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# On Prime labeling of $P_n$ (+) $N_m$, $< K_{1,n}^{(1)} : K_{1,n}^{(2)} >$, $< K_{1,n}^{(1)} : K_{1,n}^{(2)} : K_{1,n}^{(3)} >$, $P_n$ by attaching $C_3$, Alternate quadrilateral snake

**X. William [1] , N.Ramya [2]**

1. Mphil Scholar in mathematics Bharath Institute of Higher Education and Research, Selaiyur, Chennai-73

2. Associate Professor in Mathematics Bharath Institute of Higher Education and Research, Selaiyur, Chennai-73

**Abstract:** labeling is an assignment of integers, either to the vertices or edges, or both , subject to certain conditions. Prime labeling is , in an edge adjacent vertices are relatively prime to each other. In this paper we discussed prime labeling of $P_n$ (+) $N_m$ graph, star graphs, and Alternate quadrilateral snake by unique way.

**Keywords:** Prime graph, prime labeling, $< K_{1,n}^{(1)} : K_{1,n}^{(2)} > < K_{1,n}^{(1)} : K_{1,n}^{(2)} : K_{1,n}^{(3)} >$, Alternate quadrilateral snake

## Introduction:

Graph theory was presented by the French mathematician Euler in the year 1736, while managing the renowned issue of seven extensions known as Konigsberg connect issue. The issue is two islands; C and D, framed by the Pregel River in the city of Konigsberg were associated with one another and to the banks An and B with seven extensions. The issue was to begin at any of the four land regions of the city A, B, C or D stroll over every one of the seven scaffolds precisely once, and come back to the beginning stage. Euler spoke to this circumstance by methods for a diagram. The vertices speak to the land regions and the edges speak to the scaffolds. Euler demonstrated that an answer for this issue does not exist.[5]

### Preliminaries:

### Definition 1

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (edges) then the labeling is called a vertex labeling (an edge labeling).[3]

**Definition 2.**

Let G = G(V, E) be a finite simple and undirected graph with V vertices and E edges.

A bijection $f : V \to \{1, 2, \dots |V|\}$ is called a prime labeling if for each edge $e = \{u, v\} \in E$, and GCD{f(u), f(v)) = 1. A graph that admits a prime labeling is called a prime graph.[1,2]

**Definition 3**

An alternate quadrilateral snake $A(QS_n)$ is obtained from a path $u_1, u_2, \dots u_n$ by joining $u_i, u_{i+1}$ to new vertices $v_i, w_i$ respectively and then joining $v_i$ and $w_i$ [6]

**Definition 4**

In graph theory, a star $S_k$ is the complete bipartite graph $K_{1,k}$ : a tree with one internal node and k leaves (but no internal nodes and k + 1 leaves when $k \le 1$). Alternatively, some authors define $S_k$ to be the tree of order k with maximum diameter 2; in which case a star of $k > 2$ has $k - 1$ leaves.[4]

**Main results**

**Theorem 1**

The graph $P_n$ (+) $N_m$ is a prime graph.

**Proof.**

Let $V(P_n (+) N_m) = \{u_i, u_j; 1 \le i \le n, 1 \le j \le m\}$ and

$E(P_n (+) N_m) = \{v_i, v_{i+1}; 1 \le i \le n, e_j = v_1u_j, e_k = v_nu_j, 1 \le j \le m\}$.

We give the prime labeling for the vertices, can be classified in to three cases.

**Case i.** In $P_n$ (+) $N_m$, when m = n.

Let us define $f : V(G) \to \{1, 2, \dots, m+n\}$, as follows

f(v1) = 1

$f(v_n)$ = largest prime number

$f(u_i)$ = rest of the number which satisfy the prime labeling.

**Illustration:**

When m = n = 3, $P_3$ (+) $N_3$ can be labeled as

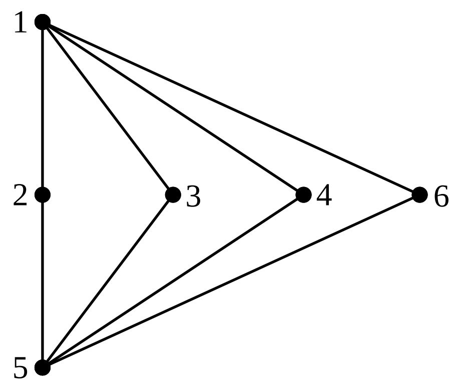


**Figure 1**

**Case ii.** In $P_n$ (+) $N_m$, when m > n, the prime labeling of ($P_n$ (+) $N_m$) has given by

$f(v_1) = 1$

$f(v_n)$ = largest prime number

$f(u_i)$ = rest of the number which satisfy prime labeling.

