e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.12, 2022
Research Paper

SOME FUNCTIONS RELATED TO Š α^* - OPEN SET IN SOFT TOPOLOGICAL SPACES

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ABSTRACT: In this paper, we introduce some soft functions like Š Perfectly α^* - continuous function, Š Totally α^* - continuous function. We study the connections of these function with other Š function . Also, we establish the relationships in between the above functions and also investigate various aspects of these functions.

Keywords and phrases: Š Perfectly α^* - continuous^s function, Š Totally α^* - continuous^s function.

1. INTRODUCTION

Molodtsov introduced the concept of soft sets from which the difficulties of fuzzy sets, intutuionistic fuzzy sets,vague sets , interval mathematics and rough sets have been rectified. Application of soft sets in decision making problems has been found by Maji et al. whereas Chen gave a parametrization reduction of soft sets and a comparison of it with attribute reduction in rough set theory. Further soft sets are a class of special information. Shabir and Naz introduced soft topological spaces in 2011 and studied some basic properties of them. Meanwhile generalized closed sets in topological spaces were introduced by Levine in 1970 and recent survey of them is in which is extended to soft topological spaces in the year 2012. Recently papers about soft sets and their applications in various fields have increased largely. Modern topology depends strongly on the ideas of set theory. Any Research work should result in addition to the existing knowledge of a particular concept.

2. PRELIMINARIES

In this section, this project X be an initial universe and Ê be a set of parameters. Let P (X) denote the power set of X and A be a non – empty subset of ξ. A pair ($F^š$, A) denoted by $F^š{}_a$ is called a soft set over X, where $F^š$ is a mapping given by $F^š : A \rightarrow P(X)$.

Definition 2.1.1 [8]: For two soft sets ($F^š$, A) and (G , B) over a common universe X, we say that ($F^š$, A) is a soft subset of (G , B) denoted by ($F^š$, A) $\subseteq_s$ (G , B), ifA $\subseteq_s$ B and $F^š$ (e) $\subseteq_s$ G(e) for all e $\in$ ξ

Definition 2.1.2 [8]: The complement of a soft set ($F^š$, A) denoted by ($F^š$, A)^c , is defined by (($F^š$, A))^c = (($F^{š\,c}$, A) , where $F^{š\,c}$: A $\rightarrow$ P (X) is a mapping given by $F^{š\,c}$(e) = X - $F^š$ (e), for all e $\in$ ξ.

Definition 2.1.3 [7]: Let a Š set ($F^š$, A) over X .

a. Null Š Set denoted by φ if for all e $\in$ A, $F^š$ (e) = φ.; b. Absolute Š Set denoted by X, if for all e $\in$ A, $F^š$ (e) = X. Clearly, X^c= φ and $φ^c$= X.

Definition 2.1.4 [4]: The Union of two Š sets of ($F^š$, A) and (G , B), over the common universe X is the Š set(H ,C). where C = A $\cup_s$ B, and for all e $\in$ C

H (e) = $F^{š}$ (e) if e ∈ A - B , H (e) = G (e) , if e ∈ B -A and H (e) = $F^{š}$ (e) $\cup_s$ G (e), if e ∈ A $\cap_s$ B . and is denoted as ($F^{š}$, A) $\cup_s$ G (e) = (H ,C).

Definition 2.1.5 [4]: The Intersection (H ,C) of two Š sets of (F , A) and (G , B), over the common universe X , denoted ($F^{š}$, A) (G , B) is the Š set(H ,C). where C = A $\cap_s$ B, and H (e) = $F^{š}$ (e) $\cap_s$ G (e), for all e ∈ C .

***Definition 2.1.6 [10]*:** Let τ be a collection of Š sets over X with the fixed set ξ of parameters.

Then τ is called a Š Topology on X, if

i. Φ and X belongs to τ_s.
ii. The union of any number of Š sets in τ_s belongs to τ_s.
iii. The intersection of any two Š sets in τ_s belongs to τ_s.

