))=\frac{7.6}{6}=7C_3s$$

For $n=14$, C_3 decomposition is given by

$$V(L_2(P_{14}))=\left\{\begin{array}{l} v_{1,3},v_{1,4},v_{3,4};v_{1,6},v_{1,5},v_{2,10};v_{1,7},v_{1,8},v_{2,11};v_{1,9},v_{1,10},v_{1,2};v_{2,4},v_{2,5},v_{1,13};v_{2,6},v_{2,7},v_{1,12};v_{2,8},v_{2,9},v_{1,11};\\ v_{3,5},v_{3,6},v_{2,13};v_{3,7},v_{3,8},v_{4,12};v_{4,6},v_{4,7},v_{3,13};v_{4,8},v_{4,9},v_{3,12};v_{5,7},v_{5,8},v_{4,13};v_{8,6},v_{6,9},v_{5,13};v_{7,9},v_{7,10},v_{6,13};\\ v_{8,10},v_{8,11},v_{7,13};v_{9,10},v_{11,13},v_{9,12};v_{9,11},v_{2,12},v_{11,12};v_{10,12},v_{10,13},v_{12,13};v_{7,9},\\ v_{9,13},v_{10,11},v_{8,9};v_{3,10},v_{3,11},v_{2,3};v_{4,10},v_{4,11},v_{4,5};v_{5,11},v_{5,12},v_{5,6};v_{6,10},v_{6,11},v_{6,7};\\ v_{7,11},v_{7,12},v_{7,8};v_{8,12},v_{8,13},v_{3,9};v_{5,9},v_{5,10},v_{6,12} \end{array}\right\}$$

Thus there are $\frac{13\times 12}{6}=26\ \ C_3s.$

For $n\geq 20$: In this case the disjoint cycles that contained in $RL_2(P_n)$ are partitioned as S_1,S_k,S_l that are given as below.

$$S_1=\{v_{1,3},v_{1,4},v_{3,4}\}\cup\{v_{i,i+2},v_{i,i+3},v_{i-1,n-1}\},2\leq i\leq n-6\ \Rightarrow |S_1|=n-6$$

$$S_2=\left\{\begin{array}{l} v_{i,i+4},v_{i,i+5},v_{i+1,n-2}/i\ isodd,\\ v_{i,i+4},v_{i,i+5},v_{i-1,n-2}/i\ iseven\\ i\neq 2,8,14...n-18, 1\leq i\leq n-9 \end{array}\right\}\cup\left\{\begin{array}{l} v_{i,i+4},v_{i,i+5},v_{i,4q+2j}\\ 1\leq j\leq q-2\\ i=2,8,14...n-18 \end{array}\right\}\Rightarrow |S_2|=n-9$$

$$S_k=\left\{\begin{array}{l} v_{i,i+2k},v_{i,i+2k+1},v_{i+1,n-k}/i\ isodd,\\ v_{i,i+2k},v_{i,i+2k+1},v_{i-1,n-k}/i\ iseven \end{array}\right\},1\leq i\leq n-3k-3,\ 3\leq k\leq 2r-1\ \Rightarrow |S_k|=n-3k-3$$

$$\sum_{k=3}^{2q-1}|S_k|=\sum_{k=3}^{2q-1}(n-3k-3)$$
$$=(n-12)+(n-15)+.......5+2$$
$$=\frac{(n-11)(n-10)}{6}$$

Remaining vertices are partitioned as

$$S_l=\left\{\begin{array}{l} v_{1,4q+1},v_{1,4q+2},v_{2,n-2};v_{2,3},v_{3,4q+3},v_{4,5};\\ v_{3,4q+1},v_{3,4q+2},v_{1,2};v_{4,4q+2},v_{4,4q+3},v_{5,6};v_{5,4q+3},v_{5,4q+4},v_{6,7};v_{6,4q+2},v_{6,4q+3},v_{7,8}\\ v_{7,4q+3},v_{7,4q+4},v_{8,n-2};v_{8,9},v_{10,11},v_{9,4q+5};\\ v_{9,4q+3},v_{9,4q+4},v_{9,10};v_{10,4q+4},v_{10,4q+5},v_{11,12};v_{11,4q+5},v_{11,4q+6},v_{12,13};v_{12,4q+4},v_{12,4q+5},v_{13,14}\\ v_{13,4q+5},v_{13,4q+6},v_{14,n-1};v_{14,15},v_{15,4q+7},v_{16,17};\\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\ v_{n-13,n-5},v_{n-13,n-4},v_{n-12,n-2};v_{n-12,n-11},v_{n-10,n-9},v_{n-3,n-11};\\ v_{n-11,n-5},v_{n-11,n-4},v_{n-11,n-10};v_{n-10,n-4},v_{n-10,n-3},v_{n-9,n-8};v_{n-9,n-3},v_{n-9,n-2},v_{n-8,n-7};v_{n-7,n-3},v_{n-6,n-2},v_{n-6,n-7};\\ v_{n-8,n-4},v_{n-8,n-3},v_{n-7,n-6};v_{n-7,n-3},v_{n-7,n-2},v_{n-6,n-5} \end{array}\right\}$$

