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COMMON FIXED POINT THEOREM FOR COMPATIBLE MAPS OF TYPE (β) IN INTUITIONISTIC GENERALIZED FUZZY 2 METRIC SPACES

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ABSTRACT:

In this paper, we prove a common fixed point theorem for compatible map of type (β) in intuitionistic generalized fuzzy 2 metric spaces.

Keywords: Intuitionistic Generalized Fuzzy 2 Metric Spaces, G- Cauchy sequences, Weakly compatible, Point of coincidence, Complete metric space.

Mathematics subject classification: 54H25, 47H10.

1**Introduction:**In 1998, Vasuki [10] established a generalization of Grabiec's fuzzy contraction theorem wherein he proved a common fixed point theorem for a sequence of mappings in a fuzzy metric space. Thereafter, Cho [2] extended the concept of compatible mapping of type (alpha) to fuzzy metric spaces. Several years later, Singh and Chauhan introduced the concept of compatible mappings and proved two common fixed point theorems in the fuzzy metric space with the

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minimum triangular norm. In 2002, Sharma [7] further extended some known results of fixed point theory for compatible mappings in fuzzy metric spaces. At the same time, Gregori and Sapena [4] introduced the notion of fuzzy contractive mapping and proved fixed point theorems in varied classes of complete fuzzy metric spaces in the senses of George and Veeramani [5], Kramosil and Michalek [9] and Grabiec's [3]. In 1986, Atanassov [1] introduced the notion of an intuitionistic fuzzy metric space. Afterward, Park gave the notion of an intuitionistic fuzzy metric space and generalized the notion of fuzzy metric space due to George and Veeramani. Our objective of this paper is to prove a common point theorem by removing the assumption of continuity, relaxing compatibility to compatible maps of type (β) weakly compatibility and replacing the completeness of the space with a set of alternative conditions for functions satisfying an implicit it relation in IFM – space.

2Preliminaries

Definition 2.1: Let I = [0,1], * be a continuous t – norm and be continuous t – co norm and F,G be the set of all real continuous function F,G : $I^6 \to R$ satisfying the following conditions:

(2.1) F is non increasing in the fifth and sixth variables and G is non decreasing in the fifth and sixth variables,

(2.2) If, for some constant k ∈ (0,1) we have

(a) $F(u(kt),v(t),v(t),u(t),1,u(\frac{t}{2}) * v(\frac{t}{2})) \geq 1$ and

$G(u(kt),v(t),v(t),u(t),0,u(\frac{t}{2}) \Diamond v(\frac{t}{2})) \leq 1$ or

(b) $F(u(kt),v(t),u(t),v(t),u(\frac{t}{2}) * v(\frac{t}{2})) \geq 1$ and

$G(u(kt),v(t),u(t),v(t),u(\frac{t}{2}) \Diamond v(\frac{t}{2})) \leq 1$

For any fixed t > 0 and non decreasing function u, v : $(0, \infty) \to I$ with $0 \leq u(t), v(t) \leq 1$ then there exists h ∈ (0, 1) with $u(ht) \geq v(t) * u(t)$ and $u(ht) \geq (1 - v(t)) \Diamond (1 - u(t))$,

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(F-3) if, for some constant $k \in (0, 1)$ we have

$F(u(kt),u(t),1,1,u(t),u(t)) \geq 1$ and $G(u(kt),u(t),0,0,u(t),u(t)) \leq 1$

For any fixed $t > 0$ and non decreasing function $u: (0, \infty) \rightarrow I$ then $u(kt) \geq u(t)$.

