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THE CONNECTED EDGE FIXING EDGE-TO-EDGE GEODETIC NUMBER OF A GRAPH

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ABSTRACT

In this article, we introduce the concept of the connected edge fixing edge-to-edge geodetic number $g_{cefee}(G)$for an edge eof a graph G. The connected edge fixing edge-to-edge geodetic number of certain classes of graphs including path, cycles, trees, complete graphs are studied. Connected graphs of size q with $g_{cefee}(G) = q - 1$ are characterized. It is shown that for a positive integers r, $d$and $\ell$with $r < d < 2r$, there exists a connected graph G with $rad(G) = r$, $diam(G) = d$ and $g_{cefee}(G) = \ell$ or $\ell - 1$ for some $e \in E(G)$.

KEYWORDS: connected edge fixing edge-to-edge geodetic number, connected edge-to-edge geodetic number, edge-to-edge geodetic number, distance,edge-to-edge distance.

1. INTRODUCTION

By a graph $G = (V, E)$, we mean a finite undirected connected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. We consider connected graphs with at least three vertices. For basic definitions and terminologies we refer to [1]. If $e = \{u, v\}$ is an edge of a graph G, we write $e = uv$, we say that e joins the vertices u and v; uand vare adjacent vertices; u andv are incident with e. The degree of a vertex v in a graph G is the number of edges of Gincident with v and is denoted by $deg_G(v)$ or $deg(v)$.A vertex v is an extreme vertex of G if the sub-graph induced by its neighbors is complete. An edge e is an extreme edge of a graph G if at least one end of e is an extreme vertex of G. For vertices u and v in a connected graph G, the distance $d(u, v)$ is the length of a shortest $u - v$ path in G. An $u - v$path of length $d(u, v)$ is called an $u - v$geodesic.The eccentricity e(u) of a vertex u is defined by $e(v) = \max \{d(u,v) : u \in V\}$.Each vertex in V at which the eccentricity function is minimized is called a central vertex of G and the set of all central vertices of G is called the center of G and is denoted by Z(G).The radius r and diameter d of G are defined by $r = \min \{e(v) : v \in V\}$ and $d = \max \{e(v) : v \in V\}$ respectively. This gives rise the concept of the geodetic number and the edge geodetic number of a graph [2-13]. For subsets A and B of V(G), the distance $d(A, B)$ is defined as $d(A, B) = \min\{d(x, y) : x \in A, y \in B\}$. An $u - v$ path of length $d(A, B)$ is called an $A - B$geodesic joining the sets A, B where $u \in A$ and $v \in B$. A set $S \subseteq E$ is called an edge-to-edge geodetic set of G if every edge of G is an element of Sor lies on a geodesic joining a pair of edges of S. The edge-to-edge geodetic number $g_{ee}(G)$ of G is the minimum cardinality of its edge-to- edge geodetic sets and any edge-to- edge geodetic set of cardinality $g_{ee}(G)$ is said to be a g_{ee}-set of G. The edge-to-edge geodetic number of a graph was studied in[1].

The following theorems are used in sequel.

Theorem 1.1. [1] If $v$is an extreme vertex of a connected graph G, then every edge-to-edge geodetic set contains at least one extreme edge is incident with v.

Theorem 1.2. [1] For any non-trivial tree T with k end vertices,$g_{ee}(T) = k$.

II. Connected Edge Fixing Edge-to-Edge Geodetic Number of a graph

Definition 2.1. Let ebe an edge of a connected graphG. A set $M(e) \subseteq E(G) - \{e\}$ is called a connected edge fixing edge-to-edge geodetic set of e of a graph G, if every edge of Glies on an$e - f$geodesic, where $f \in M(e)$. The connected edge fixing edge-to-edge geodetic number $g_{cefee}(G)$ of G is the minimum cardinality of its connected edge fixing edge-to-edge geodetic sets and any

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connected edge fixing edge-to-edge geodetic set of cardinality $g_{cefee}(G)$is ag_{cefee}-set of G.

Example 2.2. For the graph$G$ given in Figure 2.1, the connected edge fixing edge-to-edge geodetic sets of each edge of G is given in the following Table I.

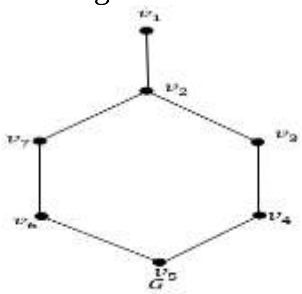

Figure 2.1

Table: I

Fixing Edge (e)	**Minimum connected edge fixing edge-to-edge geodetic sets ($M(e)$)**	$g_{cefee}(G)$
v_1v_2	$\{v_4v_5, v_5v_6\}$	2
v_2v_3	$\{v_1v_2, v_2v_7, v_6v_7, v_5v_6\}$	4
v_3v_4	$\{v_1v_2, v_6v_7, v_2v_7\}$	3
v_4v_5	$\{v_1v_2, v_2v_7\}$	2
v_5v_6	$\{v_1v_2, v_2v_3\}$	2
v_6v_7	$\{v_1v_2, v_2v_3, v_3v_4\}$	3
v_2v_7	$\{v_1v_2, v_2v_3, v_3v_4, v_4v_5\}$	4

Remark 2.3. Edge e for a connected graph G does not belong to any connected edge fixing edge-to-edge geodetic set $M(e)$. Moreover, the connected edge-fixing edge-to-edge geodetic set of an edge e is not unique.

Theorem 2.4. Let v be the most extreme vertex and e the edge of a connected graph G such that v is not incident with e.Then, irrespective of whether e is an extreme edge or not, every connected edge fixing edge-to-edge geodetic set of $e$of G contains at least one extreme edge that is incident with v.

