

## A Perturbation Approach to the Study of Unsteady MHD Periodic Flow of Viscous Fluid through a Planer Channel

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### ABSTRACT:

In this investigation, we study the periodic unsteady flow of a viscous incompressible fluid through a porous planer channel in the presence of a transverse magnetic field. Through the use of perturbation techniques, the governing equations are solved. The induced magnetic field are negligible. The fluid's velocity profiles can be computed analytically and given as expressions. Tables gives an overview of the computations that went into the response. With the aid of graphs and figures, the impacts of different parameters on the flow field are examined. Based to the findings, the flow field has been significantly affected by the transverse magnetic field's presence and its perturbation. When a magnetic field is present, it is discovered that the velocity profiles are not uniform and the flow is more unstable. In the presence of a magnetic field, it is discovered that the velocity profiles are not uniform and the flow is more unstable. There is additional discussion of the impacts of several parameters, including the Reynolds number, the strength of the magnetic field, and the perturbation parameter. The results of this work can be applied to porous channel design and improvement for many different kinds of uses, including heat transfer and chemical reaction engineering. This study will provide engineering and manufacturing professionals with alternative equations and methodologies for achieving the optimal MHD parameters, such as temperature, velocity, and pressure. This is important because MHD parameters play a critical role in a variety of engineering and manufacturing processes, such as heat transfer, fluid flow and material processing.

**KEYWORDS:** Viscous fluid, Hartmann Number, Planer Channel, Periodic Flow, Magnetohydrodynamics, and Perturbation Technique.

**MSC classification 2020 :** 76D55, 76M10, 37E35

## 1. INTRODUCTION:

It often becomes essential to construct nonlinear equations that are challenging to precisely and analytically solve in order to explain how real-world phenomena and systems behave through the use of mathematical models. As an alternative method for solving the nonlinear equations, numerical methods are therefore always used. However, it is clear that new advancements in analytic approaches continue to be crucial. It is also necessary and required to provide analytical answers to the given problems in order to demonstrate the direct correlation between the model's input parameters. If analytical solutions are available, they offer helpful insights into the importance of various system characteristics influencing the phenomenon. Contrary to pure numerical or computing methods, such solutions offer continuous physical insights. The unsteady of MHD periodic flow down a vertical channel is a common heat transfer phenomenon that has been studied extensively in thermal, industrial, and mechanical engineering. This type of flow occurs when a fluid is heated from below, causing it to become less dense and rise. The rising fluid then displaces the cooler fluid above, creating a continuous circulation loop.

Convective flows are important in a variety of real-world applications, including the following:

- Automatic cooling of heat exchangers: Heat exchangers are devices that transfer heat from one fluid to another. In many cases, the heat transfer process is enhanced by the use of convective flows.
- Cooling and heating of nuclear reactors: Nuclear reactors generate heat as a byproduct of the nuclear fission process. This heat must be removed from the reactor in order to maintain safe operating temperatures. Convective flows are often used to cool nuclear reactors.
- Many other applications: Convective flows are also used in a variety of other applications, such as the cooling of electronic devices, the drying of materials, and the mixing of fluids.

The study of free convection flow down a vertical channel has led to the development of a number of theoretical and numerical models that can be used to predict the flow behavior and heat transfer rates. These models are essential for the design and optimization of heat exchangers, nuclear reactors, and other devices that rely on convective flows. Thermal radiation plays an important role in engineering fields employing high temperatures, and comprehension of radiative heat transfer is

vital for nuclear power plants, gas turbines, and the numerous propulsion systems used in aircraft, missiles, and spacecraft. Sparrow and Cess (1961) investigated the effect of a magnetic field on a free convective heat transfer. Raptis et al. (1982) have been discussed hydro magnetic free convection flow through a porous medium between two parallel plates. Pal et al. (1984) investigated a longitudinal dispersion of tracer particles in a channel bounded by porous media using slip condition. Singh and Kumar (1993) investigated a free convective fluctuating flow through a porous medium with variable permeability. Aldoss et al. (1995) depicted a mixed convection flow from a vertical plate embedded in porous medium in the presence of magnetic field. Makinde (1995) investigated a laminar flow in a channel of varying width with permeable boundaries. Soltani and Yilmazar (1998) studied a slip velocity and slip layer thickness in flow of concentrated suspensions. Jha (1998) discussed an effect of applied magnetic field on transient free convective flow in a vertical channel.

