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Research Paper

A VIEW ON UNION OF SOME GRAPHS IN GRAPH THEORY

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ABSTRACT

In graph theory, a branch of mathematics the disjoint union of graphs is an operation that combines two or more graphs to form a larger graph. Eulerian to the Swiss mathematician Leonhard Euler, who invents graph theory in the 18th century. The disjoint union of two sets and is binary operator that combines all distinct element of a pair of given sets, while retaining the original set membership as a distinguishing characteristic of the union set.

KEYWORDS:

Union, pair sums union, edges, vertices, ladder, vertex, etc.,

Introduction:

Graph theoretical concepts are widely used to study and model various applications, in different areas. They include, study of molecules, construction of bonds in chemistry and the study of atoms. Similarly, graph theory is used in sociology for example to measure actors prestige or to explore diffusion mechanisms.

Definition: The union of two graphs G_1 and G_2 is the graph $G_1 \cup G_2$ with

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$V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$

Definition: If P_n denotes a path on n vertices, the graph $L_n = P_2 \times P_n$ is called a ladder .

Definition: The graph $C_n \hat{O} K_{1,m}$ is obtained from C_n and $K_{1,m}$ by identifying any vertex of C_n and central vertex of $K_{1,m}$.

Theorem

$K_{1,m} \cup K_{1,n}$ is a pair sum graph.

Proof

Let $x, x_1, x_2, \ldots x_m$ be the vertices of $K_{1,m}$ and

$E(K_{1,m}) = \{xx_j : 1 \le j \le m\}$.

Let $y, y_1, y_2, \ldots, y_n$ be the vertices of $K_{1,n}$ and

$E(K_{1,n}) = \{yy_j : 1 \le j \le m\}$.

when n = m.

consider $g(x) = 1$

$g(x_j) = j+1 \quad 1 \le j \le n$

$g(y) = -1$

$g(y_j) = -(j+1) \quad 1 \le j \le n$

when $n > m$

consider $g(x) = 1$

$g(x_j) = j+1 \quad 1 \le j \le m$

$g(y) = -1$

$g(y_j) = -(j+1) \quad 1 \le j \le m$

$g(y_{m+2j-1}) = -(m+1+j) \quad 1 \le j \le \frac{n-m}{2}$ if n-m is even or

$1 \le j \le \frac{n-m-1}{2}$ if n-m is odd

$g(y_{m+2j}) = m+j+3 \quad 1 \le j \le \frac{n-m}{2}$ if n-m is even or

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$1 \leq j \leq \frac{n-m-1}{2}$ if n-m is odd

Then g is a pair sum labeling.

Theorem

$P_n \cup K_{1,m}$ is a pair sum graph.

Proof

Let $x_1, x_2, \ldots x_n$ be the path P_n. Let $V(K_{1,m}) = \{y, y_j : 1 \leq j \leq m\}$ and

$E(K_{1,m}) = \{yy_j : 1 \leq j \leq m\}$.

When n = m.

Consider $g(x) = 1, \quad 1 \leq j \leq n,$

$g(y) = -1,$

$g(y_j) = -2j, \quad 1 \leq j \leq n,$

when m>n.

consider $g(x_j) = j, \quad 1 \leq j \leq n,$

$g(y) = -1,$

$g(y_j) = -2j, \quad 1 \leq j \leq n-1$

$g(y_{m+2j\_1}) = 2n+j, \quad 1 \leq j \leq \frac{m-n+1}{2}$ if m-n is odd or

$1 \leq j \leq \frac{m-n}{2}$ if m-n is even,

$g(y_{n+2j-2}) = -(2n+j-2) \quad 1 \leq j \leq \frac{m-n+1}{2}$ if m-n is odd or

$1 \leq j \leq \frac{m-n}{2} + 1$ if m-n is even

Then g is a pair sum labeling.

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Theorem

If n=m, then $C_n \cup C_m$ is a pair sum graph.

Proof

Let $x_1x_2,\dots x_mx_1$ be the first copy of the cycle in $C_m \cup C_m$ and $y_1y_2\dots y_my_1$ be the second copy of the cycle in $C_m \cup C_m$.

When n = m= 4k.