The triplet (X, τ_s, Ê) is called Š Topological Spaces over X.
The members of τ_s are called Š Open sets in X and complements of them are called Š Closed sets in X.
***Definition 2.1.6*:** A Subset of a Š topological space (X, τ_s , ξ) is said to be

1. a Š Semi-Open set [3] if ($F^{š}$, Ê) $\subseteq_s$ Š Cl(Š int($F^{š}$, Ê) and a Š Semi-Closed set if Š int(Š Cl($F^{š}$, Ê) $\subseteq_s$ ($F^{š}$, Ê).
2. a Š Pre-Open set [1] if ($F^{š}$, Ê)$\subseteq_s$ Š Int(Š Cl($F^{š}$, Ê)) and a Š Pre-Closed set if Š Cl (Š int($F^{š}$, Ê) $\subseteq_s$ ($F^{š}$, Ê) a Š α-Open set [1] if ($F^{š}$, Ê)$\subseteq_s$ Š Int(Š Cl(int($F^{š}$, Ê) and a Š α-Closed set if Š Cl(Š int(Š Cl($F^{š}$, Ê) $\subseteq_s$ ($F^{š}$, Ê)).
3. a Š β-Open set [1] if ($F^{š}$, Ê)$\subseteq_s$ Š cl(Š int(Š cl($F^{š}$, Ê))) and a Š β-Closed set if Š Int(Š Cl(int($F^{š}$, Ê)) $\subseteq_s$ ($F^{š}$, Ê)).
4. a Š - generalized Closed set (briefly Š gs - Closed) if Š Cl($F^{š}$, Ê) $\subseteq_s$ (G , ξ) whenever ($F^{š}$, Ê) $\subseteq_s$ (G , ξ) and (G , ξ) is Š Open in (X, τ_s, ξ). The complement of a Š gs -Closed set is called a Š gs-Open set.
5. a Š Semi-generalized Closed set(briefly Š Sg-Closed) if Š Cl($F^{š}$, Ê) $\subseteq_s$ (G , ξ) whenever ($F^{š}$, Ê) $\subseteq_s$ (G , ξ)and (G, ξ) is Š semi Open in (X, τ_s, ξ). The complement of a Š Sg-Closed set is called a Š Sg-Open set.
6. a generalized Š Semi-Closed set(briefly gs-Closed) if Š Cl($F^{š}$, Ê) $\subseteq_s$ (G , ξ) whenever ($F^{š}$, Ê) $\subseteq_s$ (G , ξ) and (G , ξ) is Š Open in (X, τ_s, ξ). The complement of a Š gs-Closed set is called a Š gs-Open set.
7. a Š - Closed[9] if Š Cl(F , ξ) $\subseteq_s$(G , ξ) whenever (F , ξ) $\subseteq_s$ (G , ξ) and (G , ξ) is Š semi Open in (X, τ_s, ξ)
8. a Š ω -Closed[9] if Š Cl($F^{š}$, Ê) $\subseteq_s$ (G , ξ) whenever (F , ξ) $\subseteq_s$ (G , ξ) and (G , ξ) is Š semi Open
9. αa Š alpha-generalized Closed set (briefly Š αg-Closed) if α Š Cl($F^{š}$, Ê) $\subseteq_s$(G , ξ) whenever ($F^{š}$, Ê) $\subseteq_s$ (G , Ê)and (G , Ê) is Š α Open in (X, τ_s, Ê). The complement of a Š αg-Closed set is called a Š αg-Open set.
10. a Š generalized alpha Closed set (briefly Š gα-Closed) if α Š Cl($F^{š}$, Ê) $\subseteq_s$ (G , Ê) whenever ($F^{š}$, Ê) $\subseteq_s$ (G , Ê) and (G , Ê) is Š Open in (X, τ_s, Ê). The complement of a Š gα-Closed set is called a Š gα-Open set.
11. a Š generalized pre Closed set (briefly Š gp-Closed) [1] if p Š Cl($F^{š}$, Ê) $\subseteq_s$ (G , Ê) whenever a Š gp-Open set.
12. a Š generalized pre regular Closed set (briefly Š gpr-Closed) [5] if p Š Cl($F^{š}$, Ê)$\subseteq_s$ (G , Ê) whenever ($F^{š}$, Ê) $\subseteq_s$ (G , Ê) and (G , Ê) is Š regular Open in (X, τ_s, Ê). The complement

of a Š gpr-Closed set is called a Š gpr - Open set.

Perfectly Šα*- continuous^s function

Definition 3.2.1: A Š function f : (X, τ_s, Ê) →(Ý, τ_s, K) is said to be perfectly Šα*- continuous^s function, if the inverse image of every Š α*- Ô(Ý) in (Ý, τ_s, K) is both Š - Ô(X) and Š - Ç(X) in (X, τ_s, Ê).