**Illustration:**

When m = 3, and n = 2, $P_2$ (+) $N_3$ can be labeled as

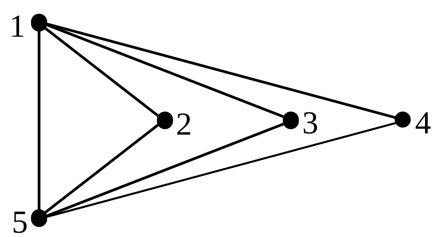


**Figure 2**

**Case iii.** In $P_n$ (+) $N_m$, when m < n, the prime labeling has given by

$f(v_1) = 1$

$f(v_2) = 2$

$f(v_3) = 3$

$f(v_n)$ = largest prime number

$f(u_i)$ = rest of the number has been labeled, which is not given in $f(v_i$'s).

**Illustration:**

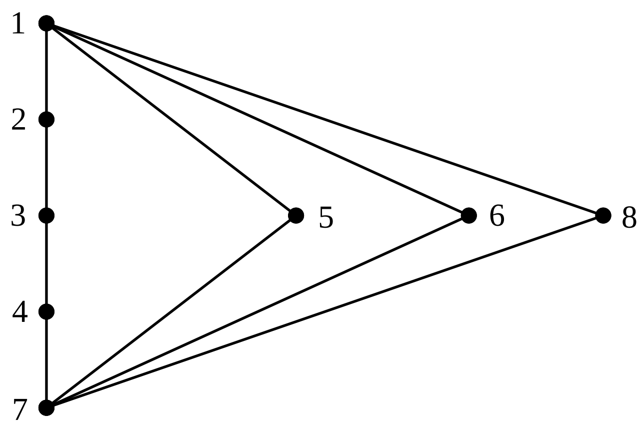


**Figure 3. $P_5$ (+) $N_3$**

**Theorem 2.**

Graph $< K_{1,n}^{(1)} : K_{1,n}^{(2)} >$ is prime.

**Proof.**

Let $u_1\ u_2 \ldots u_n$ be the pendant vertices of $K_{1,n}^{(1)}$ and $v_1, v_2 \ldots v_n$ be the pendant vertices of $K_{1,n}^{(2)}$. Let a and b are the apex vertices of $K_{1,n}^{(1)}$ and $K_{1,n}^{(2)}$ respectively and they are adjacent to a new common vertex c. Let us define $f : V(G) \rightarrow \{1, 2, \ldots 2n+3\}$, as follows,

$f(a) = 1$

$f(b)$ = largest prime number is $2n + 3$

$f(c)$ = either prime number or composite number

$f(u_i) = 1+i$, for all i

$f(v_i) = n+1+i$, for all i

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**Illustration:**

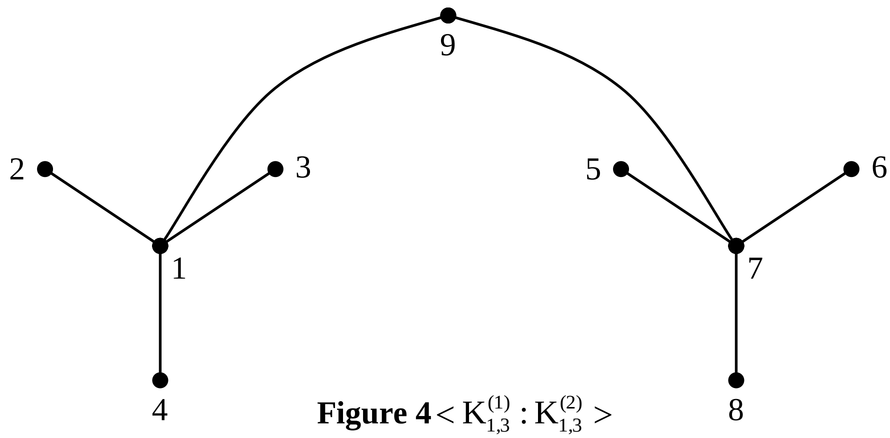


**Figure 4** $< K_{1,3}^{(1)} : K_{1,3}^{(2)} >$

**Theorem 3**

Graph $< K_{1,n}^{(1)} : K_{1,n}^{(2)} : K_{1,n}^{(3)} >$ is prime graph.

**Proof.**

Let $u_1$ $u_2$ … $u_n$ be the pendant vertices of $K_{1,n}^{(1)}$ $v_1$, $v_2$ … $v_n$ be the pendant vertices of $K_{1,n}^{(2)}$ and $w_1$ $w_2$ … $w_n$ be the pendant vertices of $K_{1,n}^{(3)}$. Let a, b, c are the apex vertices of $K_{1,n}^{(1)}$, $K_{1,n}^{(2)}$ and $K_{1,n}^{(3)}$ respectively and $K_{1,n}^{(1)}$, $K_{1,n}^{(2)}$ adjacent to new common vertex x. Similarly $K_{1,n}^{(2)}$, $K_{1,n}^{(3)}$ are adjacent to new common vertex y.

