Odds Generalized Lindley-Uniform distribution: Properties and Application

Dr. Sukanta Pramanik

Department of Statistics, Siliguri College, North Bengal University,

Siliguri-734001, West Bengal, India

email: skantapramanik@gmail.com

Abstract

Another Two parameter lifetime model is proposed for demonstrating lifetime information. An extensive record of the numerical properties of the new conveyance including assessment is introduced. An informational collection has been examined to outline its relevance.

Keywords: Lindley distribution; Uniform distribution; Maximum likelihood estimation; Odds function; Transformed-Transformer family of distributions.

1. Introduction

Displaying of any certifiable peculiarity gets confounded. Measurable circulations are significant for parametric deductions and furthermore are normally applied to portray certifiable peculiarity. Because of the convenience of measurable conveyances, their hypotheses are broadly contemplated and new appropriations are created. Various strategies have been created to produce measurable appropriations in the writing. A few strategies are created in the good 'ol days for producing univariate consistent dispersions incorporate techniques in view of differential conditions created by Pearson (1895), techniques for interpretation created by Johnson (1949), and strategies in light of quantile capabilities created by Tukey(1960). Toward the finish of 20th 100 years, McDonald (1984), Azzalini (1985), Marshall and Olkin (1997) proposed a few general techniques for producing another group of circulations. In twenty first 100 years, Eugene et al. (2002) proposed the beta-produced group of conveyances, Jones (2009) and Cordeiro and de Castro (2011) expanded the beta-created group of disseminations by involving Kumaraswamy appropriation instead of beta dispersion.

Alzaatreh et al. (2013) proposed a generalized family of distributions, called T-X (also called Transformed-Transformer) family, whose cumulative distribution function (cdf) is given by

$$F(x;\theta) = \int_a^{W[G(x)]} v(t)dt\,, \qquad (1.1)$$

where, the random variable $T \in [a,b]$, for $-\infty < a,b < \infty$ and $W[G(x)]$ be a function of the cdf $G(x)$ so that $W[G(x)]$ satisfies the following conditions:

(i) $W[G(x)] \in [a,b]$,

(ii) $W[G(x)]$ is differentiable and monotonically non-decreasing,

(iii) $W[G(x)] \to a$ as $x \to -\infty$ and $W[G(x)] \to b$ as $x \to \infty$.

ISSN PRINT 2319 1775 Online 2320 7876

Research paper © 2012 IJFANS. All Rights Reserved, UGC CARE Listed (Group -I) Journal Volume 11, Iss 12, Dec 2022

I have defined a generalized class of any distribution having positive support. Taking $W(F_\theta(x)) = \frac{F_\theta(x)}{1-F_\theta(x)}$, the odds function, the cdf of the proposed generalized class of distribution is given by

$$F(x|\lambda,\theta) = \int_0^{\frac{F_\theta(x)}{1-F_\theta(x)}} f_\lambda(t)dt. \tag{1.2}$$

In the present paper, we choose particular choice of $f_\lambda(t) = \frac{\lambda^2(1+t)}{1+\lambda}e^{-\lambda t}$ i.e. the Lindley distribution and $F_\theta(x) = \frac{x}{\theta}$ i.e. Uniform distribution in (1. 2). Hence, I call this distribution as Odds Generalized Lindley-Uniform distribution (OGLUD).

Formation of the new distribution

The c.d.f. of the distribution is given by the form as

$$F(x;\lambda,\theta) = \int_0^{\frac{x/\theta}{1-x/\theta}} \frac{\lambda^2(1+x)}{1+\lambda}e^{-\lambda x}dx = 1 - \frac{1+\lambda x}{(1+\lambda)(\theta-x)}e^{-\frac{\lambda x}{\theta-x}}$$

(2.3)

Also the p.d.f. of the distribution is given by

$$f(x;\lambda,\theta) = \frac{\lambda^2\theta^2}{(1+\lambda)(\theta-x)^3}e^{-\frac{\lambda x}{\theta-x}}$$

(2.4)

Statistical and Reliability Properties:-

Limit of the Probability Distribution Function

Since the c.d.f. of this distribution is $F(x) = 1 - \frac{1+\lambda x}{(1+\lambda)(\theta-x)}e^{-\frac{\lambda x}{\theta-x}}$

