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# A NOVEL METHOD OF NORMALIZED BROWNIAN MOTION FOR STRATONOVICH LINEAR STOCHASTIC DIFFERENTIAL EQUATIONS

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**ABSTRACT:** A stochastic differential equation is a differential equation whose coefficients are random numbers or random functions of the independent variable (or variables). This is an iterative method, namely New Iterative Method (NIM) for the solution of Stratonovich Linear Stochastic Differential Equation. The noise terms of the linear SDEs are considered based on normalized Brownian Motion with finite series. Two Illustrative examples are considered to validate the accuracy of the method, and the results showed that the approximate solutions converge faster to the exact solutions with fewer terms; though, higher terms will increase the accuracy. Hence, this method wills shows better results interms of accuracy and efficiency.

**KEYWORDS:** Stratonovich, Linear Sides, Approximate Solution, Option Pricing, Normalized Brownian Motion

## I. INTRODUCTION

Stochastic Differential equations play a very significant role in Physical sciences (applied science), and these equations are modelled in two forms: (linear and nonlinear). Stochastic Differential Equation (SDE) is a particular type of differential equation that models random effects [1]. SDE is widely used to model physical phenomena that arise in the field of Finance, Physics, Biological Sciences, and other related fields. SDEs can be obtained by adding random effects into the Ordinary or Partial Differential Equations [2]. Different areas of application of SDEs such as option pricing, population changes, and many other areas were introduced. Some authors have also presented different techniques for solving linear and nonlinear type of SDEs. Iterative methods have been the best way of obtaining exact analytical solutions. A new iterative method (NIM) was proposed by Daftardar-Geji and Jafari. This method has been widely accepted and used to solve linear and nonlinear equations of different types. The results obtained via this method converge faster to the exact solution with few terms. In our work, the NIM is used to solve some linear SDEs in terms of the normalized Brownian Motion approach [3]. Linear Stochastic Differential Equations (LSDEs) of the following form will be considered:

It is well known that, under suitable Lipschitz and growth conditions on the coefficients, a classical to stochastic differential equation

$$x(t) = \varepsilon + \int_0^t b\left(s, X(s)\right) ds + \int_0^t \sigma\left(s, X(s)\right) dW(s)$$

----- (1)

Where W is a Wiener process and ξ is a F0-measurable random variable for a given nonanticipating filtration {Ft , t ≥ 0} of W, has a unique strong solution which is a Markov process. If ξ is not F0-measurable or the coefficients b, σ are random and non-adapted, then any reasonable interpretation of X in will not be an Ft-adapted process and, unless σ is a constant, we need to use some anticipating stochastic integral to give a sense to the equation [4]. In these cases, the solution is not a Markov process in general. Still another setting that leads to anticipation is the case of boundary conditions. That means, the first variable of the solution process is no longer a datum of the problem, time runs in a bounded interval, say from 0 to 1, and we impose a relation h(X(0), X(1)) = 0 between the first and the last variables of the solution [5]. In this situation, the fact that the solution will not be Markovian is quite intuitive, since the

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strong relationship between X(0) and X(1) will prevent the independence of X(0) and X(1) from holding, even when conditioning to X(a), a ∈ ]0, 1[, except maybe in some very particular cases.

On the other hand, it may also seem intuitive that the following weaker conditional independence property can hold true: For any 0 ≤ a < b ≤ 1, the σ-fields σ{X(t), t ∈ [a, b]} and σ{X(t), t ∈ ]a, b[ c} are conditionally independent given σ{X(a), X(b)}. We will denote it by

$$\sigma\{X(t), t \in [a,b]\} \coprod_{\sigma\{X(a),X(b)\}} \sigma\{X(t), t\epsilon\, [a,b]^c\} \quad \text{----- (2)}$$

Lets consider linear stochastic differential equations of arbitrary order with additive white noise. Our boundary conditions will not be restricted to involve the solution process at the endpoints of the time interval, but we will allow them to involve the values at finitely many points inside the interval [6]. They are usually called functional or lateral boundary conditions. Our main goal is to seek which kind of conditional independence properties can be established for the solution.