Consider

$g(x_j)=j, \quad 1 \le j \le 2k-1$

$g(x_{2k})=2k+1,$

$g(x_{2k+j})=-j, \quad 1 \le j \le 2k-1,$

$g(x_m)= -2k-1,$

$g(y_j)=2k+2j, \quad 1 \le j \le 2k,$

$g(y_{2k+j})=-2k-2j, \quad 1 \le j \le 2k.$

When n = m = 4k+2

Consider

$g(x_j) =j , \quad 1\le j\le 2k+1$

$g(x_{2k+1+j}) =-j \quad 1\le j\le 2k+1$

$g(y_j) =2k+2j, \quad 1\le j\le 2k+1$

$g(y_{2k+1+j}) = -2k-2j, \quad 1\le j\le 2k+1$

when n=m=2k+1.

$g(x_j) =-j$ and $g(y_j)= j$ we have a pair sum labeling.

Theorem

If $m \le 4$, then nK_m is a pair sum graph.

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Proof

Obviously m=1,the result is true.

Case 1: m=2.

Assign the label j and j+1 to the vertices of j^{th} copy of K_2 for all odd j. For even values of j,label the vertices of the j^{th} copy of K_2 by−j+1and−j.

Case 2: m=3.

Subcase1 m is even.

Label the vertices of first n/2 copies by 3j − 2, 3j − 1, 3j(1 ≤ j ≤ n/2). Remaining n/2 copies are labeled by -3j + 2, -3j + 1, -3j.

Subcase 2 n is odd.

Label the vertices of first (n − 1) copies as in Subcase (a). In the last copy label the vertices by $\frac{3(n-1)}{2}$ +1, $\frac{-3(n-1)}{2}$ -2, $\frac{3(m-1)}{2}$ +3 respectively.

Theorem

Any triangular snake T_m is a pair sum graph.

Proof

Let $V(T_m)=\{x_i,y_j:1 \le i \le m+1, 1 \le j \le m\}$,

$E(T_m)=\{x_ix_{i+1},x_iy_j,y_iy_{j+1}:1 \le i \le m, 1 \le j \le m-1\}$.

The proof consider three cases

Case1: m= 4n−1

Define

$$g(x_j)=2j-1, \quad 1 \le j \le 2n,$$

$$g(x_{2n+j})=-2j+1, \quad 1 \le j \le 2n,$$

$$g(y_j)=2j, \quad 1 \le j \le 2n-1,$$

$$g(y_{2n})=-8n+3,$$

$$g(y_{2n+j})=-2j, \quad 1 \le j \le 2n-1.$$

Case2: m=4n+1

Define

$g(x_i)=-8n-3+2(j-1),\quad 1 \leq j \leq 2n+1,$

$g(x_{2n+1+j})=8n+3-2(j-1),\quad 1 \leq j \leq 2n+1,$

$g(y_j)=-2+2(j-1),\quad 1 \leq j \leq 2n$

$g(y_{2n+1})=3,$

$g(y_{2n+j+1})=8n+2-2(j-1),\quad 1 \leq j \leq 2n.$

Case3 $m=2n$

Define

$g(x_{n+1})=1,$

$g(x_{n+1+j})=2j,\quad 1 \leq j \leq n,$

$g(x_{n+1-j})=-2j,\quad 1 \leq j \leq n$

$g(y_n)=3,$

$g(y_{n+1})=-5,$

$g(y_{n+1+j})=5+2j,\quad 1 \leq j \leq n-1,$

$g(y_{n-j})=-(5+2j),\quad 1 \leq j \leq n-1.$

Clearly T_m is a pair sum labeling.

Theorem

The crown $C_m \odot K_1$ is a pair sum graph.

Proof

Let C_m be the cycle given by $x_1x_2,\ldots,x_mx_1$ and let $y_1,y_2,\ldots,y_m$ be the pendent vertices adjacent to $x_1,x_2,\ldots,x_m$respectively.

Case1: m is even.

Subcase(a): $m=4n$.

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Define

$g(x_j)=2j-1, \quad 1 \leq j \leq 2n$

$g(x_{2n+j})= -2j+1, \quad 1 \leq j \leq 2n,$

$g(y_j)=4n+(2j-1), \quad 1 \leq j \leq 2n,$

$g(y_{2n+j})=-(4n+2j-1), \quad 1 \leq j \leq 2$

Subcase(b) $m=4n+2$.

Obviously g is a pair sum labeling.

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