Let us define $f : V(G) \rightarrow \{1, 2, \ldots 3n+5\}$ as follows,

f(a) = 1

f(b) = any prime number

f(c) = largest prime number among the vertex set

$f(u_i)$ = 2, 3 … n

$f(v_i)$ = n+1 … n+k where k is last pendant vertex in $v_i$

$f(w_i)$ = n+k+1 … n+j where j is the last pendant vertex in $w_i$

f(x) = either composite or prime number

f(y) = either composite or prime number that should satisfy the prime labeling

  

**Example.**

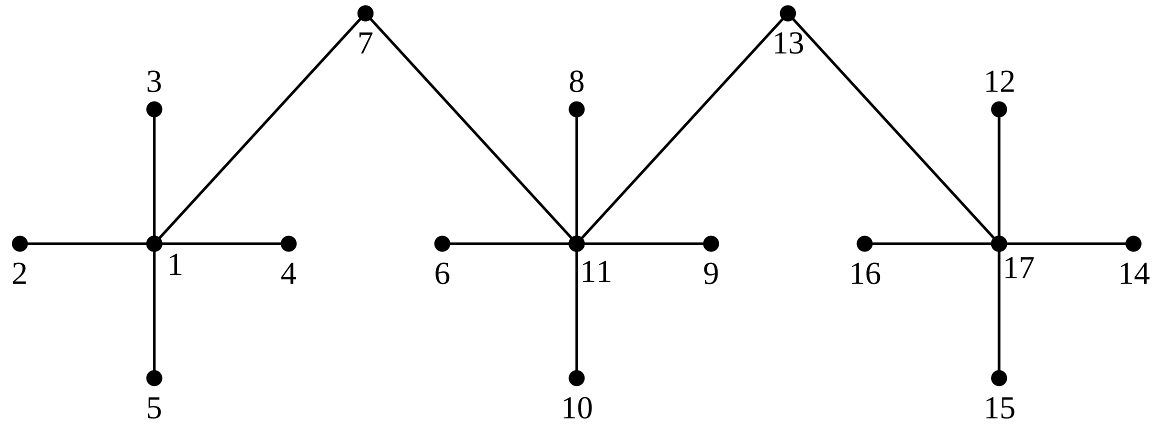


**Figure 5.** $< K_{1,4}^{(1)} : K_{1,4}^{(2)} : K_{1,4}^{(3)} >$

**Theorem 4**

Let $P_n$ be the path and G be the graph obtained from $P_n$ by attaching $C_3$ in both end edges of $P_n$. Then G is a prime graph.

**Proof.**

Let $P_n$ be the path $u_1 u_2 \dots u_n$ vertices.

Add two new vertices $v_1$ and $v_2$.

Joint $v_1u_1$, $v_1u_2$, $v_2u_{n-1}$ and $v_2u_n$

The resultant graph is G, with

$V(G) = \{u_1, u_2, \dots u_n, v_1, v_2\}$ and

$E(G) = \{u_iu_{i+1}, u_1v_1, u_2v_1, u_{n-1}v_2, u_2v_2 \,/\, 1 \leq i \leq n-1\}$

Then G has n+2 vertices and n+3 edges.

Let $f : V(G) \to \{1, 2, \dots n\}$ as follows.

$f(v_1) = 1$

$f(u_1) = 2$

$f(u_2) = 3$

$f(v_2) = 6$

$f(u_n) = 7$

$f(u_{n-1}) = 5$

$f(u_i) = i, 4 \leq i \leq n.$

which satisfy the prime labeling.

**Illustration.**

Prime labeling of G when n = 5

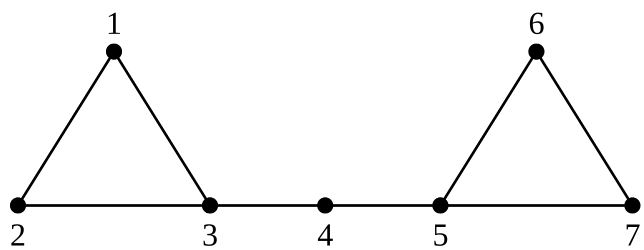


**Figure 6**

**Theorem 5**

Alternate quadrilateral snake $AQ_n$ is prime graph.

