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THE UPPER CONNECTED EDGE FIXING EDGE-TO-VERTEX STEINER NUMER OF A GRAPH

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ABSTRACT: Let G be a connected graphand $e \in E(G)$. An edge fixing edge-to-vertex Steiner set W is called connected edge fixing edge-to-vertex Steiner set G if $< W >$ is connected. The minimum cardinality of a connected edge fixing edge-to-vertex Steiner set of G is the connected edge fixing edge-to-vertex Steiner number of e of G and to denoted by $s_{cefev}(G)$. The edge fixing edge-to-vertex Steiner set of e of G of cardinality $s_{cefev}(G)$ is denoted by s_{cefev}-set of G. Some general properties satisfied by this concept is studied. It is shown that for positive integers r, d and $n \geq 2$ with $r \leq d \leq 2r$, there exists a connected graph G with rad$G = r$, diam$G = d$ and $s_{cefev}(G) = n$ for some edge e in G. For any positive integers $a$and b with $2 \leq a \leq b$, there exists a connected graph G such that $s_{cev}(G) = a$ and $s_{cefev}(G) = b$ for some edge e in G.
Keywords: the edge fixing edge-to-vertex Steiner number, the connected edge fixing edge-to-vertex Steiner number, the upper connected edge fixing edge-to-vertex Steiner number.
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1. INTRODUCTION

By a graph $G = (V, E)$, we mean a finite undirected connected graph without loops or multiple edges. The order and size of $G$are denoted by p and q respectively. The distance $d(u, v)$ between two vertices u and v in a connected graph G is the length of a shortest $u - v$ path in G. An $u - v$ path of length $d(u, v)$is called an $u - v$ geodesic. For basic graph theoretic terminology, we refer to Harary [2]. For a non-empty set W of vertices in a connected graph G, the Steiner distance $d(W)$ of W is the minimum size of a connected subgraph of G containing W. Necessarily, each such subgraph is a tree and is called a Steiner tree with respect to W or a Steiner W −tree. It is to be noted that $d(W) = d(u, v)$ when $W = \{u, v\}$. The set of all vertices of G that lie on some Steiner W-tree is denoted by $S(W)$. If $S(W) = V$, then W is called a Steiner set for G. A Steiner set of minimum cardinality is a minimum Steiner set or simply a s-set of G and this cardinality is the Steiner number $s(G)$ of G. The Steiner number of a graph was introduced and studied in [3] and further studied in [4,5,6,7]. When $W = \{u, v\}$, every Steiner W-tree in G is a $u - v$ geodesic. Also $S(W)$ equals the set of vertices lying in $u - v$ geodesic, inclusive of u, v. Hence Steiner sets, Steiner numbers can be consider as extensions of geodesic concepts. Let G be a connected graphand $e \in E(G)$. An edge fixing edge-to-vertex Steiner set W is called connected edge fixing edge-to-vertex Steiner set G if $< W >$ is connected. The minimum cardinality of a connected edge fixing edge-to-vertex Steiner set of G is the connected edge fixing edge-to-vertex Steiner number of e of G and to denoted by $s_{cefev}(G)$. The edge fixing edge-to-vertex Steiner set of e of G of cardinality $s_{cefev}(G)$ is denoted by s_{cefev}-set of G.

Theorem 1.1[1] Let e be an edge of G. Let v be an extreme vertex of a connected graph G such that v is not incident with e. Then every connected edge fixing edge-to-vertex Steiner set of an edge e of G contains at least one extreme edge that is incident with v. (Whether e is an extreme edge or not).

Corollary 1.2[1] Let e be an edge of G and f be an end edge of a connected graph G such that $e \neq f$. Then f belongs to every connected edge fixing edge-to-vertex Steiner set of an edge e of G.

Theorem 1.3[1]. For a connected graph G with size $q \geq 3$,$s_{cefev}(G)$= q if and only if e is an internal edge of a tree.

2. The Upper Connected Edge Fixing Edge-To-Vertex Steiner Number of a Graph

Definition 2.1. An edge fixing edge-to-vertex Steiner W in a connected graph G is called a *minimum connected fixing edge-to-vertex Steiner set of*G if no proper subset W is an edge fixing edge-to-vertex Steiner set of G. The upper connected edge fixing edge-to-vertex Steiner number of G is the *minimum cardinality of a minimum connected edge fixing edge-to-vertex Steiner number of* G. It is denoted by $s^+_{cefev}(G)$.

Remark 2.3. Every minimum connected edge fixing edge-to-vertex Steiner set of G is a minimal connected edge fixing edge-to-vertex Steiner set of G and the converse need not be true.

For the graph G given in Figure 2.1, let $e = v_1v_2$ and $W_2 = \{v_6v_7, v_7v_9, v_5v_9, v_5v_4\}$. Then W_2 is a miminal connected edge fixing edge-to-vertex Steiner set but not a minimumedge fixing edge-to-vertex Steiner set of an edge $e$of G.

Theorem 2.4. For every connected graph G, $1 \leq s_{cefev}(G) \leq s^+_{cefev}(G) \leq q$ for some edge $e \in G$.