Helmy (1998) investigated MHD unsteady free convective flow past a vertical porous plate. Hossain et al. (1998) investigated heat transfer response of MHD free convective flow along a vertical plate to surface temperature oscillation. Kim (2000) discussed an unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. Yu and Ameer (2002) investigated a slip low heat transfer in rectangular micro channels. Derek et al. (2002) established an apparent fluid slip at hydrophobic microchannel walls. Sharma et al. (2002) investigated the flow between annular spaces surrounded by coaxial cylindrical porous medium. Steinruck (2003) investigated about the physical relevance of similarity solution of the boundary layer flow equation describing mixed convection flow along a vertical plate. Khaled and Vafai (2004) discussed the effect of slip condition on Stokes and Couette flows due to an oscillating wall. Makinde and Mhone (2005) investigated a heat transfer to MHD Oscillatory flow in a channel filled with porous medium. Makinde and Osalusi (2006) discussed a MHD steady flow in a channel with slip at permeable boundaries. Kumar et al. (2007) investigated a viscous flow coaxial cylinder in the presence of magnetic field. Mahmood and Ali (2007) established the effect of slip condition on unsteady MHD oscillatory flow of a viscous fluid in a planer channel. Mishra et al. (2008). investigated a flow and heat transfer of a MHD viscoelastic fluid in a channel with stretching walls. Kumar et al. (2010) investigated a Finite difference technique for reliable MHD steady flow through channels permeable boundaries and applications. Kumar et al. (2010): made MHD free convective fluctuating flow through a porous effect with variable permeability parameter. Kaur et al. (2014):

Analysis of heat transfer in hydro magnetic rotating flow of viscous fluid through a non homogenous porous medium with constant heat source/sink. Samuel et al.(2018) investigated Natural convection flow of heat generating hydromagnetic couple stress fluid with time periodic boundary conditions and their applications. Agarwal and Kumar (2020): investigated an effect of viscous dissipation on MHD unsteady flow through vertical porous medium with constant suction. Kumar and Verma (2021) investigated mathematical model for Newtonian and non-Newtonian flow and their applications. Kumar (2022): profounded a periodic on convective heat transfer past a Semi-infinite vertical porous wall a computational study and applications. Onyancha et al. (2023) investigated optimization of magnetohydrodynamic parameters in two- dimensional incompressible fluid flow on a porous channel.

## 2. Mathematical Analysis :

Consider for instance the fully developed flow of an incompressible connect stress fluid through a vertically positioned channel that becomes heated at the walls on a steady-periodic basis. This is illustrated in Figure 1. We are considering an unsteady periodic flow of viscous incompressible fluid through a planer channel with saturated porous medium in the presence of transverse magnetic field. Perturbation techniques are a class of analytical methods for determining approximate solutions of nonlinear equations for which exact solutions cannot be obtained. They are useful for demonstrating, predicting, and describing phenomena in vibrating systems that are caused by nonlinear effects.

The governing equations of the motion are given below:

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho} \frac{\partial P^*}{\partial x^*} + \nu \frac{\partial^2 u^*}{\partial y^{*2}} - Ku^* - Mu^* + g\beta(T^* - T_0^*), \quad \dots (1)$$

$$\frac{\partial T^*}{\partial t^*} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho C_p} \frac{\partial q^*}{\partial y^*}, \quad \dots (2)$$

Subject to the conditions are:

$$u^* = h \frac{\partial u}{\partial y^*}, \quad T^* = T_w^* \quad \text{at } y^* = 1, \quad \dots (3)$$

$$u^* = 0, T^* = T_0^* \quad \text{at } y^* = 0, \quad \dots (4)$$

where  $u^*$  is the velocity,  $\rho$  the fluid density,  $P^*$  the pressure,  $t^*$  the time,  $\nu$  the kinematics viscosity coefficient,  $K^*$  the porous medium permeability coefficient,  $\sigma_\varepsilon$  the conductivity of the

fluid,  $B_0$  the electromagnetic induction,  $g$  the acceleration due to gravity,  $\beta$  the coefficient of volume expansion due to temperature,  $T^*$  the fluid temperature,  $K$  the thermal conductivity,  $C_p$  the specific heat at constant pressure and  $q$  is the radiative heat flux. Consider the fluid is optically thin with a relatively low density and the radiative heat flux is given by

$$\frac{\partial q^*}{\partial y^*} = 4\alpha^2(T^* - T_0^*), \quad \dots (5)$$

where  $\alpha$  is the mean radiation absorption coefficient.

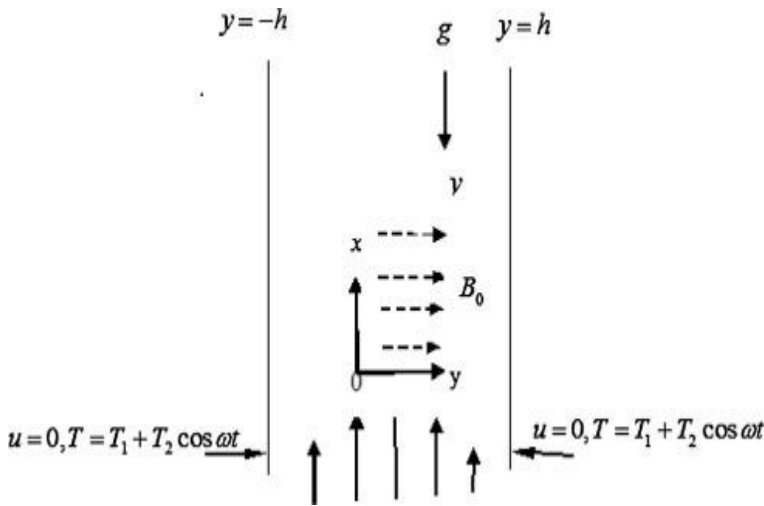


Fig. 1 Physical model of the problem.