## II. LITERATURE SURVEY

Xuehui Chen, Liang Wei, Jizhe Sui, Xiaoliang Zhang and Liancun Zheng, et.al [7] generalized differential transform method is implemented for solving several linear fractional partial differential equations arising in fluid mechanics. This method is based on the two-dimensional Differential Transform Method (DTM) and generalized Taylor's formula. Numerical illustrations of the time-fractional diffusion equation and the time-fractional wave equation are investigated to demonstrate the effectiveness of this method. Results obtained by using the scheme presented here agree well with the analytical solutions and the numerical results presented elsewhere. The results reveal the method is feasible and convenient for handling approximate solutions of linear or nonlinear fractional partial differential equations.

N. Kumaresan, K. Ratnavelu and B. R. Wong, et.al [8] optimal control for Fuzzy linear Partial Differential Algebraic Equations (FPDAE) with quadratic performance is obtained using Simulink. By using the method of lines, the FPDAE is transformed into a Fuzzy Differential Algebraic Equations (FDAE). Hence, the optimal control of FPDAE can be found out by finding the optimal control of the corresponding FDAE. The goal is to provide optimal control with reduced calculus effort by the solutions of the Matrix Riccati Differential Equation (MRDE) obtained from Simulink. Accuracy of the solution of the Simulink approach to the problem is qualitatively better. The advantage of the proposed approach is that, once the Simulink model is constructed, it allows to evaluate the solution at any desired number of points spending negligible computing time and memory and the solution curves can be obtained from the model without writing any code. An illustrative numerical example is presented for the proposed method.

K. Kittipeerachon, N. Hori and Y. Tomita, et.al [9] exact method is presented for discretizing a constant-coefficient, non-square, matrix differential Riccati equation, whose solution is assumed to exist. The resulting discrete-time equation gives the values that have no error at discrete-time instants for any discrete-time interval. The method is based on a matrix fractional transformation, which is more general than existing ones, for linearizing the differential Riccati equation. A numerical example is presented to compare the proposed method with that based on gage invariance and bilinearization, which has better

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performances than the conventional forward-difference method.

X. Mao, X. Zhang and H. Zhou, et.al [10] well-known R0 implication is developed to pseudo-De Morgan algebras, which is called generalized pseudo-R0 implication. The notion of strong pseudo-De Morgan algebras is introduced, and its elementary properties are discussed. Secondly, two necessary and sufficient conditions are proved as follows: (1) A pseudo-De Morgan algebra A with generalized pseudo-R0 implication becomes a pseudo-involutive pseudo-BCK algebra if and only if A is a strong pseudo-De Morgan algebra. (2) A pseudo-De Morgan algebra A with generalized pseudo-R0 implication and corresponding operator becomes a pseudo-regular residuated lattice if and only if A is a strong pseudo-De Morgan algebra. Finally, all pseudo-De Morgan algebras, strong pseudo-De Morgan algebras and proper pseudo-involutive pseudo-BCK algebras are obtained by MATLAB software when the order number is smaller than or equal to 8. Furthermore, starting with bounded distributive lattices, we discussed the classification problem of lower-order pseudo-involutive pseudo-BCK algebras.

K. S. Achary, P. R. Murarka and M. Reza, et.al [11] To solve complex and large mathematical expression manually using pen and paper is a time taking task which in most cases ends up in an erroneous result. This is a major drawback which may lead to heavy losses to people dealing in numbers. Henceforth we have come up with a vision of Symbolic computation which provides a quick, efficient and user friendly environment to its users. Symbolic Computation is a computer algebra system which has been designed in Java. The Object oriented Programming(OOP) concept and predefined packages of the language have been used to solve expressions consisting of differentiation, integration, series and many more symbols. Moreover these features have also been brought to use to enhance the computation time and provide a better outlook to the application.

