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Research Paper

ON THE ACHARYA INDEX WITH RESPECT TO D-DISTANCE

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ABSTRACT— We define Acharya Index $AI_\lambda(G)$ of a connected graph with respect to D-distance as the sum of the D-distance between all pair of vertices of degree *d*, which is inspired by research on the Acharya index of connected graphs, its chemical implications, and its mathematical traits. In this study, we calculated the AI(G) for standard graphs.

Keywords —Wiener index, Terminal Wiener index, distance, D-distance

I. INTRODUCTION

Chemist Harold Wiener invented the Wiener index in 1947 and used it to ascertain the physico-chemical properties of paraffin-containing alkanes. It is today regarded as one of the most well-known and practical topological indices in molecular graph theory. Alkanes are purely chemical substances made of carbon and hydrogen . One can see for further details on the Wiener index in chemical graph theory and its mathematical uses. See [1] for terms that are not defined. In, the concept of distance of graphs can be found. *d(u, v)* stands for the distance between two vertices in a graph, *n* for the order, **Δ**(G) for the maximum degree, and *p=diam(G)* for the diameter of G. In this paper, only simple connected graphs were taken into account.

The concept of detour distance in graphs was first presented by Chartrand et al. [2] as follows: The D-distance $d^D(u,v)$ between two vertices *u,v* of a connected graph G is defined as $d^D(u,v) = \min \{l^D(s)\}$ where the minimum is taken over all u-v paths s in G. Alternatively, $d^D(u,v)$ can be written as $d^D(u,v) = \min \{d(u,v)+deg(u)+deg(v)+ \sum deg(w)$ That is, $TW(G) = \sum_{1<i<j<k} d(v_i, v_j/G)$ where $d(v_i, v_j/G)$ the distance between pendent vertices $v_i$and v_j in graph. G.Sharilaja et.al defined the Acharya Index $AI_\lambda(G)$ of a graph G as the sum of the distance between all pairs of vertices with a degree *d*, that is, $AI_\lambda(G) = \sum_{\substack{1\le d\le n-1 \\ 1\le i\le p}} \vartheta(d,G).i$ where $\vartheta(d,G)$ represents the pair of vertices of degree d at distance *i*, p=diam(G). In this paper, we defined Acharya Index with respect to D-distance as the sum of the D-distance between all pairs of vertices with a degree *v*. That is, $AI_\lambda^D(G) = \sum_{\substack{1\le v\le n-1 \\ 1\le i\le p}} (d^D(v,G).i)$ where $d^D(v,G)$ represents the pair of vertices of degree *v* at D-distance *i*, p=diam(G).

II. MAIN RESULTS

This section presents the Acharya index with respect to D-distance $AI_\lambda^D(G)$ of certain significant class of graphs.*Observation:* If G is a connected graph then $AI_\lambda^D(G) > AI_\lambda(G)$.

Theorem 2.1: If $G = P_n$, $n \ge 4$ is a path on *n* vertices then $AI_\lambda^D(G) = 3(n-1) + \sum_{k=2}^{n-2}\{(3k-1)(n-(k+1))\}$.

Proof : Let *u* and *v* be any two vertices of P_n , n≥ 4 is a biregular graph with degrees of vertices either 1 and 2. We must calculate the D-distances between vertices with the same degree in order to locate the Acharya polynomial. We have the following distances and the number of pairs of vertices with these distances, if the D-distance between the vertices is $d^D(u,v) = i$.

(i)There is one pair of degree 1 vertices at D-distance $3(n-1)$.

(ii)For $0 \le k \le n-2$, there are $(n-k-1)$ pair of degree 2 vertices are at D-distance i, where $i = 3k - 1$.

We have the Acharya index w.r.t D-distance as the sum of all the mentioned distances as $AI_\lambda^D(G) = 3(n-1) + \sum_{k=2}^{n-2}\{(3k-1)(n-(k+1))\}$.

Theorem 2.2: If G = C_n , n ≥ 4 is a path on *n* vertices then

$$AI_\lambda^D(G) = \begin{cases} n\displaystyle\sum_{k=1}^{\lfloor \frac{n}{2} \rfloor}(3k+2) & \text{, n is odd} \\ n\displaystyle\sum_{k=1}^{\lfloor \frac{n-1}{2} \rfloor}(3k+2) + \frac{n}{2}\left(\frac{n}{2} + 2\left\lfloor \frac{n-1}{2} \right\rfloor + 4\right) & \text{, n is even} \end{cases}$$

Proof : Let u and v be any two vertices of C_n, n≥ 4 is a regular graph with degree of vertices 2. We must calculate the D-distances between vertices with the same degree in order to locate the Acharya polynomial. We have the following D-distances and the number of pairs of vertices with these distances, if the distance between the vertices is $d^D(u,v)$ is denoted by *i*.