The corresponding appropriate non- dimensionless quantities are given below :

$$x = \frac{x^*}{a}, y = \frac{y^*}{a}, u = \frac{u^*}{V}, \theta = \frac{T^* - T_0^*}{T_w^* - T_0^*}, t = \frac{t^* V}{a}, P = \frac{a P^*}{\rho \nu V}, Da = \frac{K^*}{a^2}, N^2 = \frac{4\alpha^2 a^2}{\kappa},$$

$$Gr = \frac{g\beta(T_w^* - T_0^*)a^2}{\nu V} \text{ (Grashoff number)}, \quad Pe = \frac{Va\rho C_p}{\kappa} \text{ (Peclet number)}$$

$$Re = \frac{Va}{\nu} \text{ (Reynolds number)}, \quad M = \sqrt{\frac{a^2 \sigma_\epsilon B_0^2}{\rho \nu}} \text{ (Hartmann number)}$$

Now using the above non dimension quantities into the equations (1) to (4), we have

$$Re \frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \frac{\partial^2 u}{\partial y^2} - (K^2 + M^2)u + Gr\theta \quad \dots (6)$$

$$Pe \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} - N^2 \theta \quad \dots (7)$$

Subject to condition

$$u = h \frac{\partial u}{\partial y}, \quad \theta = 0 \quad \text{at} \quad y = 0 \quad \dots (8)$$

$$u = 0, \quad \theta = 1 \quad \text{at} \quad y = 1 \quad \dots (9)$$

where  $k^2 = \frac{1}{Da} = \frac{a^2}{K^*}$

### 3. Numerical Method

For purely periodic flow, let the pressure gradient is of the form  $-\frac{\partial P}{\partial x} = \lambda e^{int}$ , where  $\lambda$  is a constant and  $n$  is a frequency of oscillation. In order to solve the equations (6)-(7) with appropriate boundary conditions (8)-(9), we are apply Perturbation technique, we have

$$\text{Let } u(y, t) = u_0(y) + \varepsilon u_1(y) e^{int} + o(\varepsilon^2) + \dots \dots \dots (10)$$

$$\theta(y, t) = \theta_0(y) + \varepsilon \theta_1(y) e^{int} + o(\varepsilon^2) + \dots \dots \dots (11)$$

The equations (6) and (7) as given below:

$$u_1'' - m_1^2 u_1 = -\frac{\lambda}{\varepsilon} - G_r \theta_1 \quad \dots (12)$$

$$u_0'' - m_2^2 u_0 = -G_r \theta_0 \quad \dots (13)$$

$$\theta_1'' + m_3^2 \theta_1 = 0 \quad \dots (14)$$

$$\theta_0'' + N^2 \theta_0 = 0 \quad \dots (15)$$

The corresponding conditions are given by

$$u = \gamma \frac{\partial u}{\partial y}, \quad \theta = 0 \quad \text{at} \quad y = 0$$

$$u = 0, \quad \theta = 1 \quad \text{at} \quad y = 1$$

where  $m_1 = \sqrt{K^2 + M^2 + inR_\varepsilon}$ ,  $m_2 = \sqrt{K^2 + M^2}$ ,  $m_3 = \sqrt{N^2 - inP_\varepsilon}$ .

On solving equations (12) – (15) and using the above boundary conditions, we get

$$\theta_0(y) = 0,$$

$$\theta_1(y) = \left( \frac{\sin m_3 y}{\varepsilon \sin m_3} \right),$$

$$u_0(y) = 0$$

$$u_1(y) = \frac{C_1}{\varepsilon} \cosh m_1 y + \frac{C_2}{\varepsilon} \sinh m_1 y + \frac{\lambda}{\varepsilon m_1^2} + \frac{G_r}{\varepsilon (m_1^2 + m_3^2)} \frac{\sin m_3 y}{\sin m_3}$$

where  $C_1 = \frac{-\lambda}{m_1^2} + \gamma \left\{ C_2 m_1 + \frac{m_3 G_r}{(m_1^2 + m_3^2) \sin m_3} \right\}$

$$C_2 = \frac{1}{\sinh m_1 + \gamma m_1 \cosh m_1} \left[ (\cosh m_1 - 1) \frac{\lambda}{m_1^2} + \frac{G_r}{(m_1^2 + m_3^2)} \left\{ 1 + \gamma m_3 \frac{\cosh m_1}{\sinh m_1} \right\} \right]$$

$$\text{Therefore } u(y,t) = \left\{ C_1 \cosh m_1 y + C_2 \sinh m_1 y + \frac{\lambda}{m_1^2} + \frac{G_r}{(m_1^2 + m_3^2)} \frac{\sin m_3 y}{\sin m_3} \right\} e^{int}$$

$$\theta(y,t) = \left( \frac{\sin m_3 y}{\sin m_3} \right) e^{int}$$

The shear stress is given by,  $\tau = \left( \frac{\partial u}{\partial y} \right)_{y=0}$

$$\tau = \left\{ m_1 C_2 + \frac{m_3 G_r}{(m_1^2 + m_3^2) \sin m_3} \right\} e^{int}$$