**Proof.**

Let G be the graph $AQ_n$, consider a path $u_1 u_2 \ldots u_n$. To construct G, join $u_i$, $u_{i+1}$ with two new vertices $v_i$, $w_i$, $1 \leq i \leq n-1$.

Let $f : V(G) \rightarrow \{1, 2, \ldots n\}$, as follows,

**Base Case i.** $AQ_1$

$f(u_1) = 1$

$f(u_2) = 2$

$f(u_3) = 5$

$f(u_4) = 6$

$f(v_1) = 3$

$f(w_1) = 4$

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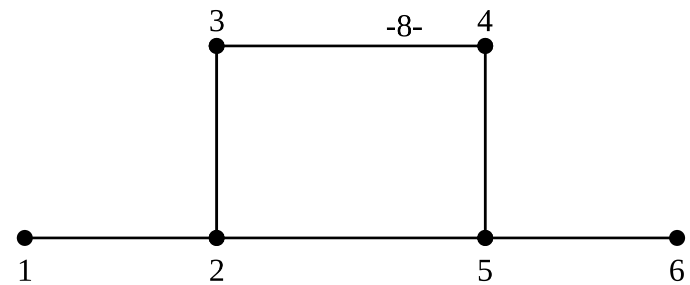


**Figure 7**

**Case i.** Prime labeling of $AQ_n$ when n = 2.

$f(u_1) = 1$

$f(u_2) = 2$

$f(u_3) = 5$

$f(u_4) = 8$

$f(u_5) = 7$

$f(u_6) = 6$

$f(v_1) = 3$

$f(v_2) = 9$

$f(w_1) = 4$

$f(w_2) = 10$.

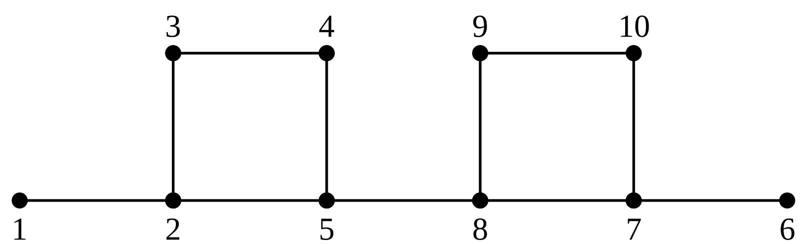


**Figure 8**

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**Case ii.** In addition to $AQ_2$, the prime labeling of $AQ_3$, is illustrated as

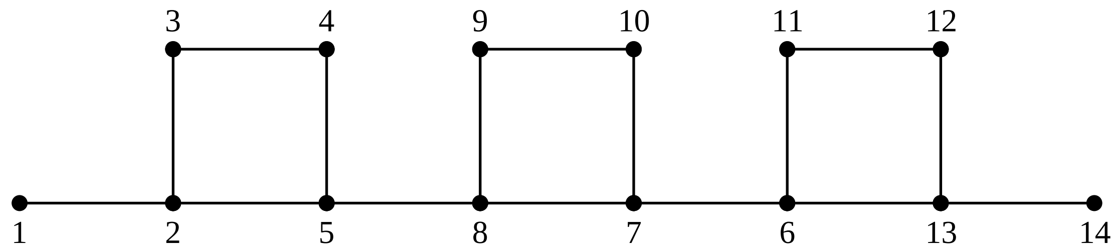


**Figure 9**

In addition to $AQ_3$, prime labeling of $AQ_4$, is given by,

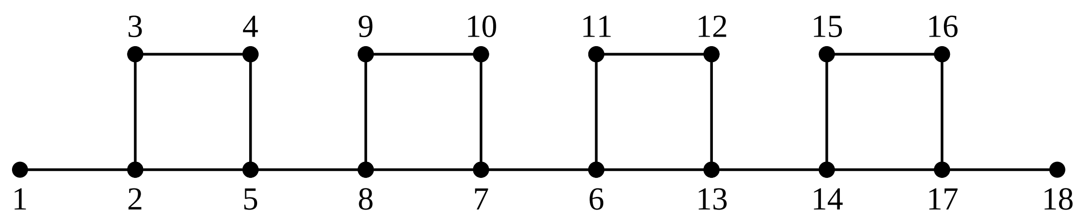


**Figure 10**

Hence the $AQ_n$ admits prime labeling.

## Conclusion:

In this paper discussed the Prime labeling of Pn+Nm, K1,n ⊙ K1,m, K1,n$^{(1)}$ ⊙ K1,n$^{(2)}$ ⊙ K1,n$^{(3)}$,and Alternate quadrilateral snake, It is of interest to consider these classes of directed graphs to find labeling.

It is of interest to look in certain kind of graphs where in total prime labeling is possible up to a large prime and verify this by programming concepts

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