$$\cup\left\{\begin{array}{l} v_{n-6,n-1},v_{n-5,n-4},v_{n-5,n-1};v_{n-5,n-3},v_{n-5,n-2},v_{n-3,n-2};\\ v_{n-4,n-2},v_{n-1,n-4},v_{n-2,n-1};v_{n-3,n-1},v_{n-4,n-3},v_{n-6,n-2}; \end{array}\right\}$$

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Thus $|S_l| = (n-7)+4 = n-3$

Clearly all cycles in above sets S1,S2,Sk,Sl are disjoint and hence the total number of cyclic subgraphs =

$$|S_1|+|S_2|+\sum_{k=3}^{2q-1}|S_k|+|S_l| = (n-6)+(n-9)+\frac{(n-11)(n-10)}{6}+(n-3) = (3n-18)+\frac{(n-11)(n-10)}{6} = \frac{(n-1)(n-2)}{6}$$

Thus the vertex set is partitioned into $\frac{(n-1)(n-2)}{6} C_3 s$

So, $\tau(L_2(P_n)) \geq \left\lfloor \frac{(n-1)(n-2)}{6} \right\rfloor$

Case 4: For n≡ 3(mod 6), n= 9, 15, 21……

In this case $\left|\binom{E(pn)}{2}\right|$ is not a multiple of 3. So we partitioned all the vertices with one C_4 and the remaining vertices with C_3s.

The distinct cycles of length 3 of $L_2(P_9)$ is given by

For $n \geq 15$: The set of $C_3 s$ that contained in $L_2(P_n)$ are partitioned as S_1, S_k, S_l that are given as below.

$$V(L_2(P_9)) = \left\{ \begin{matrix} v_{1,3}, v_{1,4}, v_{28}; v_{1,5}, v_{1,6}, v_{2,1}; v_{17}, v_{18}, v_{6,7}; v_{24}, v_{2,5}, v_{3,8}; v_{1,5}, v_{3,6}, v_{48}; \\ v_{4,5}, v_{4,6}, v_{58}; v_{37}, v_{47}, v_{68}; \end{matrix} \right\} U\{v_{26}, v_{23}, v_{27} v_{34}, v_{26}\}$$

$$S_1 = \{v_{1,3}, v_{1,4}, v_{4,3}\} \cup \left\{ \begin{matrix} v_{i,i+2}, v_{i,i+3}, v_{i-1,n-1} \\ 2 \leq i \leq n-6 \\ i \neq 2,7,13 \ldots n-8 \end{matrix} \right\} \cup \left\{ \begin{matrix} v_{i,i+2}, v_{i,i+3}, v_{i-1,4q+2j} \\ i = 2,7,13 \ldots n-8 \\ 1 \leq j \leq q \end{matrix} \right\} \Rightarrow |S_1| = n-6$$

$$S_k = \left\{ \begin{matrix} v_{i,i+2k}, v_{i,i+2k+1}, v_{i+1,n-k} / i \ isodd, \\ v_{i,i+2k}, v_{i,i+2k+1}, v_{i-1,n-k} / i \ iseven \end{matrix} \right\}, 1 \leq i \leq n-3k-3,\ 2 \leq k \leq 2q-1 \ \Rightarrow |S_k| = n-3k-3$$

$$\sum_{k=2}^{2q-1}|S_k| = \sum_{k=2}^{2q-1}(n-3k-3)$$
$$= (n-9)+(n-12)+(n-15)+\ldots\ldots.3 \ = 3+6+\ldots\ldots.+(n-9)$$
$$= \frac{(n-9)(n-6)}{6}$$

The remaining vertices can be decomposed into $C_3 s$ in the following way.

