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Q-I VAGUE IDEALS IN NEAR-RING

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Abstract. In this paper author defines the concepts about Q-I vague sets, Q-I vague subnear-ring, Q-I vague ideals, homomorphic image and pre-image of Q-I vague ideals in a near-ring R.

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Key Words: I-Vague subnear-ring, I-Vague ideals, Q-Vague sets.

1. INTRODUCTION

Solairaju and Nagarajan [1] constructed Q-fuzzy groups by defining membership functions on an ordered pair R Q to unit interval [0, 1] where $\boldsymbol{R}$ is near-ring and Q is any non-empty set. Then various re- searchers defined these concepts using Q set. Again, both of them along with Muruganantham [2] extended this to Q-vague set and defined Q-vague value, Q-vague cut, Q-vague groups, Q-vague normal subgroups, Q- vague normalizer, and centralizers. K. L. N. Swamy [9,10,11] introduced DRL-semigroups and T. Zelalem [21] defined I-vague sets from DRL- semigroup $\boldsymbol{I}$ to near ring $\boldsymbol{R}$. Pritam [18] extended this concept to define I-vague ideals in near-ring $\boldsymbol{R}$. So here in this paper, author extending that work to Q-I vague concepts.

2. PRELIMINARIES

Let us see some required definitions as follows:

Definition 2.1 [9] A system A=(A,+, ≤, —) is called a dually residuated lattice ordered semigroup (in short DRL-semigroup) if and only if

(1) A=(A,+) is a commutative semigroup with zero "0".

(2) A=(A, ≤) is a lattice such that a+(b∪c)=(a+b) ∪ (a+c) &a+(b∩c)=(a+b)∩(a+c) for all a,b,c ∈ A

(3) Given a,b ∈ A then there exists a least x in A such that b+x ≥ a, and we denotes this a by a-b (for a given a,b it is uniquely determined),

(4) (a-b) ∪0+b≤ a∪b for all a,b ∈ A,

(5) a – a ≥ 0 for all a ∈ A.

In this research, let I = (I, +,—, ∨, ∧, 0,1) be a DRL-semigroup satisfying 1— (1—a)=a for all a ∈ I.

Definition 2.2 [20] Let f be a mapping from set X into a set Y. Let B be a vague set in Y, Then the inverse image of B i.e. $f^{-1}[B]$ is the vague set in X given by, $V_{f-1}[B](x) = V_B[f(x)]$ for all $x \in X$.

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Definition 2.3. [20] Let f be a mapping from a set X into a set Y. Let A be a vague set in X. Then the image of A i.e. $f[A]$ is the vague set in Y is given by,
$V_{f[A]}(y)=\{isup\ V_A(z)/\ z \in f^{-}1(y)\}$ if $f^{-}1(y)$ is non-empty and $V_{f[A]}(y)=[0,0]$ otherwise.

Definition 2.4 [2] A Q-Vague set A in the universe of discourse X is characterized by two membership functions given by:
(1) a truth membership function $t_A : X \times Q \rightarrow [0, 1]$,
(2) a false membership function $f_A : X \times Q \rightarrow [0, 1]$,
Such that $t_A(x,p) + f_A(x,p) \leq 1$ for all $x \in X$ and $p \in Q$.

Definition 2.5 [21] An I-vague set A on a non-empty set X is a pair (t_A, f_A) where $t_A : X \times Q \rightarrow I$ and $f_A : X \times Q \rightarrow I$ with $t_A(x) \leq 1 - f_A(x)$ for all $x \in X$.

Definition 2.6 [18] Let A be an I-vague set in a near-ring R. Then A is called I-vague subnear-ring in a near-ring R if it satisfies the following conditions for all $x,y \in R$,
(1) $V_A(x-y) \geq iinf\ \{V_A(x),V_A(y)\}$
(2) $V_A(xy) \geq iinf\ \{V_A(x),V_A(y)\}$.

Definition 2.7 [18] Let A be an I-vague set in a near-ring R, then A is said to be an I-vague ideal in a near-ring R if and only if for all $x, y, z \in R$, it satisfies
(1) $V_A(x-y) \geq iinf\{V_A(x),V_A(y)\}$,
(2) $V_A(xy) \geq iinf\{V_A(x),V_A(y)\}$,
(3) $V_A(y+x-y) \geq V_A(x)$,
(4) $V_A(xy) \geq V_A(x)$,
(5) $V_A[x(y+z)-xy] \geq V_A(z)$.
Here A is said to be right I-vague ideal in a near-ring R if it satisfies (1), (2), (3) and (4) and A is said to be left I-vague ideal in a near-ring R if it satisfies (1), (2), (3) and (5).

3. *Q-I* VAGUE IDEALS IN NEAR-RING

In this section author defines and discuss about Q-I vague sets, ideals and some of the properties of it in a near-ring R. Here in this paper I be a unit interval $[0,1]$ of real numbers, with $a \oplus b= \min\{1,a+b\}$. With the usual ordering $(I, \oplus, \leq, -)$ is an involuntary DRL-semigroup. The definition is as follows:

Definition 3.1 An I vague set A in a near-ring R which can be denoted as Q-I vague set A in a near-ring R by defining membership functions t_A and f_A from an ordered pain $R \times Q$ to codomain I such that $t_A(x,p)+f_A(x,p)\leq 1$ for all $x \in R$ and $p \in Q$ where Q is a non-empty set.