B. Erabadda, S. Ranathunga and G. Dias, et.al [12] This paper presents a system that automatically assesses multi-step answers to algebra questions. The system requires teacher involvement only during the question set-up stage. Two types of algebra questions are currently supported: questions with linear equations containing fractions, and questions with quadratic equations. The system evaluates each step of a student's answer and awards full/partial marks according to a marking scheme. The system was evaluated for its performance using a set of student answer scripts from a government school in Sri Lanka and also by undergraduate students. The system accuracy was over 95.4%, and over 97.5%, respectively for the aforementioned data sets.

Jianping Yan, et.al [13] In this paper, we define a logical algebra named MP-algebra and discuss its algebraic properties. We find that MP-algebra not only takes Boole algebra, MV-algebra and R 0 algebra as its special examples but also holds the subdirect representation theorem same as that of on Boole algebra, MV algebra and R 0 algebra. We also explore the basic properties of implication operation of MP-algebra. We prove that an MP-algebra is also a residuated lattice with many good properties. The conclusions we got show that MP-algebra is a well-structure logic algebra when it is taken as the logic truth degree set.

S. A. Matos, C. R. Paiva and A. M. Barbosaet.al [14] It is well-know that conical refraction occurs for electric

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anisotropic biaxial crystals when the wave vector has the direction of the medium optic axes. In this paper, we show that conical refraction occurs - in an analogous away - for a more general type of biaxial media that have simultaneously electric and magnetic anisotropies. Furthermore, the new coordinate-free approach based on geometric algebra, developed by the authors in previous papers to address anisotropy, is shown to shed new light on this classic topic of optics that is conical refraction.

M. T. Pham, T. Yoshikawa, T. Furuhashi and K. Tachibana, et.al [15] Most conventional methods of feature extraction for pattern recognition do not pay sufficient attention to inherent geometric properties of data, even in the case where the data have spatial features. This paper introduces geometric algebra to extract invariant geometric features from spatial data given in a vector space. Geometric algebra is a multidimensional generalization of complex numbers and of quaternions, and it ables to accurately describe oriented spatial objects and relations between them. This paper proposes to combine several geometric features using Gaussian mixture models. It applies the proposed method to the classification of hand-written digits.

## III. METHODOLOGY

In order to describe the NIM, consider the general functional equation.

$S = f + L(s) + N[s]$ ---- (3)

where f is a known function, L[.] and N[.] are the linear and nonlinear operators respectively. Suppose we define M s[ .] as

$M[s] = L[s] + N[s]$ ---- (4)

Then (1) becomes

$S = f + M[s]$ ---- (5)

Now consider a solution, s having the series form:

$$\begin{cases} s = \sum_{i=0}^{\infty} S_i \\ M[s] = N[\sum_{i=0}^{\infty} S_i \end{cases}$$

---- (6)

the nonlinear operator M can be decomposed as

$$M\left(\left(\sum_{i=0}^{\infty} Si\right)\right) = M[s_0] + \sum_{i=0}^{\infty} M\left(\sum_{i=0}^{m} Si\right) - M\left(\sum_{i=0}^{m-1} si\right), m = 1,2,\ldots$$

----- (7)

Therefore, putting (6) and (7) into (5), we obtain

$$\sum_{i=0}^{\infty} Si = f + M[s_0] + \sum_{i=1}^{\infty} M\left(\sum_{i=0}^{m} Si\right) - M\left(\sum_{i=0}^{m-1} Si\right), m = 1,2,\ldots,$$

--- (8)

Hence, the recurrence relation is:

$$\begin{cases} S_{0=f} \\ S_1 = M(S_0) \\ Sm + 1M\left[\sum_{i=0}^{m} s_i\right] - M\left[\sum_{i=0}^{m-1} s_i\right], m = 1,2,\ldots \end{cases}$$

--- (9)

Such that

$$S = f + \sum_{i=1}^{\infty} S_i = \sum_{i=0}^{\infty} s_i$$

----- (10)

By using this method, the solution is found in the form of convergent finite series such that the components are easily computed; often time, the convergence of this series is very fast with few iterations needed to describe the behaviour of the solution.

