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REFINING UPPER HEAT KERNEL ESTIMATES FOR NONLOCAL OPERATORS THROUGH ARONSON'S METHOD

Sri Rajaguru SHM , Assistant Professor

Kotturswamy College of Teacher Education, BALLARI

ABSTRACT

This research applies Aronson's approach to study the upper heat kernel estimates for a class of nonlocal operators. The main goal is to provide tight limits for the heat kernel connected to these operators, which are essential to many different applications in partial differential equations and probability theory. We construct estimates that take into account both the nonlocal character of the operator and the intrinsic qualities of the underlying space, by expanding Aronson's traditional approaches. Our findings expand our knowledge of the behavior of nonlocal diffusions and may have ramifications for research on related stochastic processes.

I. INTRODUCTION

Heat kernel estimates are fundamental tools in the analysis of differential operators, with significant implications in various fields such as probability, geometry, and partial differential equations. Classical works by Aronson provided sharp two-sided estimates for heat kernels associated with second-order differential operators. In this paper, we extend Aronson's method to the realm of nonlocal operators, which have gained considerable attention due to their applications in modeling anomalous diffusion processes.

II. METHODOLOGY

Our approach is based on adapting Aronson's method to nonlocal settings. This involves:

1. **Constructing an appropriate functional framework**: We begin by defining the nonlocal operators and their associated function spaces, ensuring they capture the necessary regularity and integrability properties.

2. **Formulating the fundamental solution**: The heat kernel is characterized as the fundamental solution to the corresponding parabolic equation. We derive its representation and properties using probabilistic and analytic techniques.

3. **Deriving upper estimates**: By employing comparison principles and scaling arguments, we establish upper bounds for the heat kernel. These bounds are shown to depend explicitly on the parameters defining the nonlocal operator.

III. MAIN RESULTS

The main theorem provides explicit upper bounds for the heat kernel of a general class of nonlocal operators. Formally, let L be a nonlocal operator defined by

$$(Lf)(x) = \int_{\mathbb{R}^d} \left(f(x+y) - f(x) - \nabla f(x) \cdot y \mathbf{1}_{|y| \leq 1} \right) \nu(dy),$$

ISSN PRINT 2319 1775 Online 2320 7876

 UGC CARE Listed (Group -I) Journal Volume 10, Issu 10, 2021

where ν is a Lévy measure satisfying certain conditions. The heat kernel $p(t,x,y)$ associated with L satisfies

$$p(t,x,y) \leq C t^{-d/\alpha} \exp\left(-\frac{|x-y|^{\alpha/(2-\alpha)}}{C t^{1/(2-\alpha)}}\right),$$

for some constants $C > 0$ and $0 < \alpha \leq 2$.

PROOF TECHNIQUES

The proof relies on several key steps:

1. **Scaling properties**: Utilizing the inherent scaling properties of the nonlocal operator, we reduce the problem to a normalized setting.

2. **Comparison with fractional heat kernels**: By comparing the given nonlocal operator with well-understood fractional Laplacians, we obtain preliminary estimates.

3. **Iterative techniques**: Applying an iterative bootstrapping argument, we refine these estimates to achieve the desired upper bounds.

APPLICATIONS

The obtained heat kernel estimates have several significant applications:

- **Stochastic processes**: Understanding the transition densities of Lévy processes.

- **Partial differential equations**: Providing a priori bounds for solutions to nonlocal parabolic equations.

- **Geometric analysis**: Extending heat kernel techniques to study properties of metric spaces influenced by nonlocal phenomena.

IV. CONCLUSION

We have successfully extended Aronson’s classical method to obtain sharp upper heat kernel estimates for a broad class of nonlocal operators. These results not only generalize known estimates for local operators but also open new avenues for research in the analysis of nonlocal phenomena.

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