Definition 3.2 Let A be a Q-I vague set in a near-ring R, Then A is said to be Q-I vague subnear-ring in a near-ring R *if* for all $x, y \in R$ and $p \in Q$, it satisfies
(1) $V_A(x-y,p) \geq iinf\{V_A(x,p),V_A(y,p)\}$,
(2) $V_A(xy,p) \geq iinf\{V_A(x,p),V_A(y,p)\}$.

Definition 3.3 Let A be a Q-I vague set in a near-ring R, then A is said to be a Q-I vague ideal in a near-ring R if and only if for all $x, y, z \in R$, $p \in Q$ *it satisfies*
(1) $V_A(x-y,p) \geq iinf\{V_A(x,p),V_A(y,p)\}$,
(2) $V_A(xy,p) \geq iinf\{V_A(x,p),V_A(y,p)\}$,
(3) $V_A(y+x-y,p)\geq V_A(x,p)$,

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(4) $V_A(xy,p) \geq V_A(x,p)$,
(5) $V_A[x(y+z)-xy,p] \geq V_A(z,p)$.
Here A is said to be right Q-I vague ideal in a near-ring R if it satisfies (1), (2), (3) and (4) and A is said to be left Q-I vague ideal in a near-ring R if it satisfies (1), (2), (3) and (5). If A satisfies (1) to (5) then A is called both sided Q-I vague ideal in a near-ring R.

Remark 3.4. If A is a Q-I vague ideal in a near-ring R, then it holds commutative property. i.e., $V_A(x+y,p) = V_A(y+x,p)$ for all $x, y \in R, p \in Q$.

Remark 3.5. If A is a Q-I vague ideal in a near-ring R, then $V_A(0,p)$ is an upper bound for $V_A(x,p)$ for all $x \in R, p \in Q$.

Example 3.6. Let $Z_3=\{0,1,2\}$ be a near-ring under residue classes of addition and multiplication modulo-3 and $Q=\{p,q\}$. An I–vague set $A=(t_A,f_A)$ of R defined as $t_A:Z_3\times Q\rightarrow I$ and $f_A:Z_3\times Q\rightarrow I$ such that $t_A(x,p)=1$ if $x=0$, $t_A(x,p)=0.3$ if $x=1,2$; $f_A(x,p)=0$ if $x=0$, $f_A(x,p)=0.6$ if $x=1,2$; $t_A(x,q)=1$ if $x=0$, $t_A(x,q)=0.4$ if $x=1,2$; and $f_A(x,q)=0$ if $x=0$, $f_A(x,q)=0.5$ if $x=1,2$.

Here let us prove that A is a vague ideal in Z_3. all $x, y, z \in Z_3$, $p,q \in Q$. Let us verify the first property of Q-I vague ideal through following tables:

x	y	$r = x - y$	$t_A(r)$	$iinf\{t_A(x,p), t_A(y,p)\}$	$1-f_A(r,p)$	$iinf\{1-f_A(x,p), 1-f_A(y,p)\}$
0	0	0	1	1	1	1
0	1	2	0.3	0.3	0.4	0.4
0	2	1	0.3	0.3	0.4	0.4
1	0	1	0.3	0.3	0.4	0.4
1	1	0	1	0.3	1	0.4
1	2	2	0.3	0.3	0.4	0.4
2	0	2	0.3	0.3	0.4	0.4
2	1	1	0.3	0.3	0.4	0.4
2	2	0	1	0.3	1	0.4

TABLE 1. $V_A(x-y,p) \geq iinf\{V_A(x,p), V_A(y,p)\}$, $x, y \in Z_3$.

x	y	$r = x - y$	$t_A(r)$	$iinf\{t_A(x,q), t_A(y,q)\}$	$1-f_A(r,q)$	$iinf\{1-f_A(x,q), 1-f_A(y,q)\}$
0	0	0	1	1	1	1
0	1	2	0.4	0.4	0.5	0.5
0	2	1	0.4	0.4	0.5	0.5
1	0	1	0.4	0.4	0.5	0.5
1	1	0	1	0.4	1	0.5
1	2	2	0.4	0.4	0.5	0.5
2	0	2	0.4	0.4	0.5	0.5
2	1	1	0.4	0.4	0.5	0.5
2	2	0	1	0.4	1	0.5

TABLE 2. $V_A(x-y,q) \geq iinf\{V_A(x,q), V_A(y,q)\}$, $x, y \in Z_3$.

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From Table 1, by 4th & 5th columns we get $t_A(x-y, p) \geq iinf\{t_A(x, p), t_A(y, p)\}$ and by columns 6th & 7th columns we get $1\text{-}f_A(x-y, p) \geq iinf\{1\text{-}f_A(x, p), 1\text{-}f_A(y, p)\}$. Hence we get $V_A(x-y, p) \geq iinf\{V_A(x, p), V_A(y, p)\}$.