Definition 2.2:A 5-tuple (X, M, N, *, ◊) is said to be an intuitionistic generalized fuzzy 2 metric space if X is an arbitrary set, * is a continuous t-norm, ◊ is continuous t-co norm and M, N are fuzzy sets on $X^3 \times [0, \infty)$ satisfying the following conditions:

(i) $M(x, y, z, \theta, t) + N(x, y, z, \theta, t) \leq 1$ for all $x, y, z \in X$ and $s, t > 0$;

(ii) $M(x, y, z, \theta, t) > 0$ for all $x, y, z \in X$;

(iii) $M(x, y, z, \theta, t) = 1$ for all $x, y, z \in X$ and $t > 0$ if and only if $x = y = z$;

(iv) $M(x, y, z, \theta, t) = M(p\{x, y, z\}, \theta, t) \leq 1$ for all $x, y, z \in X$ and $t > 0$;

(v) $M (x, y, u, \theta, t_1) * M (x, u, z, \theta, t_2) * M (u, y, z, \theta, t_3) \leq M (x, y, z, \theta, t_1 + t_2 + t_3)$ for all $x, y, z \in X$ and $s, t > 0$;

(vi) For all $x, y, z \in X$, $M (x, y, z, \theta, .) : (0, \infty) \rightarrow (0, 1]$ is continuous;

(vii) $\lim_{n \rightarrow \infty} M(x, y, z, \theta, t) = 1$

(viii) $N (x, y, z, \theta, t) < 0$ for all $x, y, z \in X$;

(ix) $N (x, y, z, \theta, t) = 0$ for all $x, y, z \in X$ and $t > 0$ if and only if $x = y = z$;

(x) $N (x, y, z, \theta, t) = N (p\{x, y, z\}, \theta, t) \leq 1$ for all $x, y, z \in X$ and $t > 0$;

(xi) $N (x, y, u, \theta, t_1) \lozenge N (x, u, z, \theta, t_2) \lozenge N (u, y, z, \theta, t_3) \geq N (x, y, z, \theta, t_1 + t_2 + t_3)$ for all $x, y, z \in X$ and $s, t > 0$;

(xii) For all $x, y, z \in X$, $N (x, y, z, \theta, .) : (0, \infty) \rightarrow (0, 1]$ is continuous;

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(vii) $\lim_{n \to \infty} N(x, y, z, \theta, t) = 0$

The M and N is called a intuitionistic generalized fuzzy 2 metric space on X. The function M (x, y, z, θ, t) and N (x, y, z, θ, t) denote the degree of nearness between x, y, z and θ with respect to t.

Definition 2.3.A sequence $\{x_n\}$ in a intuitionistic generalized fuzzy 2 metric space (X, M, N, *, ◊) is said to be a converges to x iff for ach $\varepsilon > 0$ and each t > 0, $n_o \in$ N such that M (x_n, x, x, θ, t) > 1 – ε and N (x_n, x, x, θ, t) <ε for all t > n_0 .

Definition 2.4.A sequence $\{x_n\}$ in a intuitionistic generalized fuzzy 2 metric space (X, M, N, *, ◊) is said to be a G- Cauchy sequences converges to x iff for ach $\varepsilon > 0$ and each t > 0, $n_o \in$ N such that M (x_m, x_n, x_n, θ, t) > 1 – ε and N (x_m, x_n, x_n, θ, t) <ε for all n ≥ n_0 .

A intuitionistic generalized fuzzy 2 metric space (X, M, N, *, ◊) is said to be a complete if every G - Cauchy sequences in it converges to a point in it.

3Main Results

Theorem 3.1: Let (X, M, N, *, ◊) be a complete intuitionistic generalized fuzzy 2 metric space and let A,B,S,T,P and Q be mapping from X into itself such that the following conditions are satisfied:

(3.1.1) P(X) ⊂ST(X), Q(X) ⊂ AB(X),

(3.1.2) (P, AB) is compatible of type (β) and (Q, ST) is weakly compatible,

(3.1.3) there exists k ∈ (0 ,1) such that for every x, y ∈ X, t > 0

$$F\begin{pmatrix} M^2(Px, Qy, Qy, \theta, kt), M^2(ABx, Sty, Sty, \theta, t), M^2(Px, ABx, ABx, \theta, t), \\ M^2(Qy, STy, STy, \theta, t), M^2(Px, STy, STy, \theta, t), M^2(ABx, Qy, Qy, \theta, t) \end{pmatrix} > 1$$

$$G\begin{pmatrix} N^2(Px, Qy, Qy, \theta, kt), N^2(ABx, Sty, Sty, \theta, t), N^2(Px, ABx, ABx, \theta, t), \\ N^2(Qy, STy, STy, \theta, t), N^2(Px, STy, STy, \theta, t), N^2(ABx, Qy, Qy, \theta, t) \end{pmatrix} < 1$$

Then A, B, S, T, P and Q have a unique common fixed point in X.