**Table 01:** Velocity profiles for different values of parameter  $G_r = 1, R_\varepsilon = 1, M = 1, N = 1, P_\varepsilon = 0.7, n = 1, \lambda = 1, t = 0, K = 1.02$

$y$	$\gamma = 0$	$\gamma = 0.5$	$\gamma = 1.0$	$\gamma = 1.5$	$\gamma = 2.0$	$\gamma = 2.5$
0	0.0000008	10.639726	15.205435	17.743453	19.359122	20.477912
0.2	0.913037	8.106225	11.545581	13.457473	14.674559	15.517346
0.4	0.141078	5.871913	8.331123	9.698166	10.568409	11.171018
0.6	0.146172	3.43654	5.420148	6.300455	6.860848	7.248896
0.8	0.101535	1.908315	2.683639	3.114632	3.388999	3.518983
1	0.0000014	0.0000001	0.0000004	0.0000008	0.0000041	0.00000037

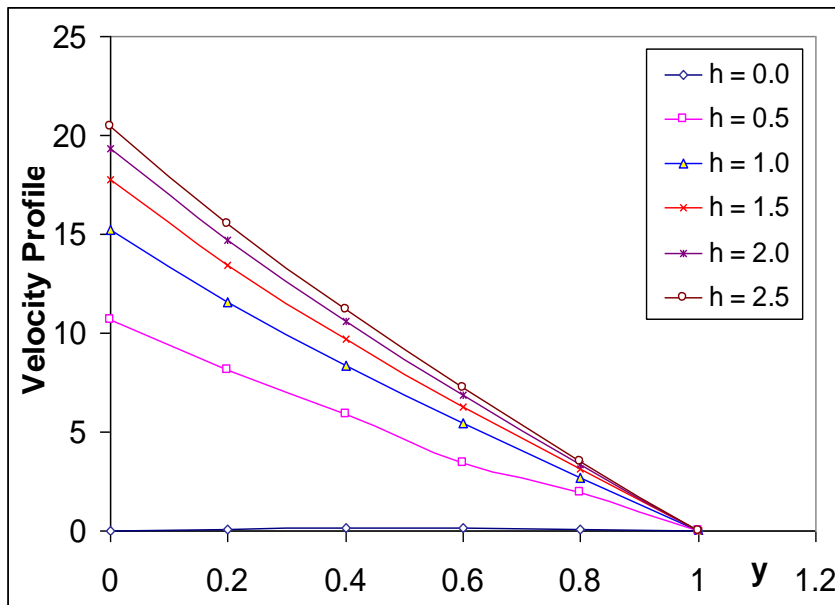


Fig.2: Velocity profiles for various values of wall slip parameter h.

**Table 02:** Shear stress for different values of parameter  $G_r = 1, R_g = 1, M = 1, N = 1, P_g = 0.7, n = 1, \lambda = 1, K = 1.02$

T	$\gamma = 0$	$\gamma = 0.5$	$\gamma = 1.0$	$\gamma = 1.5$	$\gamma = 2.0$	$\gamma = 2.5$
0	35.43408	21.27945	15.20543	11.82896	9.67955	8.19116
1	35.42868	21.27621	15.20312	11.82716	9.67808	8.18991
2	35.41249	21.26649	15.19617	11.82176	9.67366	8.18617
3	35.38552	21.25029	15.18459	11.81275	9.66629	8.17993
4	35.34776	21.22761	15.16839	11.80015	9.65597	8.171211
5	35.29924	21.19847	15.14757	11.78395	9.64272	8.15999
6	35.23997	21.16288	15.12213	11.76416	9.62653	8.14629
7	35.16996	21.12084	15.09209	11.74079	9.6074	8.1301



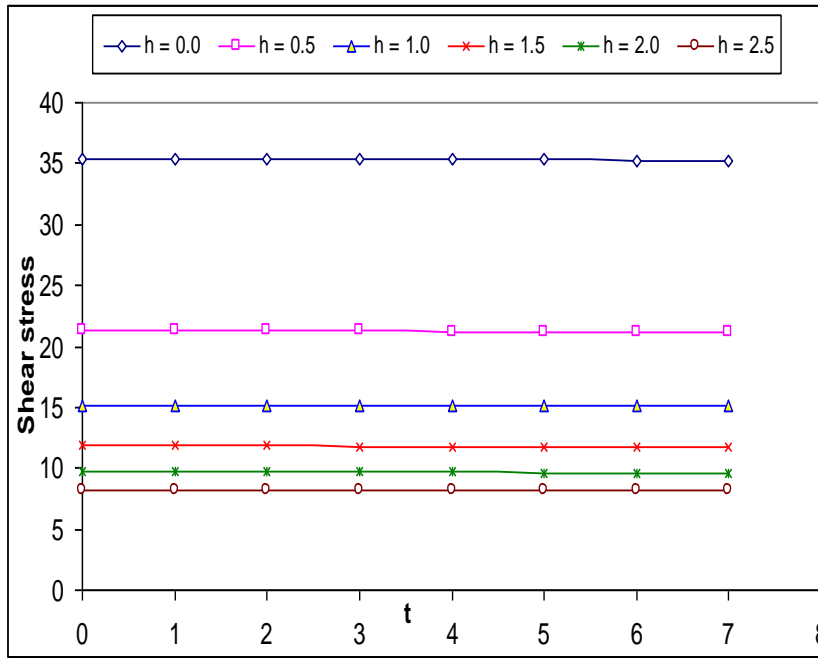


Fig.03: Shear stress for various values of wall slips parameter h.