So $\lim_{x\to 0} F(x) = 0$ i.e. $F(0) = 0$

Now $\lim_{x\to\theta} F(x) = 1$ i.e. $F(\theta) = 1$

Descriptive Statistics of the Distribution

The mean of this distribution is as follows:

$$\mu_1' = E(X) = \int_0^\theta xf(x)dx = \frac{\theta}{(1+\lambda)}$$

The median of the distribution is calculated by the equation

$$1 - \frac{1+\lambda\left(\frac{M}{a}\right)^\theta}{(1+\lambda)}e^{-\lambda\left(\left(\frac{M}{a}\right)^\theta - 1\right)} = \frac{1}{2}$$

ISSN PRINT 2319 1775 Online 2320 7876

Research paper © 2012 IJFANS. All Rights Reserved, UGC CARE Listed (Group -I) Journal Volume 11, Iss 12, Dec 2022

The mode of the distribution is $a\left(\frac{2\theta-1}{\lambda\theta}\right)^{\frac{1}{\theta}}$

The r^{th} **order raw moment** of the distribution is as follows:

$$E(X^r)=\frac{\lambda^2\theta e^{\lambda}}{(1+\lambda)a^{2\theta}}\int_a^{\infty}x^{r+2\theta-1}e^{-\lambda\left(\frac{x}{a}\right)^{\theta}}dx=\frac{a^r e^{\lambda}}{(1+\lambda)\lambda^{\frac{r}{\theta}}}\Gamma\left(\frac{r}{\theta}+2,\lambda\right)$$

Now putting suitable values of r in the above equation, I get Variance, Skewness, Kurtosis and Coefficients of variation of the Odds Generalized Lindley- Uniform Distribution (OGLUD).

Moment Generating Function (MGF):

$$M_X(t)=E(e^{tX})=\sum_{r=0}^{\infty}\frac{t^r}{r!}\frac{a^r e^{\lambda}}{(1+\lambda)\lambda^{\frac{r}{\theta}}}\Gamma\left(\frac{r}{\theta}+2,\lambda\right) \tag{3.5}$$

Characteristic Function (CF):

$$\Psi_X(t)=E(e^{itX})=\sum_{r=0}^{\infty}\frac{(it)^r}{r!}\frac{a^r e^{\lambda}}{(1+\lambda)\lambda^{\frac{r}{\theta}}}\Gamma\left(\frac{r}{\theta}+2,\lambda\right) \tag{3.6}$$

Cumulant Generating Function (CGF):

$$K_X(t)=\ln_e M_X(t)=\ln_e\left[\sum_{r=0}^{\infty}\frac{t^r}{r!}\frac{a^r e^{\lambda}}{(1+\lambda)\lambda^{\frac{r}{\theta}}}\Gamma\left(\frac{r}{\theta}+2,\lambda\right)\right] \tag{3.7}$$

Mean Deviation:

The **mean deviation** about the **mean** and the mean deviation about the **median** is defined by

$$MD_{\mu}=\int_0^{\theta}|x-\mu|\,f(x)dx$$

and

$$MD_M=\int_0^{\theta}|x-M|\,f(x)dx$$

respectively, where $\mu=E(X)$ and $M=Median(X)$ denotes the mean and median rspectively.

Thus

$$MD_{\mu}=2\left[1-e^{-\frac{\lambda\mu}{\theta-\mu}}\right]-2\mu+2\lambda\theta\sum_{r=0}^{\infty}(-1)^r\frac{\Gamma(r+2,\frac{\lambda\mu}{\theta-\mu})}{\lambda^{r+2}} \tag{3.14}$$

and

ISSN PRINT 2319 1775 Online 2320 7876

Research paper © 2012 IJFANS. All Rights Reserved, UGC CARE Listed (Group -I) Journal Volume 11, Iss 12, Dec 2022

$$MD_M = -\mu + 2\lambda\theta \sum_{r=0}^{\infty} (-1)^r \frac{\Gamma(r+2, \frac{\lambda M}{\theta - M})}{\lambda^{r+2}} \qquad (3.15)$$