Proof: Let$M(e)$ be any connected edge fixing edge-to-edge geodetic set of $e$of G, and let$e_1, e_2,\cdots, e_l$ be the edges incident with v. We claim $e_i \in M(e)$ for some $i(1 \leq i \leq l)$. Suppose that $e_i \notin M(e)$ for all $i(1 \leq i \leq l)$.The vertex $v$is lying on the connected edge-to-edge geodetic path connecting a vertex, say x, incident with e and $y \in V, M(e)$, since $M(e)$ is a connected edge fixing edge-to-edge geodetic set of $e$of G. $v$is not an extreme vertex of G since it is an internal vertex of a connected edge-to-edge geodetic path, $x - y$, which is a contradiction.Hence $e_i \in M(e)$ for some i $(1 \leq i \leq k)$.

Corollary: 2.5. Let e be an edge of G such that e is not an end edge of G.Then, every end edge of Gother than e is a part of every connected edge that fixes the edge-to-edge geodetic set$e$of G.

Proof: This follows from Theorem 2.4.

Theorem: 2.6. Let $M(e)$ be a connected edge fixing edge-to-edge geodetic set of G and let Gbe a connected graph. Let $f$be a cut-edge of G, which is not an end edge of G and let G_1 and G_2be the two components of $G - \{f\}$.

(i) If $e = f$,then an element of $M(e)$ is contained in each of the two components of$G - \{f\}$.

(ii) If $e \neq f$, then $M(e)$ contains at least one edge of components of $G - \{f\}$ where e does not lie.

Proof: Let $f = uv$.Let G_1 and G_2 be the two component of $G-\{f\}$ such that$u \in V(G_1)$ and $v \in V(G_2)$. Let$e = f$. Assume that $M(e)$ does not contain any element of G_1. Then$M(e) \subseteq E(G_2)$. Suppose $h$is an edge of$E(G_1)$. Then $h$must lie on an$e-f'$ geodetic path $P: v, v_1, v_2, \dots, v_l, v, u, u_1, u_2, \dots, u_s, u, v,$
$v_1, v_2, \dots,, v'$Where$v_1, v_2, \dots, v_l \in V(G_2), u_1, u_2, \dots, u_s \in V(G_1)$,
Wherev' is an end of $f' \in M(e)$. Hence v lies twice in P, which is a contradiction to P a geodetic path.By using a similar justification, we may demonstrate that if $e \neq f$, then $M(e)$ contains at least one edge of $G - \{f\}$ components where $e$does not lie.

Theorem: 2.7. Let $M(e)$be a minimum connected edge fixing edge-to-edge geodetic set of an edge $e$of $G$and G be a connected graph and f be a cut-edge.

(i)If $e = f$ is an end-edge of G then $e \notin M(e)$.

(ii) If f is not an end-edge of G then $e \in M(e)$.

Proof: Let $M(e)$ represent a minimum connected edge fixing edge-to-edge geodetic set for an edge$e = uv$ of G. Let $f = u'v'$ be an edge G_1 contains an edge xy and G_2 contains an edge $x'y'$ where $xy, x'y' \in M(e)$. Since$G[M(e)]$ is connected, $f \in M(e)$.

Theorem: 2.8. For any non-trivial tree T with $k$end edges, $g_{cefee}(T) = \begin{cases} k & \text{if } e \text{ is an internal edge of } T \\ k-1 & \text{if } e \text{ is an end edge of } T \end{cases}$

Proof: This follows from Corollary 2.5 and Theorem 2.7.

Corollary: 2.9. For a star $G = K_{1,q}$, $g_{cefee}(G) = q - 1$for any edge e of G.

Theorem: 2.10. For a positive integer$r, d$and $\ell > d - r + 2$ and $l \geq d$with r ≤ d ≤ $2r$, there exists a connected graph G with $rad(G) = r, diam(G) = d$ and $g_{cefee}(G) = \ell$.

Proof: If $l = 2$, consider G to be any path with at least three vertices. Let $l \geq 3$ and let $P_{d-r+1}: u_0, u_1, u_2, \dots . u_{d-r}$ be a path of length $d - r + 1. C_{2r}: v_1, v_2, \dots . v_{2r}, v_1$ be a cycle of length $2r$. By locating v_1 in C_{2r} and u_0 in P_{d-r+1}, we may construct the graph H from C_{2r} and u_0 in P_{d-r+1}. In order to create the graph G, join each vertex $w_i (1 \leq r \leq l - d + r - 2)$ to the vertex u_{d-r+1}, add $(l - d + r - 2)$ new vertices $w_1, w_2, \dots . w_{\ell-d+r-2}$ to H.By Theorem 2.4 and Corollary 2.5, Z is a subset of everyconnected edge fixing the edge-to-edge geodetic set of an edge $e$of G and so $g_{cefee}(G) \geq l - 1$. It is clear that Z is not an connected edge fixing the edge-to-edge geodetic set of an edge $e$of G and so $g_{cefee}(G) \geq l$. Let $S = Z \cup \{v_1 v_2\}$. Then S is a connected edge fixing the edge-to-edge geodetic set of an edge $e$of G so that $g_{cefee}(G) = l$.

3. CONCLUSIONS

With the contribution of the connected edge fixing edge-to-edge geodetic number of a graph, we can introduce the forcing connected edge fixing edge-to-edge geodetic number$f_{g_{cefee}}(G)$ of an edge $e$of G.The forcing connected edge fixing edge-to-edge geodetic *number* of certain graphs can be studied.

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