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$$S_l = \left\{ \begin{array}{l} v_{1,2}, v_{1,4q+1}, v_{1,4q+2}; v_{2,4q+2}, v_{2,4q+2}, v_{1,n-1}; v_{3,2}, v_{3,4q+3}, v_{3,4q+4}; v_{4,5}, v_{4,4q+2}, v_{4,4q+3}; v_{5,6}, v_{7,4q+3}, v_{7,4q+4}; \\ v_{6,n-1}, v_{5,4q+3}, v_{5,4q+4}; \\ \qquad v_{7,4q+5}, v_{6,7}, v_{8,9}; \\ v_{8,4q+4}, v_{8,4q+5}, v_{7,8}; v_{9,4q+5}, v_{9,4q+6} v_{9,10}; v_{10,4q+4}, v_{10,4q+5} v_{10,11}; v_{11,4q+5}, v_{11,4q+6}, v_{12,n-1}; v_{12,11}, v_{13,4q+5}, v_{13,4q+6}; \\ \qquad v_{13,4q+7}, v_{14,15}, v_{12,13}; \\ v_{14,4q+6}, v_{14,4q+7}, v_{13,14}; v_{15,4q+7}, v_{15,4q+8} v_{15,16}; v_{16,4q+6}, v_{16,4q+7} v_{16,17}; v_{17,4q+7}, v_{17,4q+8}, v_{18,n-1}; v_{18,17}, v_{19,4q+7}, v_{19,4q+8}; \\ \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \qquad v_{n-8,n-2}, v_{n-9,n-8}, v_{n-7,n-6}; \\ v_{n-7,n-8}, v_{n-6,n-1}, v_{n-6,n-2}; v_{n-6,n-2}, v_{n-6,n-1}, v_{n-5,n-4}; \end{array} \right\}$$

$$\cup \{v_{n-7,n-8}, v_{n-7,n-3}, v_{n-7,n-2}; v_{n-3,n-5}, v_{n-5,n-2}, v_{n-3,n-2};\} U \{v_{n-5,n-6}, v_{n-4,n-2}, v_{n-4,n-3}; v_{n-4,n-1}, v_{n-5,n-6}\}$$

$$|S_l| = (n-6) + 2 + 1 = n-3$$

Total number of cyclic subgraphs = $|S_1| + \sum_{k=2}^{2q-1} |S_k| + |S_l| = (n-6) + \frac{(n-9)(n-6)}{6} + (n-3) = \frac{n(n-3)}{6}$

Clearly, all cycles in above sets are distinct and the vertex set is partitioned into $\left(\frac{n(n-3)}{6} - 1\right) C_3 s$ and a C_4

So, $\tau(L_2(P_n)) \geq \frac{n(n-3)}{6} \geq \left\lfloor \frac{(n-1)(n-2)}{6} \right\rfloor$

***Case* 5:** $n \equiv 4 (\mathrm{mod}\, 6)$

In this case $\left|\binom{E(Pn)}{2}\right|$ is multiple of 3. So we partitioned all the vertices with C_3s.

For n=10, the C_3 decomposition is given by

$$V(L_2(P_{10})) = \left\{ \begin{array}{l} v_{1,3}, v_{1,4}, v_{3,4}; v_{1,6}, v_{1,5}, v_{2,9}; v_{1,7}, v_{1,8}, v_{2,4}; v_{2,5}, v_{2,6}, v_{1,9}; v_{2,7}, v_{2,8}, v_{3,5}; v_{3,6}, v_{3,7}, v_{4,9}; v_{3,8}, v_{3,9}, v_{1,2}; \\ v_{4,5}, v_{4,7}, v_{2,3}; v_{4,6}, v_{5,6}, v_{5,9}; v_{5,7}, v_{5,8}, v_{7,8}; v_{6,7}, v_{7,9}, v_{4,8}; v_{6,8}, v_{6,9}, v_{8,9} \end{array} \right\}$$

Thus there are $\frac{9 \times 8}{6} = 12 \quad C_3 s.$

For $n \geq 16$: The set of $C_3 s$ that contained in $RL_2(P_n)$ are partitioned as S_1, S_k, S_l that are given as below.

$$S_1 = \{v_{1,3}, v_{1,4}, v_{3,4}\} \cup \left\{ \begin{array}{l} v_{i,i+2}, v_{i,i+3}, v_{i-1,n-1} \\ i \neq 5,11,17 \ldots n-17 \\ 2 \leq i \leq n-6 \end{array} \right\} \cup \left\{ \begin{array}{l} v_{i,i+2}, v_{i,i+3}, v_{i-1,4q+2j} \\ i \neq 5,11,17 \ldots n-17 \\ 2 \leq j \leq q-1 \end{array} \right\}, \quad \Rightarrow |S_1| = n-6$$