Conditional Moments:
The residual life and the reversed residual life play an important role in reliability theory and other branches of statistics. Here, the r-th order raw moment of the **residual life** is given by

$$\mu'_r(t) = E[(X-t)^r | X > t] = \frac{1}{\bar{F}(t)} \int_t^{\theta} (x-t)^r f(x) dx$$
$$= \frac{1}{e^{-\frac{\lambda t}{\theta - t}}} \sum_{j=0}^{r} \sum_{k=0}^{\infty} (-1)^{j+k} \binom{r}{j} \binom{r+k-j-1}{k} t^j \theta^{r-j} \frac{\Gamma(r+k-j+1, \frac{mbdat}{\theta - t})}{\lambda^{r+k-j}}$$

The r-th order raw moment of the **reversed residual life** is given by

$$m_r(t) = E[(t-X)^r | X < t] = \frac{1}{F(t)} \int_0^{t} (t-x)^r f(x) dx$$
$$= \frac{1}{1 - e^{-\frac{\lambda t}{\theta - t}}} \sum_{j=0}^{r} \sum_{k=0}^{\infty} (-1)^{j+k} \binom{r}{j} \binom{r+k-1}{k} \theta^j t^{r-j} \frac{\gamma(j+k+1, \frac{\lambda t}{\theta - t})}{lambda^{j+k}}$$

L- Moments:
Define $X_{k:n}$ be the k^{th} smallest moment in a sample of size n. The L-moments of X are defined by

$$\lambda_r = \frac{1}{r} \sum_{k=0}^{r-1} (-1)^k \binom{r-1}{k} E[X_{r-k:r}], \quad r = 1,2,..$$

Now for OGLUD with parameter λ and θ, we have

$$E[X_{j:r}] = \frac{r!}{(j-1)!(r-j)!} \int_0^{\infty} x [F(x)]^{j-1} [1-F(x)]^{r-j} dF(x)$$
$$= \frac{r!}{(j-1)!(r-j)!} \int_0^{\infty} x [1 - e^{-\lambda(e^{\theta x}-1)}]^{j-1} [e^{-\lambda(e^{\theta x}-1)}]^{r-j} \lambda\theta e^{\theta x} e^{-\lambda(e^{\theta x}-1)} dx$$
$$= \frac{r!}{(j-1)!(r-j)!} \lambda\theta \int_0^{\infty} x\, e^{\theta x} e^{-\lambda(r-j+1)(e^{\theta x}-1)} [1 - e^{-\lambda(e^{\theta x}-1)}]^{j-1} dx$$

So the first four L- Moments are,

$$\lambda_1 = E[X_{1:1}] = \theta \sum_{j=0}^{\infty} (-1)^j \frac{\Gamma(j+2)}{\lambda^{j+1}}$$

ISSN PRINT 2319 1775 Online 2320 7876

Research paper © 2012 IJFANS. All Rights Reserved, UGC CARE Listed (Group -I) Journal Volume 11, Iss 12, Dec 2022

$$\lambda_2 = \frac{1}{2}E[X_{2:2} - X_{1:2}] = \theta \sum_{j=0}^{\infty} (-1)^j \frac{\Gamma(j+2)}{\lambda^{j+1}} \left[1 - \frac{1}{2^{j+1}}\right]$$

$$\lambda_3 = \frac{1}{3}E[X_{3:3} - 2X_{2:3} + X_{1:3}]$$
$$= \theta \sum_{j=0}^{\infty} (-1)^j \frac{\Gamma(j+2)}{\lambda^{j+1}} \left[1 - \frac{3}{2^{j+1}} + \frac{2}{3^{j+1}}\right]$$

$$\lambda_4 = \frac{1}{4}E[X_{4:4} - 3X_{3:4} + 3X_{2:4} - X_{1:4}]$$
$$= \theta \sum_{j=0}^{\infty} (-1)^j \frac{\Gamma(j+2)}{\lambda^{j+1}} \left[1 - \frac{6}{2^{j+1}} + \frac{10}{3^{j+1}} - \frac{5}{4^{j+1}}\right]$$

Quantile function:

Let X denote a random variable with the probability density function 2.2. The quantile function, say $Q(p)$, defined by $F(Q(p)) = p$ is the root of the equation

$$1 - e^{-\frac{\lambda Q(p)}{\theta - Q(p)}} = p$$

$$\Rightarrow e^{-\frac{\lambda Q(p)}{\theta - Q(p)}} = 1 - p$$
$$\Rightarrow -\frac{\lambda Q(p)}{\theta - Q(p)} = ln(1-p)$$
$$\Rightarrow \frac{Q(p)-\theta}{\lambda Q(p)} = \frac{1}{ln(1-p)}$$
$$\Rightarrow \frac{1}{\lambda} - \frac{\theta}{\lambda Q(p)} = \frac{1}{ln(1-p)}$$
$$\Rightarrow \frac{\theta}{\lambda Q(p)} = \frac{1}{\lambda} - \frac{1}{ln(1-p)}$$

Order Statistics

Suppose $X_1, X_2, X_3, \ldots\ldots, X_n$ is a random sample from Eq. Let $X_{(1)}, X_{(2)}, X_{(3)}, \ldots\ldots, X_{(n)}$, denote the corresponding order statistics. It is well known that the probability density function and the cumulative distribution function of the k^{th} order statistic, say $Y = X_{(k)}$, are given by

$$f_Y(y) = \frac{n!}{(k-1)!\,(n-k)!} F^{k-1}(y)[1 - F(y)]^{n-k} f(y)$$
$$= \frac{n!}{(k-1)!\,(n-k)!} \left[1 - e^{-\frac{\lambda y}{0a - y}}\right]^{k-1} \left[e^{-\frac{\lambda y}{\theta - y}}\right]^{n-k} \frac{\lambda\theta}{(\theta - y)^2} e^{-\frac{\lambda y}{\theta - y}}$$
$$= \frac{n!}{(k-1)!\,(n-k)!} \frac{\lambda\theta}{(\theta - y)^2} e^{-\frac{\lambda(n-k+1)y}{\theta - y}} \left[1 - e^{-\frac{\lambda y}{\theta - y}}\right]^{k-1} \tag{1.14}$$

and

$$F_Y(y) = \sum_{j=k}^{n} \binom{n}{j} F^j(y)[1 - F(y)]^{n-j}$$

ISSN PRINT 2319 1775 Online 2320 7876
Research paper © 2012 IJFANS. All Rights Reserved, UGC CARE Listed (Group -I) Journal Volume 11, Iss 12, Dec 2022

$$= \sum_{j=k}^{n} \binom{n}{j} e^{-\frac{\lambda(n-j)y}{\theta-y}} \left[1 - e^{-\frac{\lambda y}{\theta-y}}\right]^{j} \tag{1.15}$$

Entropies

An entropy of a random variable X is a measure of variation of the uncertainty. A popular entropy measure is **Renyi entropy** (Renyi 1961). If X has the probability density function f(x), then Renyi entropy is defined by

$$H_R(\gamma) = \frac{1}{1-\gamma} log\left\{\int_0^{\theta} f^{\gamma}(x)dx\right\} \tag{1.16}$$

where $\gamma > 0$ and $\gamma \neq 1$. Suppose X has the probability density function Eq. . Then, one can calculate

$$\int_0^{\theta} f^{\gamma}(x)dx = \int_0^{\theta} \frac{\lambda^{\gamma}\theta^{\gamma}}{(\theta-x)^{2\gamma}} e^{-\frac{\lambda\gamma x}{(\theta-x)}} dx$$

Put $u = \frac{x}{(\theta-x)} \Rightarrow du = \frac{\theta}{(\theta-x)^2}dx$, with $x = 0 \Rightarrow u = 0$ and $x = \theta \Rightarrow u = \infty$ So

$$\int_0^{\theta} f^{\gamma}(x)dx = \lambda^{\gamma}\theta^{1-\gamma}\int_0^{\infty}(1+u)^{-2(1-\gamma)}e^{-\lambda\gamma u}du$$

$$= \lambda^{\gamma}\theta^{1-\gamma}\sum_{j=0}^{\infty}(-1)^j\binom{j+2(1-\gamma)-1}{j}u^j e^{-\lambda\gamma u}du$$

$$= \lambda^{\gamma}\theta^{1-\gamma}\sum_{j=0}^{\infty}(-1)^j\binom{j-2\gamma+1}{j}\frac{\Gamma(j+1)}{(\lambda\gamma)^{j+1}}$$