Normalized Brownian Motion and the NIM

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Let $Y_0$ $Y_1$, ... be mutually independent random variables with identically, independent Gaussian distribution with N (0,1 ). The random process,

$$w(t) = \frac{Y}{\sqrt{2\pi}} = t + \sum_{k=1}^{\infty} \sqrt{\frac{2}{\pi}} \frac{Y_k}{K} \sin(kt), \text{for } t \in [0, \pi]$$

---- (11)

is a called a normalized Brownian Motion on the interval [0,Π].

$$dW_t = \frac{Y_0}{\sqrt{2\pi}} + \sum_{k=1}^{\infty} \sqrt{\frac{2}{\pi}} Y_k \cos(kt)$$

---- (12)

By replacing the finite terms of the normalized Brownian Motion in SDE

$$\sum \begin{matrix} dS = a(S,t)dt + \sigma(S,t)\frac{Y_0}{\sqrt{2\pi}} + \sum_{k=1}^{5} \sqrt{\frac{2}{\pi}} Y_k \cos(kt), \\ S(0) = S_0, \end{matrix} .$$

--- (12)

where L=5 and $Y_k$ are randomly generated random variables.

## IV. RESULT ANALYSIS

In this performance analysis Linear stochastic differential equations NIM is observed in this section.

**Table.1: Performance Analysis**

| Parameters | NIM | LSDE |
|---|---|---|
| Accuracy | 96.7 | 81.3 |
| Efficiency | 92.8 | 87.6 |

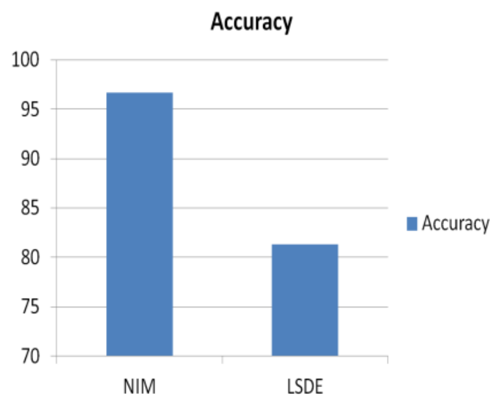


**Fig.1: Accuracy Comparison Graph**

In Fig.1 accuracy comparison graph is observed between NIM and Linear Stochastic Differential Equations (LSDEs).

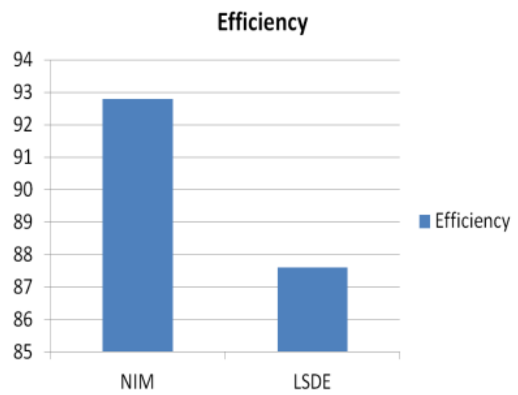


**Fig.2: Efficiency Comparison Graph**

In Fig.2 efficiency comparision graph is observed between NIM and Linear Stochastic Differential Equations (LSDEs).

## V. CONCLUSION

The application of a New Iterative Method (NIM) for approximate analytical solution of linear Stratonovich Stochastic Differential Equations. The white noise was handled using Normalized Brownian Motion (NBM) in terms of finite series form. Part of the advantages of the NIM includes its ease of application and less iteration. Therefore, recommend these approaches for higher order and nonlinear models of Itô or Stratonovich Stochastic Differential Equations (SDEs).Hence, this

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model achieves better results interms of accuracy and efficiency.

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