Proof. For an edge e of G, any connected edge fixing edge-to-vertex Steiner set needs at least one edge and so $s_{cefev}(G) \geq 1$. For an edge$e$of G, since every minimal connected edge fixing edge-to-vertex Steiner set $s_{cefev}(G) \leq s^+_{cefev}(G)$. Also for an edge e, since $E(G)$ is a connected edge fixing edge-to-vertex Steiner set of an edge $e$of G, it is clear that $s^+_{cefev}(G) \leq q$. Then $1 \leq s_{cefev}(G) \leq s^+_{cefev}(G) \leq q$ for some edge $e \in G$.

Remark 2.5. The bound in Theorem 2.4 can be sharp.

For the path $G = P_p$ $(p \geq 3)$, for an end edge e in $E(G)$, $s_{cefev}(G) = 1$. For an internal edge of a tree, $s_{cefev}(G) = s^+_{cefev}(G) = q$.

The bound in Theorem2.4 can be strict.

For the graph G given in Figure 2.1. Let $e = v_1v_2$. Then $W_1 = \{v_4v_5, v_5v_6, v_5v_9\}$ is a s_{cefev}-set of $G s_{cefev}(G)$= 3. Also$W_2 = \{v_6v_7, v_7v_9, v_5v_9, v_5v_4\}$ is the minimal connectededge fixing edge-to-vertex Steiner set of G so that $s^+_{cefev}(G) \geq 4$. It is easily verified that there is no minimal connected edge fixng edge-to-vertex Steiner set of cardinality more than 4. Therefore$s^+_{cefev}(G) = 4$. Thus $1 < s_{cefev}(G) < s^+_{cefev}(G) < q$.

Theorem 2.6. For a connected graph G, $s^+_{cefev}(G) = q$if and only if $s_{cefev}(G) = q$, for some edge e in $E(G)$.

Proof. Let $s^+_{cefev}(G) = q$for an edge e of G. Then $W = V(G)$ is the unique minimal connected edge fixing edge-to-vertex Steiner set of an edge e of G. Since no proper subset of W is a connected edge fixing edge-to-vertex Steiner set of an edge e of G, W is the unique minimum connected edge fixing edge-to-vertex Steiner set of G and so that $s_{cefev}(G) = q$. The converse is clear.

Theorem 2.7. For the connected graph G, $s_{cefev}(G) = q - 1$ if and only if $s^+_{cefev}(G) = q - 1$for every edge e in $E(G)$.

Proof. Let $s_{cefev}(G) = q - 1$. Then $S = E(G) - \{e\}$ is the unique minimal connected edge fixing edge-to-vertex Steiner set of an edge e of G. Since no proper subset of S is an edge fixing edge-to-vertex Steiner set of an edge e of G, it is clear that S is the unique minimum connected edge fixing edge-to-vertex Steiner set of G and so $s^{+}_{cefev}(G) = q - 1$.The converse follows from Theorem 2.4.**Corollary 2.8.** For the connected graph G of size $q \geq 4$, the following are equivalent for the some edge e in G.

(i) $s_{cefev}(G) = q$

(ii)$s^{+}_{cefev}(G) = q$

(iii) e is an internal edge of a tree.

Proof. Itfollows from Theorems 1.3[1] and 2.6.

Corollary 2.9. For the connected graph G of size $q \geq 4$, the following are equivalent for the some edge e in G.

(i) $s_{cefev}(G) = q - 1$

(ii)$s^{+}_{cefev}(G) = q - 1$

(iii) $G = K_{1,q-1}$

Proof. It follows from Theorem 2.7 and Theorem 1.4[1].

Theorem 2.10. For the complete graph $G = K_p$ $(p \geq 4)$, $s^{+}_{cefev}(K_p)$ = p - 2 for any edge e of G.

Proof. Let $e = uv$ be an edge of G. Let W be any set of $p - 2$ adjacent edges of K_p incident at the vertex v. Since each vertex of K_p lies on Steiner W_{ev}-tree of G, it follows that W is a connected edge fixing edge-to-vertex Steiner set of an edge e of G. If W is not a minimal connected edge fixing edge-to-vertex Steiner set of an edge e of G, then there exists a proper subset of W' of W such that W' is a connectededge fixing edge-to-vertex Steiner set of an edge e of G. Therefore there exists atleast one vertex, say u of K_p such that u is not incident with any edge of W'. Hence u does not belong to any Steiner ${W'}_{ev}$-tree of G, Which is a contradiction. Hence W is a minimal connected edge fixing edge-to-vertex Steiner set of an edge e of G. Therefore $s^{+}_{cefev}(G) \geq p - 2$. Suppose that there exists a minimal connected edge fixing edge-to-vertex Steiner set of an edge e of G of M such that $|M| \geq p - 1$. Since $M \cup \{e\}$contains atleast p edges, $<M>$ contains at least one cycle. Let $M' = M - \{f\}$, where f is an edge of a cycle which lies in $< M >$. It is clear that M' is a connected edge fixing edge-to-vertex Steiner set with $W' \subset W$, Which is a contradiction.

Therefore$s^{+}_{cefev}(G)$= $p - 2$.

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