Similarly from Table 2, by 4th & 5th columns we get $t_A(x-y, q) \geq iinf\{t_A(x, q), t_A(y, q)\}$ and by columns 6th & 7th columns we get $1\text{-}f_A(x-y, q) \geq iinf\{1\text{-}f_A(x, q), 1\text{-}f_A(y, q)\}$. Hence we get $V_A(x-y, q) \geq iinf\{V_A(x, q), V_A(y, q)\}$.

Similarly, we can prove the remaining properties given below:
$V_A(xy, p) \geq iinf\{V_A(x, p), V_A(y, p)\}$ and $V_A(xy, q) \geq iinf\{V_A(x, q), V_A(y, q)\}$. $V_A(y + x - y, p) \geq V_A(x, p)$ and $V_A(y + x - y, q) \geq V_A(x, q)$.
$V_A(xy, p) \geq V_A(x, p)$ and $V_A(xy, q) \geq V_A(x, q)$.
$V_A[(x + z)y - xy, p] \geq V_A(z, p)$ and $V_A[(x + z)y - xy, q] \geq V_A(z, q)$, for $x, y, z \in Z_3$ and $p, q \in Q$.

We know that unit interval $[0,1]$ is DRL-semigroup satisfying $1-(1-a)=a$ for all a in I. As here A is defined over an ordered pair $R\times Q$. So we get A is a Q-I vague ideal in a near-ring R.

Theorem 3.7 Let A be a Q-I vague ideal in a near-ring R, Then the condition $V_A(xt\text{-}xy, p) \geq V_A(t\text{-}y, p)$ is equivalent to the condition $V_A[x(y+z)\text{-}xy,p] \geq V_A(z,p)$ for all $x, y, z, t \in R$, $p \in Q$. (we can prove this by considering $t = y+z$).

Theorem 3.8 Let R be a near-ing and A be a Q-I vague set in a near-ring R satisfies the condition $V_A(x\text{-}y,p) \geq iinf\{V_A(x,p),V_A(y,p)\}$, then for all $x, y \in R$, $p \in Q$ the following properties are hold
(a) $V_A(0,p) \geq V_A(x,p)$, (b) $V_A(-x,p) = V_A(x,p)$, (c) $V_A(x,p) = V_A(y,p)$ if $V_A(x\text{-}y,p) = V_A(0,p)$. (Proof is obvious).

Definition 3.9 Let A be a Q-I vague set in a near-ring R and g be a well-defined function defined on R. Then a Q-I vague set B *in* g(R) such that, $V_B(y,p)=\{isup\ V_A(x,p)/\ x \in f^{-1}(y)\}$ for all $y \in g(R)$ and $p \in Q$ is the image of A under the function g. Similarly if A is a Q-I vague set in $g(R)$ then the $B=A\circ g$ is a Q-I vague set in a near-ring R *i.e.*, $V_B(x,p)=V_A[g(x),p]$, for all x in R and p in Q.

Theorem 3.10 A pre-image of onto homomorphic function of a Q-I vague ideal in a near-ring R is a Q-I vague ideal in a near-ring R in the respective near-ring.

Proof. Let ψ be an onto homomorphic function defined in a near-ring R to a near-ring S and A be a Q-I vague ideal in a near-ring S, where $B=\psi^{-1}(A)$ in a near-ring R.

Let us show A is Q-I vague ideal in near-ring R. Now, for all $x,y,z \in R$, $p \in Q$.

$$V_A(x\text{-}y,p) = V_B[\psi(x\text{-}y),p]= V_B[\psi(x)\text{-}\psi(y)),p] \geq iinf\{V_B(\psi(x),p),\ V_B(\psi(y),p)\}$$

$$\geq iinf\{V_A(x,p),\ V_A(y,p)\}.$$

$$V_A(xy,p) = V_B[\psi(xy),p]= V_B[\psi(x)\psi(y)),p] \geq iinf\{V_B(\psi(x),p),\ V_B(\psi(y),p)\}$$

$$\geq iinf\{V_A(x,p),\ V_A(y,p)\}.$$

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$V_A(xy,p) = V_B[\psi(xy),p]= V_B[\psi(x)\psi(y)),p] \geq V_B(\psi(x),p) = V_A(x,p).$

$V_A[(x+z)y-xy,p] = V_B[\psi[(x+z)y-xy],p]= V_B[\psi(x)\psi(y)),p]$

$$\geq iinf\{V_B(\psi(x),p), V_B(\psi(y),p)\} \geq iinf\{V_A(x,p), V_A(y,p)\}.$$

It shows A is a *Q-I* vague ideal in near-ring R, for all $x, y, z \in R$, $p \in Q$.

5. CONCLUSION

In this paper, the concepts of *Q-I* vague sub near-ring and *Q-I* vague ideals of near-ring are discussed. Also properties related to *Q-I* vague ideals of near-ring are discussed. Then we have observed what happens with the homomorphic image and pre-image of *Q-I* vague ideals with the helpof some previous concepts.

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