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Proof: Let $x_0 \in X$, then from 3.1 (a) we have $x_1, x_2 \in X$ such that $Px_0 = STx_1$ and $Qx_1 = ABx_2$

Inductively, we construct sequences $\{x_n\}$ and $\{y_n\}$ in X such that for $n \in N$

$Px_{2n-2} = STx_{2n-1} = Y_{2n-1}$ and $Qx_{2n-1} = ABx_{2n} = Y_{2n}$

Put $x = x_{2n}$ and $y = x_{2n+1}$ in (3.1.3) then we have

$$F\begin{pmatrix} M^2(Px_{2n}, Qx_{2n+1}, Qx_{2n+1}, \theta, kt), M^2(ABx_{2n}, STx_{2n+1}, STx_{2n+1}, \theta, t) \\ M^2(Px_{2n}, ABx_{2n}, ABx_{2n}, \theta, t), M^2(Qx_{2n+1}, STx_{2n+1}, STx_{2n+1}, \theta, t) \\ M^2(Px_{2n}, STx_{2n+1}, STx_{2n+1}, \theta, t), M^2(ABx_{2n}, Qx_{2n+1}, Qx_{2n+1}, \theta, t) \end{pmatrix} > 1$$

$$G\begin{pmatrix} N^2(Px_{2n}, Qx_{2n+1}, Qx_{2n+1}, \theta, kt), N^2(ABx_{2n}, STx_{2n+1}, STx_{2n+1}, \theta, t) \\ N^2(Px_{2n}, ABx_{2n}, ABx_{2n}, \theta, t), N^2(Qx_{2n+1}, STx_{2n+1}, STx_{2n+1}, \theta, t) \\ N^2(Px_{2n}, STx_{2n+1}, STx_{2n+1}, \theta, t), N^2(ABx_{2n}, Qx_{2n+1}, Qx_{2n+1}, \theta, t) \end{pmatrix} < 1$$

$$F\begin{pmatrix} M^2(y_{2n+1}, y_{2n+2}, y_{2n+2}, \theta, kt), M^2(y_{2n}, y_{2n+1}, y_{2n+1}, \theta, t) \\ M^2(y_{2n+1}, y_{2n}, y_{2n}, \theta, t), M^2(y_{2n+2}, y_{2n+1}, y_{2n+1}, \theta, t) \\ M^2(y_{2n+1}, y_{2n+1}, y_{2n+1}, \theta, t), M^2(y_{2n}, y_{2n+2}, y_{2n+2}, \theta, t) \end{pmatrix} > 1 \text{ and}$$

$$G\begin{pmatrix} N^2(y_{2n+1}, y_{2n+2}, y_{2n+2}, \theta, kt), N^2(y_{2n}, y_{2n+1}, y_{2n+1}, \theta, t) \\ N^2(y_{2n+1}, y_{2n}, y_{2n}, \theta, t), N^2(y_{2n+2}, y_{2n+1}, y_{2n+1}, \theta, t) \\ N^2(y_{2n+1}, y_{2n+1}, y_{2n+1}, \theta, t), N^2(y_{2n}, y_{2n+2}, y_{2n+2}, \theta, t) \end{pmatrix} < 1$$

$$F\begin{pmatrix} M^2(y_{2n+1}, y_{2n+2}, y_{2n+2}, \theta, kt), M^2(y_{2n}, y_{2n+1}, y_{2n+1}, \theta, t) \\ M^2(y_{2n+1}, y_{2n}, y_{2n}, \theta, t), M^2(y_{2n+2}, y_{2n+1}, y_{2n+1}, \theta, t) \\ M^2(y_{2n+1}, y_{2n+1}, y_{2n+1}, \theta, t), \\ M^2\left(y_{2n}, y_{2n+1}, y_{2n+1}, \theta, \frac{t}{2}\right) * M^2\left(y_{2n+1}, y_{2n+2}, y_{2n+2}, \theta, \frac{t}{2}\right), \end{pmatrix} > 1 \text{ and}$$