So **Renyi entropy** is

$$H_R(\gamma) = \frac{1}{1-\gamma} log\left\{\lambda^{\gamma}\theta^{1-\gamma}\sum_{j=0}^{\infty}(-1)^j\binom{j-2\gamma+1}{j}\frac{\Gamma(j+1)}{(\lambda\gamma)^{j+1}}\right\}$$

$$= \frac{mma}{1-\gamma}log\lambda - log\theta + \frac{1}{1-\gamma}log\sum_{j=0}^{\infty}(-1)^j\binom{j-2\gamma+1}{j}\frac{\Gamma(j+1)}{(\lambda\gamma)^{j+1}} \tag{1.17}$$

Shannon measure of entropy is defined as

$$H(f) = E[-logf(x)] = -\int_0^{\theta} f(x)logf(x)dx$$

$$= -log\lambda\int_0^{\theta} f(x)dx - log\theta\int_0^{\theta} f(x)dx + 2\int_0^{\theta} log(\theta-x)f(x)dx + \int_0^{\theta}\frac{\lambda x}{(\theta-x)}f(x)dx$$

ISSN PRINT 2319 1775 Online 2320 7876
Research paper © 2012 IJFANS. All Rights Reserved, UGC CARE Listed (Group -I) Journal Volume 11, Iss 12, Dec 2022

$$= -logmbda - log\theta + 2\lambda\theta\int_0^\theta \frac{log(\theta-x)}{(\theta-x)^2}e^{-\frac{\lambda x}{\theta-x}}dx$$

$$+ \lambda^2\theta\int_0^\theta \frac{x}{(\theta-x)^3}e^{-\frac{\lambda x}{\theta-x}}dx$$

$$= -log\lambda\theta + 2\lambda\theta\int_0^\theta \frac{log(\theta-x)}{(\theta-x)^2}e^{-\frac{\lambda x}{\theta-x}}dx$$

$$+ \lambda^2\theta\int_0^\theta \frac{x}{(\theta-x)^3}e^{-\frac{\lambda x}{\theta-x}}dx$$

Put $u = \frac{x}{(\theta-x)} \Rightarrow du = \frac{\theta}{(\theta-x)^2}dx$, with $x = 0 \Rightarrow u = 0$ and $x = \theta \Rightarrow u = \infty$

So

$$H(f) = -log\lambda\theta + 2\int_0^\infty \lambda\, log(\frac{\theta}{1+u})e^{-\lambda u}du + \int_0^\infty \lambda^2\, ue^{-\lambda u}du$$

$$= -log\lambda\theta + 2\lambda log\theta\int_0^\infty e^{-\lambda u}\,du - 2da\int_0^\infty l\,og(1+u)e^{-\lambda u}du + \lambda^2\frac{\Gamma(2)}{\lambda^2}$$

$$= 1 - log\lambda\theta + 2log\theta - 2\lambda\int_0^\infty\sum_{r=0}^{\infty}(-1)^r\frac{u^{r+1}}{r+1}e^{-\lambda u}du$$

$$= 1 + log\frac{\theta}{\lambda} - 2\lambda\sum_{r=0}^{\infty}(-1)^r\frac{1}{r+1}\int_0^\infty u^{r+1}\,e^{-\lambda u}du$$

$$= 1 + log\frac{\theta}{\lambda} - 2\lambda\sum_{r=0}^{\infty}(-1)^r\frac{1}{r+1}\frac{\Gamma(r+2)}{\lambda^{r+2}}$$

$$= 1 + log\frac{\theta}{\lambda} - 2\sum_{r=0}^{\infty}(-1)^r\frac{\Gamma(r+1)}{\lambda^{r+1}} \quad (1.18)$$

Reliability and related properties

The **Reliability** function of OGLUD is given by the form as:

$$R(x) = 1 - F(x) = e^{-\frac{\lambda x}{\theta-x}} \quad (1.19)$$

and the **Hazard rate** of OGLUD is given by the form as:

$$r(t) = \frac{f(t)}{1-F(t)}$$

$$= \frac{\frac{\lambda\theta}{(\theta-t)^2}e^{-\frac{\lambda t}{\theta-t}}}{e^{-\frac{\lambda t}{\theta-t}}}$$

$$= c\lambda\theta(\theta-t)^2 \quad (1.20)$$

Now $t < \theta$, so $r(t)$ is an increasing function of t. Thus $r(t)$ is **IFR**(Increasing Failure Rate)

Now $\frac{d^2}{dx^2}lnf(x) = \frac{2[\theta(1-\lambda)-x]}{(\theta-x)^3}$

So, $\frac{d^2}{dx^2}lnf(x) > 0\ if\ \lambda < 1\ and\ < 0\ if\ \lambda > 1$

Thus the distribution is **log-convex** if $\lambda < 1$ and **log-concave** if $\lambda > 1$.

