e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.12, 2022

MINIMAL CONNECTED GEO CHROMATIC NUMBER OFSOME STANDARD GRAPHS

Q. ANLIN LOUISHA MERLAC, Department of Mathematics, St. John's College of Arts and Science Ammandivilai - 629 204, Tamil Nadu, India.merlac1996@gmail.com

G. SUDHANA, Department of Mathematics, Nesamony Memorial Christian College Marthandam - 629 165, Tamil Nadu, India. sudhanaarun1985@gmail.com

ABSTRACT:
For a connected graph *G* of order $n \geq 2$, a connected geo chromatic set S_{cg} in a connected graph *G* is called a minimal connected geo chromatic set if no proper subset of S_{cg} is a connected geo chromatic set of *G*. The minimal connected geo chromatic number $\chi^{+}_{cg}(G)$ is the maximum cardinality of a minimum connected geo chromatic set of *G*. We determined the minimum connected geo chromatic number of certain standard graphs and bounds of the minimum connected geo chromatic number is proved. It is shown that for positive integers *x*, *y* and *z* such that $2 \leq x < y \leq z$, there exists a connected graph *G* such that $g(G) = x$, $\chi_{cg}(G) = y$ and $\chi^{+}_{cg}(G) = z$.

Keywords : geodetic number, chromatic number, geo chromatic number, connected

1.INTRODUCTION

Let $G = (V, E)$ be a finite undirected connected graph without multiple edges or loops. The order and size of *G* are denoted by *n* and *m* respectively. For basic graph theoretic terminology we refer to Harary [8]. For vertices *p* and *q* in a connected graph *G*, the distance *d(p, q)* is the length of a shortest $p - q$ path in *G*. A $p - q$ path of length *d(p, q)* is called a $p-q$ geodesic. A vertex *x* is said to lie on a $p-q$ geodesic *p* ′, if *x* is a vertex of *p* ′, including the vertices of *p* and *q*. The neighborhood of a vertex *x* is the set *N(x)* consisting of all vertices *y* which are adjacent with *x*. A vertex *x* is an extreme vertex of *G* if the subgraph induced by its neighbors is complete. The closed interval *I* [*p*, *q*] consists of all vertices lying on some $p - q$ geodesic of *G*, while for $S \subseteq V$, $I[S] = \cup_{p,q \in S} I[p, q]$. If $I[S] = V$, then a set *S* of vertices is a geodetic set and the minimum cardinality of a geodetic set is the geodetic number $g(G)$. A geodetic number of a graph was introduced in [3,5] and further studied in [7,9].A connected geodetic set of *G* is a geodetic set *S*′ such that the subgraph *G* [*S*′] induced by *S* ′ is Minimal Connected Geo Chromatic Number of a Graph

Definition 2.1. A connected geo chromatic set S_{cg} in a connected graph *G* is called a minimal connected geo chromatic set if no proper subset of S_{cg} is a connected geo chromatic set of *G*. The minimal connected geo chromatic number $\chi^{+}_{cg}(G)$ is the maximum cardinality of a minimum connected geo chromatic set of *G*.

Example 2.2. For the graph *G* given in Figure (a), $S_{cg1} = \{a_2, a_4, a_5, a_6\}$, $S_{cg2} = \{a_1, a_2, a_4, a_5\}$, $S_{cg3} = \{a_1, a_2, a_4, a_6\}$, $S_{cg4} = \{a_2, a_3, a_4, a_5\}$, $S_{cg5} = \{a_2, a_3, a_4, a_6\}$ are the minimum connected geo chromatic set of G so that $\chi_{cg}(G)$=4. The set S^{+}_{cg}= $\{a_1,a_3,a_4,a_5,a_6\}$ is also a connected geo chromatic set of G . Hence $\chi^{+}_{cg}(G)$ = 5.

Remark 2.3. Every minimum connected geo chromatic set of G is a minimal connected geo chromatic set of G. The converse is not true. For the graph G given Figure 1, S^{+}_{cg}= $\{a_1,a_3,a_4,a_5,a_6\}$ is a minimal connected geo chromatic set but not a minimum connected.

3. Minimal Connected Geo Chromatic Number of Some Standard Graphs

Theorem 3.1.

For a connected graph C_n, $\chi^{+}_{cg}(C_n)$=k. *Proof.* Let $V(P_k) = h_1, h_2, \ldots, h_k$ be the vertex set of P_k. Let usconsider two cases.