Theorem 3.2.2: Let f : (X, τ_s, Ê) --→ (Ý, τ_s, K) be perfectly Šα*- continuous^s function, then it is strongly Šα*- continuous^s function.

Proof:

Let ($F^{\check{s}}$, ξ) be Š α*- Ô(X) in (Ý, τ_s, K). Since, f is perfectly Šα*- continuous^s function, then f^{-1} ($F^{\check{s}}$, ξ) is both Š - Ô(X) and Š - Ç(X) in (X, τ_s, Ê)⇒ f^{-1}($F^{\check{s}}$, Ê) is Š - Ô(X) in (X, τ_s, Ê)

Therefore, f is strongly Š α* - continuous^s.

Remark 3.2.3: The converse of the above theorem need not be true.

Example 3.2.4: Let X = Ý = { x_1, ,x_2},
τ_s = {$F^{\check{s}}_1$,$F^{\check{s}}_2$, $F^{\check{s}}_3$, $F^{\check{s}}_4$, $F^{\check{s}}_5$, $F^{\check{s}}_6$, $F^{\check{s}}_7$,$F^{\check{s}}_8$,$F^{\check{s}}_{10}$,$F^{\check{s}}_{12}$,$F^{\check{s}}_{13}$,$F^{\check{s}}_{14}$,$F^{\check{s}}_{15}$,$F^{\check{s}}_{16}$ }, τ_s^c = {$F^{\check{s}}_1$,$F^{\check{s}}_2$, $F^{\check{s}}_3$, $F^{\check{s}}_4$, $F^{\check{s}}_6$, $F^{\check{s}}_7$,$F^{\check{s}}_9$,$F^{\check{s}}_{10}$, $F^{\check{s}}_{11}$, $F^{\check{s}}_{12}$,$F^{\check{s}}_{13}$,$F^{\check{s}}_{14}$,$F^{\check{s}}_{15}$,$F^{\check{s}}_{16}$ }, and σ_s = {$F^{\check{s}}_2$, $F^{\check{s}}_{10}$, $F^{\check{s}}_{11}$, $F^{\check{s}}_{15}$,$F^{\check{s}}_{16}$},
Š α*- Ô(Ý)= {$F^{\check{s}}_1$,F_4, $F^{\check{s}}_5$,$F^{\check{s}}_7$,$F^{\check{s}}_8$,$F^{\check{s}}_6$,$F^{\check{s}}_{10}$,$F^{\check{s}}_{12}$,$F^{\check{s}}_{13}$,$F^{\check{s}}_{14}$,$F^{\check{s}}_{15}$,$F^{\check{s}}_{16}$ }. Let f : (X, τ_s, Ê)→(Ý, τ_s, K)be defined by f($F^{\check{s}}_1$) = $F^{\check{s}}_5$, f($F^{\check{s}}_2$) = $F^{\check{s}}_{11}$ f($F^{\check{s}}_3$) = $F^{\check{s}}_8$ f($F^{\check{s}}_4$) = $F^{\check{s}}_1$ f($F^{\check{s}}_5$) = $F^{\check{s}}_3$ f($F^{\check{s}}_6$) = $F^{\check{s}}_{13}$, f($F^{\check{s}}_7$) = $F^{\check{s}}_4$ f($F^{\check{s}}_8$) = $F^{\check{s}}_{14}$, f($F^{\check{s}}_9$) = $F^{\check{s}}_9$, f($F^{\check{s}}_{10}$) = $F^{\check{s}}_{10}$, f($F^{\check{s}}_{11}$) = $F^{\check{s}}_{11}$, f($F^{\check{s}}_{12}$) = $F^{\check{s}}_{12}$, f($F^{\check{s}}_{13}$) = $F^{\check{s}}_6$, f($F^{\check{s}}_{14}$) = $F^{\check{s}}_7$ f($F^{\check{s}}_{15}$) = $F^{\check{s}}_{15}$ f($F^{\check{s}}_{16}$) = $F^{\check{s}}_{16}$. Clearly f is strongly Š α*– continuous^s. Hence f is not perfectly Šα*- continuous^s function, because f^{-1}($F^{\check{s}}_{14}$) = $F^{\check{s}}_8$ is Š - Ô(X) but not Š - Ç(X) in (X, τ_s, Ê).

Theorem 3.2.5: Let f : (X, τ_s, Ê) → (Ý, τ_s, K) be perfectly Šα*- continuous^s function, then it is perfectly Š - continuous^s function.