$$G\begin{pmatrix} N^2(y_{2n+1}, y_{2n+2}, y_{2n+2}, \theta, kt), N^2(y_{2n}, y_{2n+1}, y_{2n+1}, \theta, t) \\ N^2(y_{2n+1}, y_{2n}, y_{2n}, \theta, t), N^2(y_{2n+2}, y_{2n+1}, y_{2n+1}, \theta, t) \\ N^2(y_{2n+1}, y_{2n+1}, y_{2n+1}, \theta, t), \\ N^2\left(y_{2n}, y_{2n+1}, y_{2n+1}, \theta, \frac{t}{2}\right) * N^2\left(y_{2n+1}, y_{2n+2}, y_{2n+2}, \theta, \frac{t}{2}\right), \end{pmatrix} < 1$$

For condition (3.1.2) we have $M^2(y_{2n+1}, y_{2n+2}, y_{2n+2}, \theta, kt) \geq$

$M^2\left(y_{2n}, y_{2n+1}, y_{2n+1}, \theta, \frac{t}{2}\right) * M^2\left(y_{2n+1}, y_{2n+2}, y_{2n+2}, \theta, \frac{t}{2}\right)$ and

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$N^2(y_{2n+1}, y_{2n+2}, y_{2n+2}, \theta, kt) \geq$

$$N^2\left(y_{2n}, y_{2n+1}, y_{2n+1}, \theta, \frac{t}{2}\right) \Diamond N^2\left(y_{2n+1}, y_{2n+2}, y_{2n+2}, \theta, \frac{t}{2}\right)$$

We have

$M^2(y_{2n+1}, y_{2n+2}, y_{2n+2}, \theta, kt) \geq M^2\left(y_{2n}, y_{2n+1}, y_{2n+1}, \theta, \frac{t}{2}\right)$ and

$N^2(y_{2n+1}, y_{2n+2}, y_{2n+2}, \theta, kt) \geq N^2\left(y_{2n}, y_{2n+1}, y_{2n+1}, \theta, \frac{t}{2}\right)$. That is

$M(y_{2n+1}, y_{2n+2}, y_{2n+2}, \theta, kt) \geq M\left(y_{2n}, y_{2n+1}, y_{2n+1}, \theta, \frac{t}{2}\right)$ and

$$N(y_{2n+1}, y_{2n+2}, y_{2n+2}, \theta, kt) \geq N\left(y_{2n}, y_{2n+1}, y_{2n+1}, \theta, \frac{t}{2}\right)$$

Similarly, we have

$M(y_{2n+2}, y_{2n+3}, y_{2n+3}, \theta, kt) \geq M\left(y_{2n+1}, y_{2n+2}, y_{2n+2}, \theta, \frac{t}{2}\right)$ and

$$N(y_{2n+2}, y_{2n+3}, y_{2n+3}, \theta, kt) \geq N\left(y_{2n+1}, y_{2n+2}, y_{2n+2}, \theta, \frac{t}{2}\right)$$

Thus we have

$M(y_{n+1}, y_{n+2}, y_{n+2}, \theta, kt) \geq M\left(y_n, y_{n+1}, y_{n+1}, \theta, \frac{t}{2}\right)$ and

$$N(y_{n+1}, y_{n+2}, y_{n+2}, \theta, kt) \geq N\left(y_n, y_{n+1}, y_{n+1}, \theta, \frac{t}{2}\right)$$

$M(y_{n+1}, y_{n+2}, y_{n+2}, \theta, kt) \geq M\left(y_n, y_{n+1}, y_{n+1}, \theta, \frac{t}{2^k}\right)$ and

$$N(y_{n+1}, y_{n+2}, y_{n+2}, \theta, kt) \geq N\left(y_n, y_{n+1}, y_{n+1}, \theta, \frac{t}{2^k}\right)$$

$M(y_n, y_{n+1}, y_{n+1}, \theta, kt) \geq M\left(y_0, y_1, y_1, \theta, \frac{t}{2^{nk}}\right) \to 1$ and

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$N(y_n, y_{n+1}, y_{n+1}, \theta, kt) \geq N\left(y_0, y_1, y_1, \theta, \frac{t}{2nk}\right) \rightarrow 0$ as n→ ∞, and hence

$M(y_n, y_{n+1}, y_{n+1}, \theta, t) \rightarrow 1$ and $N(y_n, y_{n+1}, y_{n+1}, \theta, t) \rightarrow 0$ as n→∞ for all t > 0.