Mean Residual Life(MRL) function is defined as

ISSN PRINT 2319 1775 Online 2320 7876
Research paper © 2012 IJFANS. All Rights Reserved, UGC CARE Listed (Group -I) Journal Volume 11, Iss 12, Dec 2022

$$e_x(t) = \frac{1}{\bar{F}(t)}\int_t^\theta \bar{F}\,(x)dx = \frac{1}{e^{-\frac{\lambda t}{(\theta-t)}}}\int_t^\theta e^{-\frac{\lambda x}{(\theta-x)}}\,dx$$

Reversed Hazard rate:

$$\mu_F(x) = \frac{f(x)}{F(x)}$$
$$= \frac{\frac{\lambda\theta}{(\theta-x)^2}e^{-\frac{\lambda x}{\theta-x}}}{1-e^{-\frac{\lambda x}{\theta-x}}}$$
$$= \frac{\lambda\theta}{(\theta-x)^2\left[e^{\frac{\lambda x}{\theta-x}}-1\right]} \quad (0.20)$$

Expected Inactivity Time (EIT) or Mean Reversed Residual Life (MRRL) function is defined as:

$$\bar{e}_x(t) = E(t-X|X<t)$$
$$= t\left[1-e^{\frac{-\lambda t}{\theta-t}}\right] - \theta\sum_{r=0}^{\infty}(-1)^r\frac{\gamma(r+2,\lambda)}{\lambda^{r+1}} \quad (0.21)$$

Stress-Strength reliability

he Stress-Strength model describes the life of a component which has a random strength X that is subjected to a random stress Y. The component fails at the instant that the stress applied to it exceeds the strength, and the component will function satisfactorily whenever $X > Y$. So, Stress-Strength Reliability is $R = P(Y < X)$.
Let $X \sim OGEUD(\lambda_1,\theta_1)$ and $Y \sim OGEUD(\lambda_2,\theta_2)$ be independent random variables. Then Stress-Strength Reliability

$$R = P(Y < X)$$
$$= \int_0^{\theta_1} G_y\,(x)f(x)dx$$
$$= \int_0^{\theta_1}\left[1-e^{-\frac{\lambda_2 x}{\theta_2-x}}\right]\frac{\lambda_1\theta_1}{(\theta_1-x)^2}e^{-\frac{\lambda_1 x}{\theta_1-x}}dxber$$
$$= 1-\lambda_1\theta_1\int_0^{\theta_1}\frac{1}{(\theta_1-x)^2}e^{-(\frac{\lambda_1 x}{\theta_1-x}+\frac{\lambda_2 x}{\theta_2-x})}dx$$

If $\theta_1 = \theta_2 = \theta$, then

$$R = 1-\lambda_1\theta\int_0^{\theta}\frac{1}{(\theta-x)^2}e^{-\frac{(\lambda_1+\lambda_2)x}{\theta-x}}dx$$
$$= 1-\frac{\lambda_1}{\lambda_1+\lambda_2}$$
$$= \frac{\lambda_2}{\lambda_1+\lambda_2}$$

Estimation of the Parameters

Using the **MLE**, we estimate the parameter of OGLUD.
Since

ISSN PRINT 2319 1775 Online 2320 7876

Research paper © 2012 IJFANS. All Rights Reserved, **UGC CARE Listed (Group -I) Journal Volume 11, Iss 12, Dec 2022**

$$f_{e.u}(x;\lambda,\theta)=\frac{\lambda\theta}{(\theta-x)^2}e^{-\frac{\lambda x}{\theta-x}}$$