Proof:

Let ($F^{\check{s}}$, Ê) be Š - Ô(X) in (Ý, τ_s, K). Then ($F^{\check{s}}$, Ê) be Š α*- Ô(X) in (Ý, τ_s, K). Since, f is perfectly Šα*- continuous^s function, then f^{-1} ($F^{\check{s}}$, Ê) is both Š - Ô(X) and Š - Ç(X) in (X, τ_s, Ê). Therefore, f is perfectly Š - continuous^s function.

Remark 3.2.6: The converse of the above theorem need not be true.

3.3: Totally Šα*-continuous function

Defnition 3.3.1: A Š function f :(X, τ_s, Ê) → (Ý, τ_s, K) is said to be totally Šα*- continuous^s function, if the inverse image of every Š - Ô(Ý) in (Ý, τ_s, K) is both Š α*- Ô(X) and Š α*- Ç(X) in (X, τ_s, Ê).

Example 3.3.2: Let X = Ý = { x_1, ,x_2}, τ_s = { $F^{\check{s}}_5$,$F^{\check{s}}_{10}$,$F^{\check{s}}_{15}$,$F^{\check{s}}_{16}$ }, and σ_s = {$F^{\check{s}}_1$,$F^{\check{s}}_{13}$,$F^{\check{s}}_{15}$,$F^{\check{s}}_{16}$},σ_s^c = {$F^{\check{s}}_2$,$F^{\check{s}}_{14}$,$F^{\check{s}}_{15}$,$F^{\check{s}}_{16}$},
Š α*- Ô(X)= {$F^{\check{s}}_1$,F_2, $F^{\check{s}}_3$, $F^{\check{s}}_4$, $F^{\check{s}}_5$, $F^{\check{s}}_6$ $F^{\check{s}}_7$,$F^{\check{s}}_8$,$F^{\check{s}}_9$,$F^{\check{s}}_{10}$,$F^{\check{s}}_{12}$,$F^{\check{s}}_{11}$, $F^{\check{s}}_{13}$,$F^{\check{s}}_{14}$, $F^{\check{s}}_{15}$,$F^{\check{s}}_{16}$ }, Š α*- Ç(X)= {$F^{\check{s}}_1$,F_2, $F^{\check{s}}_3$, $F^{\check{s}}_4$, $F^{\check{s}}_5$, $F^{\check{s}}_6$ $F^{\check{s}}_7$,$F^{\check{s}}_8$,$F^{\check{s}}_9$,$F^{\check{s}}_{10}$,$F^{\check{s}}_{12}$,$F^{\check{s}}_{11}$, $F^{\check{s}}_{13}$,$F^{\check{s}}_{14}$, $F^{\check{s}}_{15}$,$F^{\check{s}}_{16}$ }.
Let f : (X, τ_s, Ê→(Ý, τ_s, K)be defined by f($F^{\check{s}}_1$) = $F^{\check{s}}_5$, f($F^{\check{s}}_2$) = $F^{\check{s}}_{11}$ f($F^{\check{s}}_3$) = $F^{\check{s}}_{12}$ f($F^{\check{s}}_4$) = $F^{\check{s}}_1$ f($F^{\check{s}}_5$) = $F^{\check{s}}_1$ f($F^{\check{s}}_6$) = $F^{\check{s}}_{13}$, f($F^{\check{s}}_7$) = $F^{\check{s}}_5$ f($F^{\check{s}}_8$) = $F^{\check{s}}_{11}$, , f($F^{\check{s}}_9$) = f($F^{\check{s}}_9$) , f($F^{\check{s}}_{11}$) = $F^{\check{s}}_{13}$, f($F^{\check{s}}_{12}$) = $F^{\check{s}}_2$, f($F^{\check{s}}_{13}$) = $F^{\check{s}}_{11}$, f($F^{\check{s}}_{14}$) = $F^{\check{s}}_{10}$, f($F^{\check{s}}_{15}$) = $F^{\check{s}}_{15}$ f($F^{\check{s}}_{16}$) = $F^{\check{s}}_{16}$ and f^{-1}($F^{\check{s}}_1$) = $F^{\check{s}}_5$, f^{-1}($F^{\check{s}}_{13}$) = $F^{\check{s}}_{11}$ are both in Š α*- Ô(X) and Š α*- Ç(X) in (X, τ_s, Ê).

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