For each $\varepsilon > 0$ and t> 0, we can choose $n_o \in N$ such that

$M(y_n, y_{n+1}, y_{n+1}, \theta, t) > 1 - \varepsilon$ and $N(y_n, y_{n+1}, y_{n+1}, \theta, t) < \varepsilon$ for all $n > n_0$

For any m, n ∈ N we suppose that m ≥ n. Then we have

$$M(y_n, y_m, y_m, \theta, t) \geq$$

$M\left(y_n, y_{n+1}, y_{n+1}, \theta, \frac{t}{m-n}\right) * M\left(y_{n+1}, y_{n+2}, y_{n+2}, \theta, \frac{t}{m-n}\right) * \ldots\ldots *$

$M\left(y_n, y_{n+1}, y_{n+1}, \theta, \frac{t}{m-n}\right)$ and $N(y_n, y_m, y_m, \theta, t) \leq N\left(y_n, y_{n+1}, y_{n+1}, \theta, \frac{t}{m-n}\right)$

$\Diamond$ ▯ $N\left(y_{n+1}, y_{n+2}, y_{n+2}, \theta, \frac{t}{m-n}\right) \Diamond \ldots\ldots\ldots\ldots \Diamond$ ▯$N\left(y_n, y_{n+1}, y_{n+1}, \theta, \frac{t}{m-n}\right)$

$M((y_n, y_m, y_m, \theta, t) \geq (1 - \in) * (1 - \in) * \ldots\ldots *(1 - \in)$ (m – n) timesand

$N(y_n, y_m, y_m, \theta, t) \leq (\in) \Diamond (\in) \Diamond \ldots\ldots \Diamond$ ▯$(\in)$ (m – n) times

$M((y_n, y_m, y_m, \theta, t) \geq (1 - \in)$ and $N(y_n, y_m, y_m, \theta, t) \leq (\in)$, and hence $\{y_n\}$ is a Cauchy sequences in X. Since (X, M, N, *, ◊ ▯) is complete, $\{y_n\}$ converges to some point z ∈ X. That is $\{Px_{2n+2}\} \rightarrow z$ and $\{STx_{2n+1}\} \rightarrow z$

$\{Qx_{2n+1}\} \rightarrow z$ and $\{ABx_{2n}\} \rightarrow z$

As (P, AB) is compatible pair of type (β), we have

$M(PPx_{2n}, (AB)(AB)x_{2n}, (AB)(AB)x_{2n}, \theta, t) = 1$ and

$N(PPx_{2n}, (AB)(AB)x_{2n}, (AB)(AB)x_{2n}, \theta, t) = 0$, for all t > 0 Or

$M(PPx_{2n}, ABz, ABz, \theta, t) = 1$ and $N(PPx_{2n}, ABz, ABz, \theta, t) = 0$.

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Therefore, $PPx_{2n} \to ABz$.

Put $x = (AB)x_{2n}$ and $y = x_{2n+1}$ in (3.1.3) we have

$$F\begin{pmatrix} M^2\big(P(AB)x_{2n}, Qx_{2n+1,} Qx_{2n+1\,,}\, \theta, kt\big), \\ M^2(AB(AB)x_{2n}, STx_{2n+1}, STx_{2n+1}, \theta, t), \\ M^2(P(AB)x_{2n}, AB(AB)x_{2n}, AB(AB)x_{2n}, \theta, t), \\ M^2(Qx_{2n+1}, STx_{2n+1}, STx_{2n+1}, \theta, t), \\ M^2(P(AB)x_{2n}, STx_{2n+1}, STx_{2n+1}, \theta, t), \\ M^2(AB(AB)x_{2n}, Qx_{2n+1}, Qx_{2n+1}, \theta, t) \end{pmatrix} > 1 \text{ and}$$

