

d^d-DISTANCE IN SOME CORONA RELATED GRAPHS

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ABSTRACT: Let G = (V, E) be a simple graph. Let u, v be two vertices of a connected graph G. Then the *d-length* of a u-v path defined as $d^d(u, v) = d(u, v) + \deg(u) + \deg(v) + \deg(u)\deg(v)$, where $d(u, v)$ is the shortest distance between the vertices u and v. In this paper d^d – distance of some corona relaed graphs are determined.

Keywords: d^d-distance, complete graph, cycle graph and corona graph.

1. INTRODUCTION

Let G(V,E) be a simple, connected graph where V(G) is its vertex set and E(G) is its edge set. The degree of any vertex v in G is the number of edges incident with v and is denoted by deg v. The minimum degree of a graph is denoted by δ(G) and the maximum degree of a graph G is denoted by Δ(G) . A vertex of degree 0 is called an isolated vertex and a vertex of degree 1 is called a pendent vertex. The standard or usual distance d(u, v) between u and v is the length of the shortest u − v path in G. In this paper, d^d – distance of some corona relaed graphs are determined.

Definition 1.1: The corona $G_1 \odot G_2$ of two graphs G_1 and G_2 where G_1 has m vertices and n edges is defined as the graph G_1 obtained by taking one copy of G_1 and m copies of G_2, and then joining by an edge the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2.

2.Main Results:

Theorem 2.1: If $G = C_m \odot K_n$ then $d^d(u = v_i, v = v_{i+1}) = (n + 3)^2$, where $v_i \in C_m$.

Proof: Let $V(C_m) = \{v_i : 1 \le i \le m\}$, $V(K_i) = \{u_{i1}, u_{i2}, \ldots, u_{in} : 1 \le i \le m\}$ and $V(H) = V(C_m) \cup V(K_i)$, $1 \le i \le m$ and also $E(H) = E(C_m) \cup E(K_i) \cup \{ v_i u_{ij} : 1 \le i \le m, 1 \le j \le n \}$. We have $d^d(u, v) = d(u, v) + \deg(u) + \deg(v) + \deg(u)\deg(v)$. Since deg $u_{in} = n$, $1 \le i \le m$. $N(v_i)$ and $N(v_{i+1})$ are u_{im} $1 \le i \le m$ and adjacent vertices of v_i. v_i is adjaent with two vertices and v_{i+1} also adjacent with two vertices because $v_i \in C_m$. Hence deg $(u = v_i)$ = deg $(v = v_{i+1}) = n + 2$. Now, $\deg(u) + \deg(v) = n + 2 + n + 2 = 2n + 4$ and $\deg(u)\deg(v) = (n + 2)(n + 2) = (n + 2)^2$. Since u and v are adjacent vertices d(u, v) = 1. Therefore $d^d(u, v) = d(u, v) + \deg(u) + \deg(v) + \deg(u)\deg(v)$

$$= 1 + 2n + 4 + (n + 2)^2$$

$= 5 + 2n + n^2 + 4 + 4n$ $= n^2 + 6n + 9$
$= (n + 3)^2$.

Theorem 2.2: If $G = K_m \odot K_n$, m,n ≥ 2 then $d^d(u,v) = n^2 + m^2 + 2nm$, where u, v ∈ K_m.

Proof: Let $V(K_m) = \{v_i : 1 \le i \le m\}$, $V(K_i) = \{u_{i1}, u_{i2}, \dots, u_{in} : 1 \le i \le m\}$ and $V(H) = V(K_m) \cup V(K_i)$, $1 \le i \le m$ and also $E(H) = E(K_m) \cup E(K_i) \cup \{v_i u_{ij} : 1 \le i \le m, 1 \le j \le n\}$. We have $d^d(u, v) = d(u, v) + deg(u) + deg(v) + deg(u)\, deg(v)$. Since u and v ∈ K_m , d(u, v) = 1. Since deg u_{in} = n, $1 \le i \le m$. Since u and v be any vertex of K_m , $N(v) = N(u) = \{v_i, u_{in}, u_{mn} : 1 \le i \le m - 1\}$. Hence deg u and deg v is equal to n + m – 1. Now, deg(u) + deg(v) = n + m - 1 + n + m - 1 = 2m + 2n - 2 and

deg(u) deg(v) = (n + m – 1)(n +m - 1). Therefore $d^d(u, v) = d(u, v) + deg(u) + deg(v) + deg(u)deg(v)$

$= 1 + 2m + 2n - 2 + (n + m - 1)(n + m - 1) = 2n + 2m - 1 + n^2 + nm - n + nm + m^2 - m - n - m + 1 = 2n + 2m + n^2 + 2nm - 2n + m^2 - 2m + 1 = n^2 + m^2 + 2nm$.