Case A: When *n* is odd.

(i)For $1 \le k \le \left\lfloor \frac{n}{2} \right\rfloor$, there are n pair of degree 2 vertices are at D-distance $3k+2$. Thus we have $AI_\lambda^D(G)$ = n $\left[\sum_{k=1}^{\lfloor \frac{n}{2} \rfloor}(3k+2)\right]$.

Case B: When *n* is even.

(i)For $1 \le k \le \left\lfloor \frac{n-1}{2} \right\rfloor$, there are n pair of degree 2 vertices are at D-distance i, where $i = 3k+2$.

(ii)There are $\frac{n}{2}$ pair of degree 2 vertices are at D-distance $\frac{n}{2} + 2\left\lfloor \frac{n-1}{2} \right\rfloor + 4$. Thus we have the Acharya index w.r.t D-distance as the sum of all the mentioned distances as $AI_\lambda^D(G) = n\left[\sum_{k=1}^{\lfloor \frac{n-1}{2} \rfloor}(3k+2)\right] + \frac{n}{2}\left(\frac{n}{2} + 2\left\lfloor \frac{n-1}{2} \right\rfloor + 4\right)$.

Theorem 2.3: If G = W_n , n ≥ 3 is a wheel on $n+1$ vertices then $AI_\lambda^D(G)$ = 10n+ $12[(n-3) + \sum_{k=1}^{n-3} k]$

Proof : Let u and v be any two vertices of W_n, n≥ 3 is a biregular graph with degrees of vertices either 3 and n. We must calculate the D-distances between vertices with the same degree in order to locate the

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Acharya polynomial. We have the following D-distances and the number of pairs of vertices with these distances are:

(i)There is n pair of degree 3 vertices at D-distance 10.

(ii)There are $(n-3)+\sum_{k=1}^{n-3} k$ pair of degree 3 vertices are at a D-distance 12.

We have the Acharya index w.r.t D-distance as the sum of all the mentioned distances as $AI_{\lambda}^{D}(G)=$ 10n+ $12[(n-3)+\sum_{k=1}^{n-3} k]$.

Theorem 2.4: If G = K_n , n ≥ 4 is a complete graph on n vertices then $AI_{\lambda}^{D}(G)=(2n-1)\binom{n}{2}$.

Proof : Let u and v be any two vertices of K_n , n≥ 4 is a regular graph with degree of vertices $n-1$. We must calculate the D-distances between vertices with the same degree in order to locate the Acharya polynomial. There is $\binom{n}{2}$ pair of degree n-1 vertices at D-distance $2n-1$.

Thus we have the Acharya index w.r.t D-distance as $AI_{\lambda}^{D}(G)=(2n-1)\binom{n}{2}$.

Theorem 2.5: If G = $K_{m,n}$, $m<n, n=m+1$, m ≥ 2 is a bipartite graph on $m+n$ vertices then $AI_{\lambda}^{D}(G)=\binom{m}{2}[2n+2m+2]+\sum_{k=1}^{m} k\,[2m+2n+2]$.

Proof : Let u and v be any two vertices of $K_{m,n}$, $m\geq 2$ is a biregular graph with degree of vertices either n or m. We must calculate the D-distances between vertices with the same degree in order to locate the Acharya polynomial. We have the following distances and the number of pairs of vertices with these distances, if the D-distance between the vertices is $d^D(u,v)$ is denoted by i.

(i)For $1\leq k\leq\left\lfloor\frac{n-1}{2}\right\rfloor$, there are n pair of degree 2 vertices are at D-distance i, where $i=3k+2$.

(ii)There are $\binom{m}{2}$ pair of degree n vertices are at D-distance $2n+2m+2$.

(iii) For $1\leq k\leq m$, there are k pair of degree m vertices are at D-distance $2m+2n+2$. Thus we have the Acharya index w.r.t D-distance as the sum of all the mentioned distances as $AI_{\lambda}^{D}(G)=\binom{m}{2}[2n+2m+2]+\sum_{k=1}^{m} k\,[2m+2n+2]$.

Theorem 2.6: If G = $K_{n,n}$, n ≥ 2 is a complete bipartite graph on 2n vertices then $AI_{\lambda}^{D}(G)=n(n-1)[3n+2]$.

Proof : Let u and v be any two vertices of $K_{n,n}$, n≥ 2 is a regular graph with degree of vertices n. We must calculate the D-distances between vertices with the same degree in order to locate the Acharya polynomial. There are n^2-n pair of degree n vertices are at D-distance $3n+2$.

Thus we have the Acharya index w.r.t D-distance as $AI_{\lambda}^{D}(G)=n(n-1)[3n+2]$.

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