The Likelihood function is given by:

$$L(x;\lambda,\theta)=\prod_{i=1}^{n}f\,(x_i)$$

$$=\prod_{i=1}^{n}\frac{\lambda\theta}{(\theta-x_i)^2}e^{-\frac{\lambda x_i}{\theta-x_i}}$$

$$=\frac{\lambda^n\theta^n}{\prod_{i=1}^{n}(\theta-x_i)^2}e^{-\sum_{i=1}^{n}\frac{\lambda x_i}{\theta-x_i}} \qquad (1.1)$$

Then the natural logarithm of likelihood is

$$lnL(x;\lambda,\theta)=nln\lambda+nln\theta-2\sum_{i=1}^{n}l\,n(\theta-x_i)-\lambda\sum_{i=1}^{n}\frac{x_i}{(\theta-x_i)}$$

The MLEs of θ and λ are the roots of

$$\frac{\partial lnL(x;\lambda,\theta)}{\partial\theta}=0 and\frac{\partial lnL(x;\lambda,\theta)}{\partial\lambda}=0$$

Now the method of differentiation clearly fails for first equation. We can get the MLE of θ directly from the Likelihood function. Our object is to choose θ and λ so that L is maximum. From equation no. , it is obvious that whatever $\lambda(>0)$, L would be maximum when θ takes the largest possible value. Let $x_{(n)}$ denote the value of the n^{th} order statistic in the given sample of size n.

Thus $\hat{\theta}=x_{(n)}$.

Now

$$\frac{\partial lnL(x;\lambda,\theta)}{\partial\lambda}=\frac{n}{\lambda}-\sum_{i=1}^{n}\frac{x_i}{(\hat{\theta}-x_i)}=0$$

$$\Rightarrow\hat{\lambda}=\frac{n}{\sum_{i=1}^{n}\frac{x_i}{(x_{(n)}-x_i)}} \qquad (1.2)$$

Data Analysis

In this section, we fit the odds generalized Lindley uniform model to a real data set obtained from Lee and Wang [10]. The data set corresponding to remission times (in months) of a random sample of 128 bladder cancer patients as presented in Table 1. We also fit Lindley-Lindley distribution[16], Weibull Distribution and Power-Lindley distribution[6].

(i) Lindley-Lindley distribution (LED(λ,θ)):

$$f(x)=\frac{\lambda\theta^2e^{-\lambda x}(1-e^{-\lambda x})^{\theta-1}(1-log(1-e^{-\lambda x}))}{1+\theta},\qquad x,\theta,\lambda>0$$

(ii) Weibull distribution (WD(λ,θ)):

$$f(x)=\frac{\theta}{\lambda^\theta}x^{\theta-1}e^{-(\frac{x}{\lambda})^\theta},\qquad x,\theta,\lambda>0$$

(iii) Power-Lindley distribution (PLD(λ,θ)):

$$f(x)=\frac{\lambda^2\theta}{1+\lambda}(1+x^\theta)x^{\theta-1}e^{-\lambda x^\theta},\qquad x,\theta,\lambda>0$$

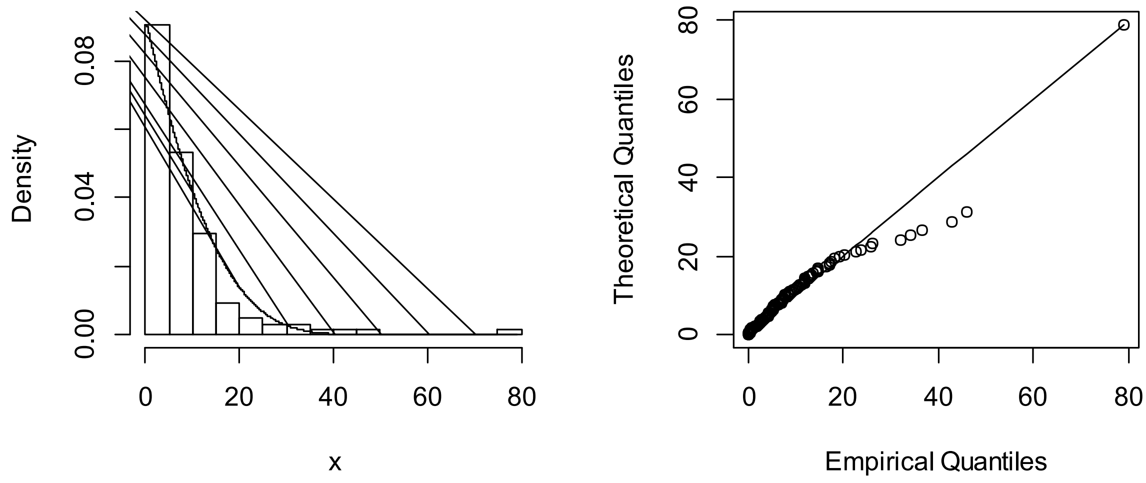

Figure 1. Plots of the fitted pdf and estimated quantiles versus observed quantiles of the OGLUD