Theorem 2.3: If $G = K_{1,m} \odot K_n$, m,n ≥ 2 then $d^d(u,v) = n^2 + 4n + 5$ where u, v ∈ $K_{1,m}$ - x.

Proof: Let $V(K_{1,m}) = \{x, v_i : 1 \le i \le m\}$, $V(K_i) = \{u_{i1}, u_{i2}, \dots, u_{in} : 1 \le i \le m\}$ and $V(H) = V(K_{1,m}) \cup V(K_i)$, $1 \le i \le m$ and also $E(H) = (K_{1,m}) \cup E(K_i) \cup \{v_i u_{ij} : 1 \le i \le m, 1 \le j \le n\}$. We have $d^d(u, v) = d(u, v) + deg(u) + deg(v) + deg(u)\, deg(v)$. u and v be any vertex of $K_{1,m}$ - x , d(u, v) = 2. Since deg u_{in} = n, $1 \le i \le m$. $N(v) = N(u) = \{x, u_{in} : 1 \le i \le m\}$. Hence deg u and deg v is equal to n + 1.

Now, deg(u) + deg(v) = n + 1 + n + 1 = 2n + 2 and deg(u) deg(v) = $(n + 1)^2$. Therefore

$d^d(u, v) = d(u, v) + deg(u) + deg(v) + deg(u)deg(v) = 2 + 2n + 2 + (n + 1)^2 = 4 + 2n + n^2 + 1 + 2n$

$= n^2 + 4n + 5$.

Theorem 2.4: If $G = K_{1,m} \odot K_n$, m,n ≥ 2 then $d^d(u,v) = (n + 2)(n + m + 1)$ where u = x, v ∈ $K_{1,m}$ - x.

Proof: Let $V(K_{1,m}) = \{x, v_i : 1 \le i \le m\}$, $V(K_i) = \{u_{i1}, u_{i2}, \dots, u_{in} : 1 \le i \le m\}$ and $V(H) = V(K_{1,m}) \cup V(K_i)$, $1 \le i \le m$ and also $E(H) = E(K_{1,m}) \cup E(K_i) \cup \{v_i u_{ij} : 1 \le i \le m, 1 \le j \le n\}$. We have $d^d(u, v) = d(u, v) + deg(u) + deg(v) + deg(u)\, deg(v)$. u = x and v be any vertex of $K_{1,m}$ - x , d(u, v) = 1. Since deg u_{in} = n, $1 \le i \le m$. $N(u) = \{v_i, u_{in} : 1 \le i \le m\}$ $N(v) = \{x, u_{in} : 1 \le i \le m\}$. Hence deg u = n + m and deg v = n + 1. Now, deg(u) + deg(v) = (n + m) + (n + 1) and deg(u) deg(v) = (n + m)(n + 1). Therefore

$d^d(u, v) = d(u, v) + deg(u) + deg(v) + deg(u)deg(v)$

$= 1 + (n + m) + (n + 1) + (n + m)(n + 1)$

$= 1 + (n + m) + (n + m)(n + 1) + (n + 1)$

= 1 + (n + m) [1 + (n + 1)]

+ n + 1

= (n + m) (n + 2) + (n + 2)

= (n + 2)(n + m + 1).

Theorem 2.5: If $G = K_m \odot K_{1,n}$ **,** m,n ≥ 2 then $d^d(u,v) = (n + m + 1)^2$, where u , v ∈ K_m.

Proof: : Let $V(K_m) = \{ v_1, v_2, \dots, v_m \}$, $V(H) = V(K_m) \cup \{u_i, u_{ij}: 1 \le i \le m, 1 \le j \le n \}$ and $E(H) = E(K_m) \cup \{v_i u_{ij}, v_i u_i, u_i u_{ij} : 1 \le i \le m, 1 \le j \le n \}$. We have $d^d(u, v) = d(u, v) + \deg(u) + \deg(v) + \deg(u)\deg(v)$. u and v be any vertex of K_m , d(u, v) = 1.. $N(u) = N(v) = \{ v_k, u_i, u_{ij}: 1 \le k \le m - 1, 1 \le i \le m, 1 \le j \le n \}$. Hence deg u = deg v = m – 1 + n + 1 = n + m. Now, deg(u) + deg(v) = (n + m) + (n + m)

= 2n + 2m and

deg(u) deg(v) = $(n + m)^2$. Therefore $d^d(u, v) = d(u, v) + \deg(u) + \deg(v) + \deg(u)\deg(v) = 1 + (2n + 2m) + (n + m)^2$

$= 1 + 2n + 2m + n^2 + m^2 + 2nm = n^2 + m^2 + 2nm + 2n + 2m + 1 = (n + m + 1)^2$.

Conclusion:

Many researchers are concentrating various distance concepts in graphs. We introduced d^d – disance in graps. In this paper we disccuss about d^d-distance of some corona related graphs.

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