For each k, $2 \leq k \leq 2q$

$$S_k = \left\{ \begin{array}{l} v_{i,i+2k}, v_{i,i+2k+1}, v_{i+1,n-k} / i \text{ is odd}, \\ v_{i,i+2k}, v_{i,i+2k+1}, v_{i-1,n-k} / i \text{ is even} \end{array} \right\}, 1 \leq i \leq n-3k-3, \quad \Rightarrow |S_k| = n-3k-3$$

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$$\sum_{k=2}^{2q}|S_k| = \sum_{k=2}^{2q}(n-3k-3)$$
$$=(n-9)+(n-12)+.......+1$$
$$=\frac{(n-7)(n-8)}{6}$$

The remaining vertices can be decomposed into C_3s in the following way.

$$S_l = \left\{\begin{array}{c} v_{1,2}, v_{1,4q+3}, v_{1,4q+4}; v_{2,3}, v_{2,4r+2}, v_{2,4q+3}; v_{3,4q+3}, v_{3,4q+4}, v_{4,n-1}; \\ v_{4,5}, v_{5,4q+5}, v_{6,7}; \\ v_{5,6}, v_{5,4q+3}, v_{5,4q+4}; v_{6,4q+4}, v_{6,4q+5}, v_{7,8}; v_{7,4q+5}, v_{7,4q+6} v_{8,9}; v_{8,4q+4}, v_{8,4q+5}, v_{9,10}; v_{9,4q+5}, v_{9,4q+6}, v_{10,n-1}; \\ v_{10,11}, v_{11,4q+7}, v_{12,13}; \\ \dots\dots\dots \\ v_{n-12,n-11}, v_{n-10,n-9}, v_{n-11,n-3}; \\ v_{n-11,n-5}, v_{n-11,n-4}, v_{n-11,n-10}; v_{n-11,n-10} v_{n-11,n-5}, v_{n-11,n-4}; v_{n-10,n-9}, v_{n-11,n-10}, v_{n-11,n-3}; v_{n-9,n-3}, v_{n-9,n-2}, v_{n-8,n-7}; \\ v_{n-8,n-3}, v_{n-7,n-6} v_{n-8,n-4}; v_{n-7,n-3}, v_{n-7,n-2}, v_{n-3,n-2} \end{array}\right\}$$

$$\cup\{v_{n-6,n-4}, v_{n-5,n-3}, v_{n-5,n-2}; v_{n-5,n-3}, v_{n-6,n-1} v_{n-1,n-2}; v_{n-4,n-2}, v_{n-5,n-4}, v_{n-3,n-2}; v_{n-3,n-1}, v_{n-4,n-3}, v_{n-4,n-1}\}$$

All cycles in the above set are distinct and
Total number of cyclic subgraphs =

$$|S_1|+\sum_{k=2}^{2q}|S_k|+|S_l| = (n-6)+\frac{(n-7)(n-8)}{6}+(n-3)=\frac{(n-1)(n-2)}{6}$$

$$\tau(L_2(P_n)) \geq \left\lfloor \frac{(n-1)(n-2)}{6} \right\rfloor$$

***Case* 6**: $n \equiv 5(\bmod 6)$

In this case $\left|\binom{E(Pn)}{2}\right|$ is multiple of 3. So we partitioned all the vertices with C_3s.

For n=11, the C_3 decomposition is given by

$$V(L_2(P_n)) = \left\{\begin{array}{l} v_{1,3}, v_{1,4}, v_{3,4}; v_{1,6}, v_{1,5}, v_{2,9}; v_{1,7}, v_{1,8}, v_{1,2}; v_{2,4}, v_{2,5}, v_{1,10}; v_{2,6}, v_{2,7}, v_{1,9}; v_{3,5}, v_{3,6}, v_{2,10}; v_{4,6}, v_{4,7}, v_{3,10}; v_{5,7}, v_{5,8}, v_{4,10}; \\ v_{2,8}, v_{2,3}, v_{3,8}; v_{4,8}, v_{4,9}, v_{4,5}; v_{5,9}, v_{5,10}, v_{5,6}; v_{6,8}, v_{6,9}, v_{8,9}; v_{7,9}, v_{7,10}, v_{9,10}; v_{7,8}, v_{6,10}, v_{3,9}; v_{6,7}, v_{3,7}, v_{8,10} \end{array}\right\}$$

Thus there are $\frac{9\times 10}{6} = 15 \ C_3 s.$

For $n \geq 17$: The set of $C_3 s$ that contained in $L_2(P_n)$ are partitioned as S_1, S_k, S_l that are given as below.