Concluding Remark

In this article, we have studied a new probability distribution called Odds Generalized Lindley-Uniform Distribution. This is a particular case of T-X family of distributions proposed by Alzaatreh et al. (2013). The structural and reliability properties of this distribution have been studied and inference on parameters has also been mentioned. The advantage is that the distribution has only two parameters that are to be estimated. The appropriateness of fitting the Odds Generalized Lindley-Uniform distribution has been established by analyzing a real life data set.

References

1. Adamidis K., and Loukas S.,(1998) A lifetime distribution with decreasing failure rate, *Statistics and Probability Letters*, Vol(39), 35-42.
2. Arnold B.C., Balakrishnan N. and Nagaraja H.N.(2013): A First Course in Order Statistics, *Wiley*, New York, 1992.
3. Bakouch H. S., Al-Zahrani B. M., Al-Shomrani A. A., Marchi V. A., and Louzada F.,(2012) An extended Lindley distribution, *Journal of the Korean Statistical Society*, Vol(41), 75-85.
4. Elbatal I., Merovci F., and Elgarhy M.(2013): A new generalized Lindley distribution, *Math- ematical Theory and Modeling*, Vol(3) no. 13.
5. Ghitany M. E., Alqallaf F., Al-Mutairi D. K., and Husain H. A., (2011) A two-parameter weighted Lindley distribution and its applications to survival data, *Mathematics and Com- puters in Simulation*, Vol. (81), no. 6,1190-1201.
6. Ghitany M. E., Al-Mutairi D. K., and Aboukhamseen S. M. , (2013) Estimation of the reliability of a stress-strength system from power Lindley distributions, *Communications in Statistics - Simulation and Computation*, Vol (78), 493-506.
7. Ghitany M. E., Atieh B., and Nadarajah, S., (2008) Lindley distribution and its application, *Mathematics and Computers in Simulation*, Vol (78), 493-506.
8. Hassan M.K.,(2014), On the Convolution of Lindley Distribution, *Columbia International*

ISSN PRINT 2319 1775 Online 2320 7876

Research paper © 2012 IJFANS. All Rights Reserved, UGC CARE Listed (Group -I) Journal Volume 11, Iss 12, Dec 2022

Publishing Contemporary Mathematics and Statistics, Vol. (2) No. 1,47-54.

9. Jor ´ da P.,(2010) Computer generation of random variables with Lindley or Poisson-Lindley distribution via the Lambert W function, *Mathematics and Computers in Simulation*, Vol(81), 851-859.
10. Lee E.T., and Wang J.W.,(2003) Statistical methods for survival data analysis, *John Wiley and Sons*, inc., Hoboken, New Jersey, 3 edition.
11. Lindley D. V.,(1958) Fiducial distributions and Bayes theorem, *Journal of the Royal Statistical Society, Series* B (Methodological),102-107.
12. Mahmoudi E., and Zakerzadeh H., (2010) Generalized Poisson Lindley distribution, *Communications in Statistics*: Theory and Methods, Vol (39), 1785-1798.
13. Miroslav M. Risti and Narayanaswamy Balakrishnan (2012): The gammaexponentiated Lindley distribution, *Journal of Statistical Computation and Simulation*, 82:8, 1191-1206, DOI: 10.1080/00949655.2011.574633
14. Shanker R., Sharma S., and Shanker R., (2013) A Two-Parameter Lindley Distribution for Modeling Waiting and Survival Times Data, *Applied Mathematics*, Vol (4), 363-368.
15. Zakerzadeh, H. and Mahmoudi, E.,(2012) A new two parameter lifetime distribution: model and properties. arXiv:1204.4248 v1 [stat.CO].
16. Bhati, D. and Malik, A, Md., (2014) On Lindley-Lindley Distribution: Properties and Application