$$G\begin{pmatrix} N^2\big(P(AB)x_{2n}, Qx_{2n+1,} Qx_{2n+1\,,}\, \theta, kt\big), \\ N^2(AB(AB)x_{2n}, STx_{2n+1}, STx_{2n+1}, \theta, t), \\ N^2(P(AB)x_{2n}, AB(AB)x_{2n}, AB(AB)x_{2n}, \theta, t), \\ N^2(Qx_{2n+1}, STx_{2n+1}, STx_{2n+1}, \theta, t), \\ N^2(P(AB)x_{2n}, STx_{2n+1}, STx_{2n+1}, \theta, t), \\ N^2(AB(AB)x_{2n}, Qx_{2n+1}, Qx_{2n+1}, \theta, t) \end{pmatrix} > 1$$

Taking $n \to \infty$ and (3.1.1) we get

$M^2\big((AB)z, z, z, \theta, kt\big) \geq M^2\big((AB)z, z, z, \theta, t\big)$ and

$N^2\big((AB)z, z, z, \theta, kt\big) \leq N^2\big((AB)z, z, z, \theta, t\big)$. That is

$M\big((AB)z, z, z, \theta, kt\big) \geq M\big((AB)z, z, z, \theta, t\big)$ and

$N\big((AB)z, z, z, \theta, kt\big) \leq N\big((AB)z, z, z, \theta, t\big)$,

So we have $ABz = z$. Put $x = z$ and $y = x_{2n+1}$ in (3.1.3) we have

$$F\begin{pmatrix} M^2(Pz, Qx_{2n+1}, Qx_{2n+1}\ \theta, kt), M^2(Qx_{2n+1}, STx_{2n+1}, STx_{2n+1}, \theta, t) \\ M^2(ABz, STx_{2n+1}, STx_{2n+1}\ \theta, t), M^2(Pz, ABz, ABz, \theta, t), \\ M^2(Pz, STx_{2n+1}, STx_{2n+1}\ \theta, t), M^2\ (ABz, Qx_{2n+1}, Qx_{2n+1}, \theta, t) \end{pmatrix} > 1$$

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$$G\begin{pmatrix} N^2(Pz, Qx_{2n+1}, Qx_{2n+1}\,\theta, kt), N^2(Qx_{2n+1}, STx_{2n+1}, STx_{2n+1}, \theta, t) \\ N^2(ABz, STx_{2n+1}, STx_{2n+1}\,\theta, t), N^2(Pz, ABz, ABz, \theta, t), \\ N^2(Pz, STx_{2n+1}, STx_{2n+1}\,\theta, t), N^2\,(ABz, Qx_{2n+1}, Qx_{2n+1}, \theta, t) \end{pmatrix} < 1$$

Taking n → ∞ and (3.1.1) we get

That is $M^2(Pz, z, z, \theta, kt) \geq M^2(Pz, z, z, \theta, t)$ and

$N^2(Pz, z, z, \theta, kt) \leq N^2(Pz, z, z, \theta, t)$. That is

$M(Pz, z, z, \theta, kt) \geq \; M(Pz, z, z, \theta, t)$ and

$N(Pz, z, z, \theta, kt) \leq \; N(Pz, z, z, \theta, t)$, we have Pz = z.

So we have ABz = Pz = z. Put x = Bz and y = x_{2n+1} in (3.1.3) we get

$$F\begin{pmatrix} M^2(PBz, Qx_{2n+1}, Qx_{2n+1}\,\theta, kt), M^2(ABBz, STx_{2n+1}, STx_{2n+1}, \theta, t) \\ M^2(PBz, ABBz, ABBz\,\theta, t), M^2(Qx_{2n+1}, STx_{2n+1}, STx_{2n+1}, \theta, t), \\ M^2(PBz, STx_{2n+1}, STx_{2n+1}\,\theta, t), M^2\,(ABBz, Qx_{2n+1}, Qx_{2n+1}, \theta, t) \end{pmatrix} > 1$$

$$G\begin{pmatrix} N^2(PBz, Qx_{2n+1}, Qx_{2n+1}\,\theta, kt), N^2(ABBz, STx_{2n+1}, STx_{2n+1}, \theta, t) \\ N^2(PBz, ABBz, ABBz\,\theta, t), N^2(Qx_{2n+1}, STx_{2n+1}, STx_{2n+1}, \theta, t), \\ N^2(PBz, STx_{2n+1}, STx_{2n+1}\,\theta, t), N^2\,(ABBz, Qx_{2n+1}, Qx_{2n+1}, \theta, t) \end{pmatrix} < 1$$