$$S_1 = \{v_{1,3}, v_{1,4}, v_{3,4}\} \cup \{v_{i,i+2}, v_{i,i+3}, v_{i-1,n-1}\}, 2 \leq i \leq n-6 \Rightarrow |S_1| = n-6$$

$$S_2 = \left\{\begin{array}{l} v_{i,i+4}, v_{i,i+5}, v_{i+1,n-2} / i \ isodd, \\ v_{i,i+4}, v_{i,i+5}, v_{i-1,n-2} / i \ iseven \\ i \neq 1,7,13...n-8, 1 \leq i \leq n-9 \end{array}\right\} \cup \left\{\begin{array}{l} v_{i,i+4}, v_{i,i+5}, v_{i,4q+2j} \\ 2 \leq j \leq q+1 \\ i = 1,7,13...n-8 \end{array}\right\} \Rightarrow |S_2| = n-9$$

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$$S_k = \begin{Bmatrix} v_{i,i+2k}, v_{i,i+2k+1}, v_{i+1,n-k} / i\ isodd, \\ v_{i,i+2k}, v_{i,i+2k+1}, v_{i-1,n-k} / i\ iseven \end{Bmatrix}, 1 \le i \le n-3k-3,\ 3 \le k \le 2q\ \Rightarrow |S_k| = n-3k-3$$

$$\sum_{k=3}^{2q} |S_k| = \sum_{k=3}^{2q} (n-3k-3)$$

$$= (n-12) + (n-15) + ...5 + 2$$

$$= \frac{(n-11)(n-10)}{6}$$

The remaining vertices can be decomposed into $C_3 s$ as

$$S_l = \begin{Bmatrix} v_{1,4q+3}, v_{1,4q+4}, v_{2,n-2}; \\ v_{2,1}, v_{3,4q+3}, v_{4,4q+4}; v_{3,4q+5}, v_{2,3}, v_{4,5}; v_{4,4q+4}, v_{4,4r+5}, v_{5,6}; v_{5,4q+5}, v_{5,4q+6}, v_{6,7}; v_{6,4q+4}, v_{6,4q+5}, v_{7,8}; \\ v_{7,4q+5}, v_{7,4q+6}, v_{8,n-2}; \\ v_{89}, v_{10,11}, v_{9,4q+7}; v_{9,10}, v_{9,4q+5}, v_{9,4q+6}; v_{10,4q+6}, v_{10,4r+7}, v_{11,12}; v_{11,4q+7}, v_{11,4q+8}, v_{12,13}; v_{12,4q+6}, v_{12,4q+7}, v_{13,14}; \\ v_{13,4q+7}, v_{13,4q+8}, v_{14,n-2}; \\ ....\ ....\ ....\ ..... \\ ....\ ....\ ....\ ..... \\ v_{n-10,n-4}, v_{n-10,n-3}, v_{n-9,n-2}; \\ v_{n-9,n-8}, v_{n-7,n-6}, v_{n-8,n-2}; v_{n-8,n-4}, v_{n-8,n-3}, v_{n-8,n-7}; v_{n-7,n-6}, v_{n-9,n-8}, v_{n-8,n-2}; v_{n-6,n-5} v_{n-7,n-3}, v_{n-7,n-2} \end{Bmatrix}$$

$$\cup \{v_{n-6,n-4}, v_{n-5,n-3}, v_{n-5,n-2}; v_{n-5,n-3}, v_{n-6,n-1} v_{n-1,n-2}; v_{n-4,n-2}, v_{n-5,n-4}, v_{n-3,n-2}; v_{n-3,n-1}, v_{n-4,n-3}, v_{n-4,n-1}\}$$

Clearly all cycles in ther above sets ate distinct and Total number of cyclic subgraphs =

$$|S_1| + |S_2| + \sum_{k=3}^{2q} |S_k| + |S_l| = (n-6) + (n-9) + \frac{(n-11)(n-10)}{6} + (n-3) = \frac{(n-1)(n-2)}{6}$$

So, $\tau(L_2(P_n)) \ge \dfrac{(n-1)(n-2)}{6}$

Now, by definition $L_2(P_n)$ has $\dfrac{(n-1)(n-2)}{2}$ vertices and hence

$$\tau(L_2(P_n)) \le \left\lfloor \frac{(n-1)(n-2)}{6} \right\rfloor \text{------------------------------(2)}$$

Thus in all the above cases, from 1,2 we have $\tau(L_2(P_n)) = \dfrac{(n-1)(n-2)}{6}$

Conclusion:

In this paper we derived the tulgeity of superline graph of path graph. Further we wish to extend this work to superline graph of wheel graph

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