Table 03: Velocity profiles for different values of parameter

$$G_r = 1, R_g = 1, M = 1, N = 1, P_g = 0.7, n = 1, \lambda = 1, K = 1.02, \gamma = 1$$

y	t=0	t=π/4	t=π/2	t=3π/4	t=π	t=5π/4
0	14.20543	10.75186	0	-10.7519	-15.2054	-10.7519
0.2	12.54558	8.16395	0	-8.16395	-11.5455	-8.16395
0.4	9.33112	5.89099	0	-5.89099	-8.33112	-5.89099
0.6	6.42014	3.83262	0	-3.83262	-5.42014	-3.83262
0.8	3.68363	1.89761	0	-1.89761	-2.68363	-1.89761
1	0	0	0	0	0	0

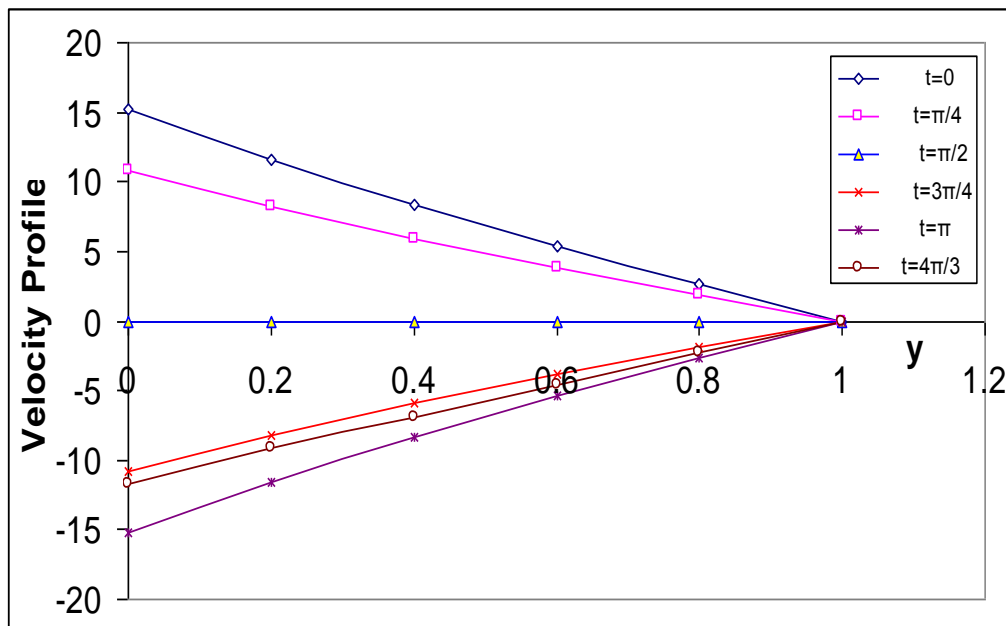


Fig.4: Velocity profile for various values of time t.

Table 04: Velocity profiles for different values of parameter

$$G_r = 1, R_g = 1, N = 1, P_g = 0.7, n = 1, \lambda = 1, K = 1, \gamma = 1, t = 0$$

y	M=0	M=0.5	M=1.0	M=1.5	M=2.0	M=2.5
0	22.40381	20.18969	15.20543	10.3673	6.89356	4.64272
0.2	17.39652	15.59154	11.54558	7.65592	4.90775	3.16674
0.4	12.76581	11.39433	8.33112	5.41026	3.3748	2.11022
0.6	8.39547	7.4736	5.42014	3.47412	2.13198	1.31034
0.8	4.17562	3.71282	2.68363	1.71199	1.04614	0.64212
1	0	0	0	0.000001	0.000001	0.000001