Case 1. Suppose that k is even. Then $S = h_1, h_k$ is the minimum geodetic set of P_k and so$g(P_k)$ = 2. Define a coloring of P_k such that the vertices $h_1, h_3, \ldots, h_{k-3}, h_{k-1}$ receive color 1 and the vertices $h_2, h_4, \ldots, h_{k-2}, h_k$ receive color 2. It is easily seen that S is a chromatic set of P_k. Therefore $S = \{h_1, h_k\}$ is also theminimum geo chromatic set S_c of P_k and so $\chi_{gc}(P_k) = 2$. Clearly, the induced subgraph $< S_c >$ is not connected so that S_c not a connected geo chromatic set of P_k. If at least one $h_i \notin S_c$ $(2 \leq i \leq k-1)$, then the subgraph induced by S_c is not connected so that $\chi_{cg}(P_k) < k$ is not possible. Hence $\chi_{cg}(P_k) = k$ and it follows that $\chi_{cg}{}^{+}(P_k) = k$.

Case 2. Suppose that k is odd. Then the set $S = \{h_1, h_k\}$ is the unique minimum geodetic set of P_k so that $g(P_k) = 2$. Define a coloring of P_k such that the vertices $h_2, h_4, \ldots, h_{k-2}, h_k$ receive color 1 and the
vertices $h_1, h_3, \ldots, h_{k-3}, h_{k-1}$ receive color 2. Let the vertices which receive color 1 and color 2 belong to the
color classes, namely C and D. No vertex from color class D belongs to S so that the minimum geodetic set S is not a chromatic set of P_k. To obtain S as a geo chromatic set S_c , choose at least one vertex from color
class D. Let $h_{k-1} \in D$. If $h_{k-1} \in S$, then S becomes $S_c = S \cup \{h_{k-1}\}$, which is a chromatic set of P_k Therefore, $S_c = S \cup \{h_{k-1}\}$ is a geo chromatic set of P_k and $\chi_{cg}(P_k)$=3. By an argument exactly similar to the one given in Case 1, it can be proved that $\chi_{cg}(P_k) = k$ and it follows that $\chi_{cg}{}^{+}(P_k) = k$. *Example 3.2:* For the path P_6 given in Figure 1, the vertexset {1,6} is a minimum geo chromatic set S_c of G and so $\chi_{gc}(P_6) = 2$. It is clear that $< S_c >$ is not connected. If the vertices 2, 3, 4, 5 ∈ S_c, then $< S_c >$ is connected and so $\chi_{cg}(P_6) = 6$. It is clear that S_c is the unique minimal geo chromatic set of maximum cardinality so that $\chi_{cg}{}^{+}(P_6) = 6$.

Example 2.3: For the path P_7 given in Figure 2, the vertexset {1,6,7} is a minimum geo chromatic set S_c of G and so $\chi_{gc}(P_7) = 3$. It is clear that $< S_c >$ is not connected. If the vertices 2, 3, 4,

$5 \in S_c$, then $< S_c >$ is connected and so $\chi_{cg}(P_7) = 7$. It is clear that S_c is the unique minimal geo chromatic set of maximum cardinality so that $\chi_{cg}^+(P_7) = 7$.

Theorem 2.4: For a connected graph K_k, $\chi_{cg}^+(K_k) = k$.

Proof. Each vertex of K_k receive distinct colors and so each vertexof K_k belong to a geo chromatic set S_c of G. It is clear thatthe induced subgraph $< S_c >$ is connected. Therefore $\chi_{cg}(K_k) = k$ and it follows that $\chi_{cg}^+(K_k) = k$.

CONCLUSION

In this paper, the minimal connected geo chromatic number $\chi_{cg}^+(G)$ of some standard graphs has been discussed. Future works can be carried out on obtaining the minimal connected geo chromatic number $\chi_{cg}^+(G)$ with some graph parameters

REFERENCES

[1] Anlin Louisha Merlac, Q & Sudhana, G 2021, 'The Minimal Connected Geo Chromatic Number of a Graph', Turkish Online Journal of Quali- tative Inquiry (TOJQI), Vol.12, no.10, pp.5237-5245, ISSN: 1309-6591.

[2] Beulah Samli, S & Robinson Chellathurai, S, 2018, 'Geo Chromatic Number of a Graph', International Journal of Scientific Research in Mathematical and Statistical Sciences,Vol.5, no.6, pp.259 - 264.

[3] Beulah Samli, S, John, J & Robinson Chellathurai, S 2021, 'The Double Geo Chromatic Number of a Graph', Bulletin of the International Mathematical Virtual Institute, Vol.11, no.1, pp.55 - 68.

[4] Beulah Samli, S, Robinson Chellathurai, S & Ashwin Shijo, M 2018, " The Connected Geo Chromatic Number of a Graph", Proceedings of Two Day International Conference on Mathematical Computer Engineering, VIT University, Chennai, pp.318-319, ISBN:978-93-81899-94-6.

[5] Buckley, F & Harary, F 1990, 'Distance in Graphs', Addison-Wesley, Redwood City, Calif.