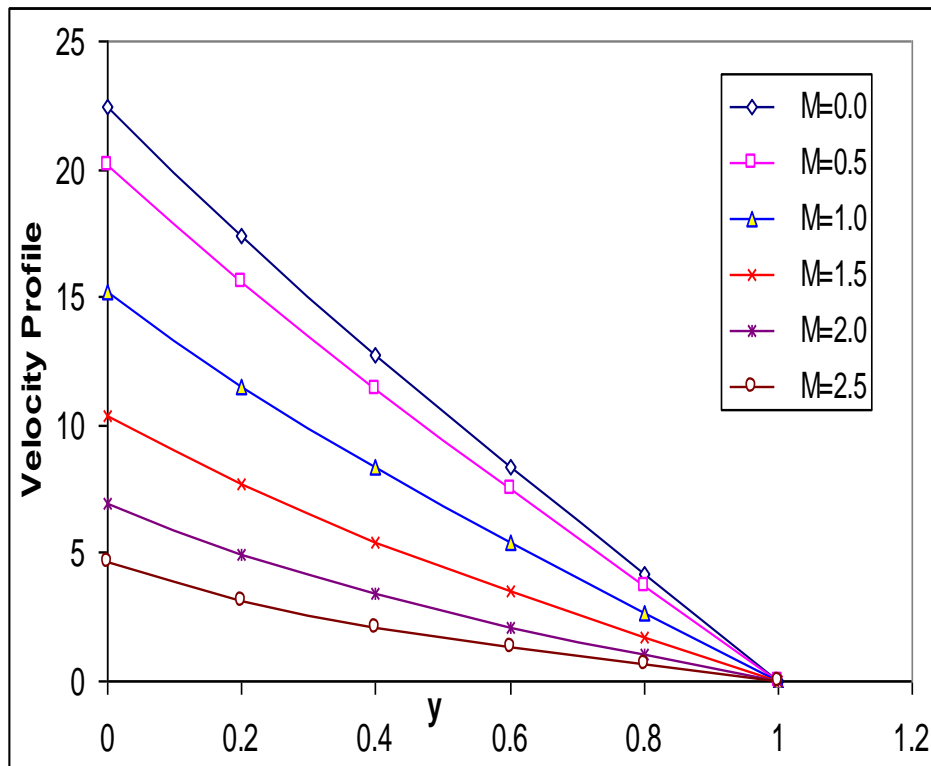


Fig.5: Velocity profile for various values Hartmann number M.

Table 05: Velocity profiles for different values of parameter

$G_r = 1, R_g = 1, M = 1, N = 1, P_g = 0.7, n = 1, \lambda = 1, \gamma = 1, t = 0$

y	K=0	K=0.5	K=1.0	K=1.5	K=2.0	K=2.5
0	22.40380	20.18960	15.20540	10.3670	6.89350	4.64270
0.2	17.39652	15.59154	11.54558	7.65592	4.90775	3.16674
0.4	12.76580	11.39430	8.33110	5.41020	3.3740	2.11020
0.6	8.39547	7.4736	5.42014	3.47412	2.13198	1.31034
0.8	4.17562	3.71282	2.68363	1.71199	1.04614	0.64212
1	0	0	0	0.000001	0.000001	0.000001

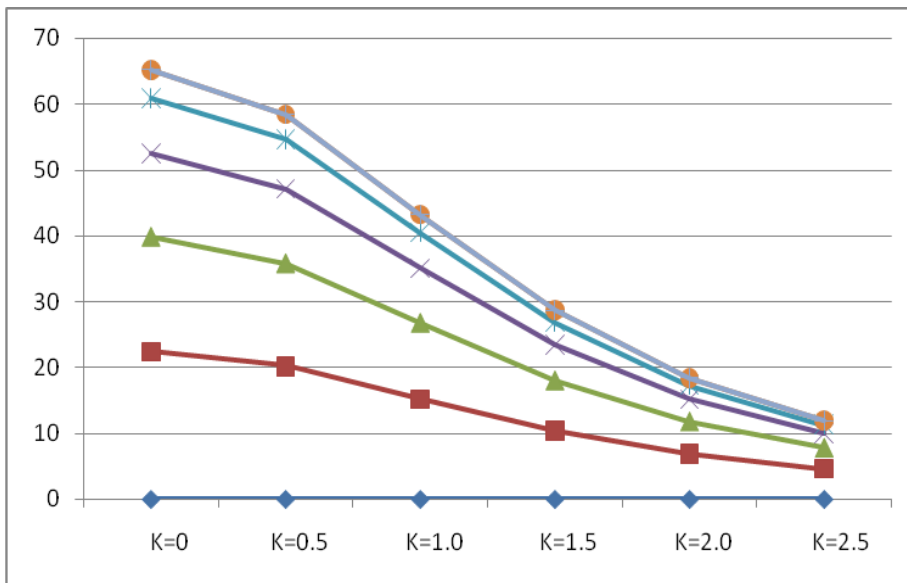


Fig 6 Velocity profiles for different values of parameter

$$G_r = 1, R_g = 1, M = 1, N = 1, P_g = 0.7, n = 1, \lambda = 1, \gamma = 1, t = 0$$

Table 6: Velocity profiles for different values of parameter

$$G_r = 1, R_g = 1, N = 1, n = 1, \lambda = 1, \gamma = 1, t = 0$$

$y$	$P_g = 0$	$P_g = 0.5$	$P_g = 1.0$	$P_g = 1.5$	$P_g = 2.0$	$P_g = 2.5$
0	15.73650	15.4470	14.76860	13.96430	13.07480	12.41940
0.2	11.94633	11.72845	11.21602	10.60913	9.91402	9.44345
0.4	8.61839	8.4622	8.09491	7.65992	7.12995	6.82442
0.6	5.60576	5.50482	5.26755	4.98654	4.59973	4.44679
0.8	2.77494	2.72526	2.60862	2.47049	2.21125	2.20517
1	0.000129	0.000001	0	0	0.14157	0.000001

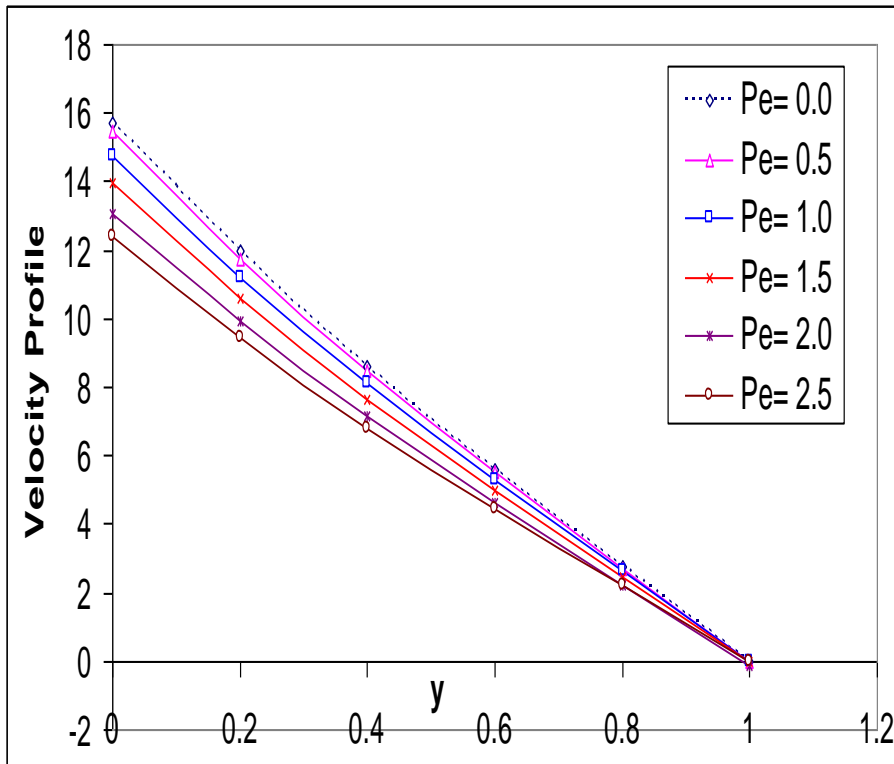


Fig.7: Velocity profile for various values Peclet number  $P_e$

Table 07: Velocity profiles for different values of parameter

$G_r = 1, M = 1, N = 1, P_e = 0.7, n = 1, \lambda = 1, \gamma = 1, t = 0$

y	Re=0	Re=0.5	Re=1.0	Re=1.5	Re=2.0	Re=2.5
0	15.8918	15.70765	15.20544	14.49766	13.69436	12.87216
0.2	12.10094	11.95187	11.54558	10.97365	10.32555	9.663441
0.4	8.750434	8.637843	8.331123	7.899777	7.411637	6.913751
0.6	5.700659	5.625317	5.420148	5.131826	4.80587	4.473808
0.8	2.824052	2.786332	2.683639	2.539392	2.376417	2.210514
1	0.000001	0.000001	0	0	0	0.000001

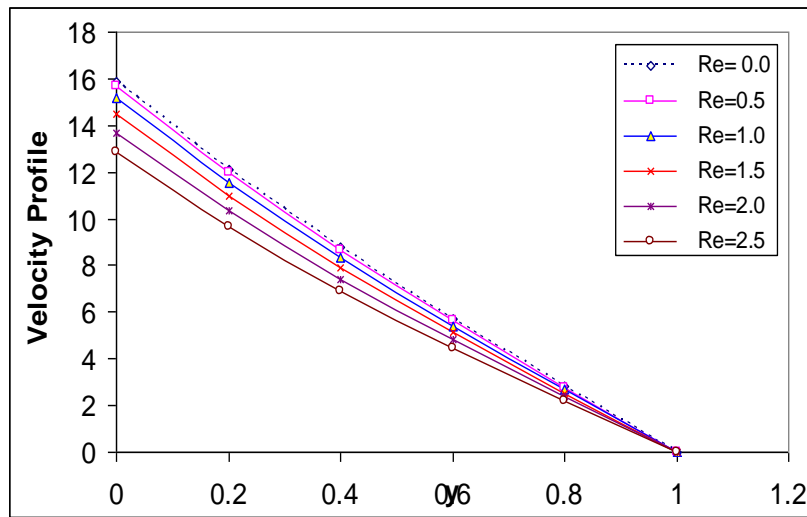


Fig.8: Velocity profile for various values Reynolds number  $R_e$ .

Table 08: Velocity profiles for different values of parameter

$$G_r = 1, M = 1, R_e = 1, P_e = 0.7, n = 1, \lambda = 1, \gamma = 1, t = 0$$

$y$	N=0	N=0.5	N=1.0	N=1.5	N=2.0	N=2.5
0	19.82857	18.727573	15.20543	11.15656	8.07829	5.97638
0.2	15.02676	14.2032	11.54558	8.49051	6.16781	4.58258
0.4	10.81671	10.23596	8.33112	6.14141	4.47662	3.34142
0.6	7.012565	6.65071	5.42014	4.00555	2.93006	2.19811
0.8	3.445713	3.28853	2.68363	1.98828	1.45961	1.10201
1	0.04236	0.00000002	0.0000004	0.0000026	0.00000001	0.004504

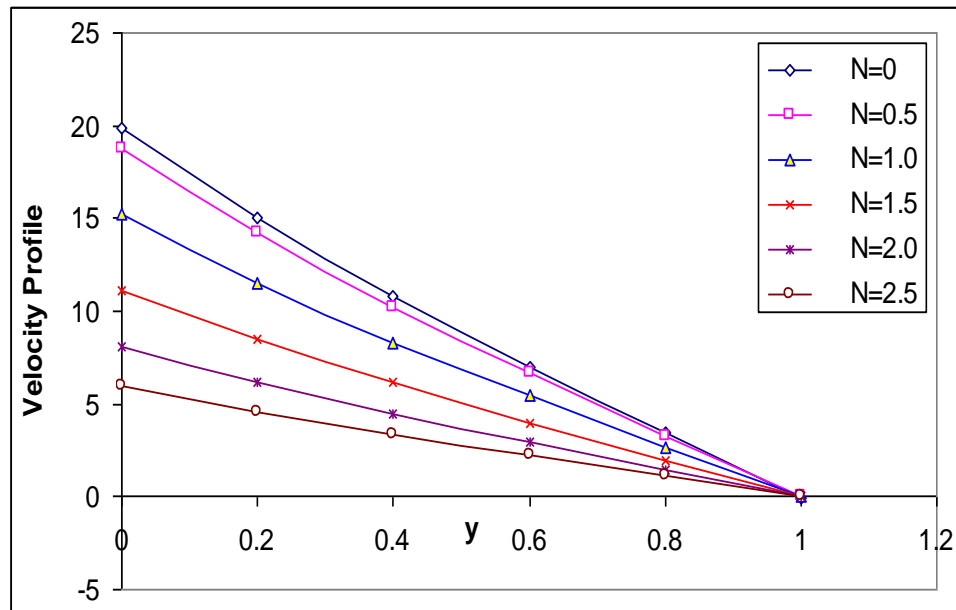


Fig.9: Velocity profile for various values radiation parameter N.

#### 4. RESULT AND DISCUSSION:

This study investigates the effect of temperature and concentration on the MHD periodic flow in a planer channel. The wall shear stress and rate of heat and mass transfer at the channel walls have been estimated using the velocity, temperature, and concentration distributions that are determined analytically. The effects of various parameters like Hartmann number  $M$ , Grashoff number  $G_r$ , Peclet number  $P_e$ , Reynolds number  $R_e$ , permeability of porous medium  $k$ , Radiation parameter  $N$ , time  $t$  and wall slip parameter  $h$  on velocity and temperature profiles have been shown in tables and graphs. The brief description is as given below:

Fig.2 shows that the velocity is everywhere increases corresponding to every increase in the wall slip parameter  $h$  in the region  $y \in [0,1]$ , which clearly tallies the natural phenomenon.

Fig.3 depicts the tendency of shear stress distribution corresponding to different values of wall slip parameter  $h$ . Hence for every smaller values of  $h$  the value of shear stress is seen much more than that of larger one. Fig.4 depicts that the velocity profile of the fluid is periodic with period  $\pi$ .

Fig.5 depicts velocity profile for various values of magnetic parameter  $M$ . It shows that the flow decelerates on the imposition of magnetic field i.e. on the increase of magnetic parameter  $M$ .

Fig.6 depicts that velocity profile for different values of porosity parameter  $k$ .

The effect of Peclet number  $P_e$  on the velocity profiles are shown in table 06 and fig.7. It is found that the fluid velocity decreases as the values of Peclet number increases corresponding to the same value of  $y$ . Table 07 & fig.8 also show similar effects of Reynolds number  $R_e$  as the Peclet number has the flow. Table 08 and fig.9 depict the velocity profiles for various values of radiation parameter  $N$ . From table 08 or fig.08, it is observed that the increasing values of radiation parameter  $N$  have the similar effects on the velocity distribution as parameter  $P_e$  and number  $R_e$  have. It has been studied how a heat-generating pair stress fluid with periodic boundary conditions behaves in natural convection in MHD.

#### NOMENCLATURE :

$u$ : Velocity along x axis

$P$ : Fluid pressure

$\rho$ : Fluid density

$\nu$  : Kinematic viscosity

$B_0$ : Electromagnetic induction

$\sigma_e$ : Fluid conductivity

$H_0$ : Intensity of magnetic field

$\beta$ : Coefficient of volume expansion due to temperature

$\mu_e$ : Magnetic permeability

$\lambda$ : A constant

$M$  : Hartmann number

$C_p$ : Specific heat at constant pressure

$g$ : Acceleration due to gravity

$M$ : Hartmann number

$K$ : Thermal conductivity

$K$ : Porous medium permeability

$N$ : Radiation parameter

$k$ : Porous medium shape factor

$T$ : Fluid temperature

$T_0$ : Temperature at the boundary  $y = 0$

$T_w$ : Temperature at the boundary  $y = a$

$n$ : Frequency of the periodic flow



$\theta$ : Non-dimensional temperature

$R_g$ : Reynolds number

$G_r$ : Grashoff number

$Da$ : Darcy number

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