Taking n → ∞ and (3.1.2) we get

$M^2(Bz, z, z, \theta, kt) \geq M^2(Bz, z, z, \theta, t), N^2(Bz, z, z, \theta, kt) \leq \; N^2(Bz, z, z, \theta, t)$.

That is $M(Pz, z, z, \theta, kt) \geq \; M(Pz, z, z, \theta, t)$ and

$N(Bz, z, z, \theta, kt) \leq \; N(Bz, z, z, \theta, t)$, we have Bz = z and also we have ABz = z

implies Az = z. Therefore Az = Bz = Pz = z.

Put x = x_{2n} and y = u in (3.1.3) we get

$$F\begin{pmatrix} M^2(Px_{2n}, Qu, Qu, \theta, kt), M^2(ABx_{2n}, STu, STu, \theta, t), \\ M^2(Px_{2n}, ABx_{2n}, ABx_{2n}\,\theta, t), M^2(Qu, STu, STu, \theta, t), \\ M^2(Px_{2n}, STu, STu, \theta, t), M^2\,(ABx_{2n}, Qu, Qu, \theta, t) \end{pmatrix} > 1$$

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$$G\begin{pmatrix} N^2(Px_{2n}, Qu, Qu, \theta, kt), N^2(ABx_{2n}, STu, STu, \theta, t), \\ N^2(Px_{2n}, ABx_{2n}, ABx_{2n}\ \theta, t), N^2(Qu, STu, STu, \theta, t), \\ N^2(Px_{2n}, STu, STu, \theta, t), N^2\ (ABx_{2n}, Qu, Qu, \theta, t) \end{pmatrix} < 1$$

Taking n → ∞ we get

$$F\begin{pmatrix} M^2(z, Qu, Qu, \theta, kt), M^2(z, STu, STu, \theta, t), \\ M^2(z, z, z, \theta, t), M^2(Qu, STu, STu, \theta, t), \\ M^2(z, STu, STu, \theta, t), M^2\ (z, Qu, Qu, \theta, t) \end{pmatrix} > 1$$

$$G\begin{pmatrix} N^2(z, Qu, Qu, \theta, kt), N^2(z, STu, STu, \theta, t), \\ N^2(z, z, z, \theta, t), N^2(Qu, STu, STu, \theta, t), \\ N^2(z, STu, STu, \theta, t), N^2\ (z, Qu, Qu, \theta, t) \end{pmatrix} < 1$$

$M^2(z, Qu, Qu, \theta, kt) \geq M^2(z, Qu, Qu, \theta, t)$, $N^2(z, Qu, Qu, \theta, kt) \leq N^2(z, Qu, Qu, \theta, t)$.

That is

$M(z, Qu, Qu, \theta, kt) \geq M(z, Qu, Qu, \theta, t)$, $N(z, Qu, Qu, \theta, kt) \leq N(z, Qu, Qu, \theta, t)$

we have Qu = z and also we have STu = z = Qu.

Hence (Q, ST) is weak compatible, therefore, we have QSTu = STQu.

Thus Qz = STz. Put x = x_{2n} and y = z in (3.1.3) weget

$$F\begin{pmatrix} M^2(Px_{2n}, Qz, Qz, \theta, kt), M^2(ABx_{2n}, STz, STz, \theta, t), \\ M^2(Px_{2n}, ABx_{2n}, ABx_{2n}\ \theta, t), M^2(Qz, STz, STz, \theta, t), \\ M^2(Px_{2n}, STz, STz, \theta, t), M^2\ (ABx_{2n}, Qz, Qz, \theta, t) \end{pmatrix} > 1$$

$$G\begin{pmatrix} N^2(Px_{2n}, Qz, Qz, \theta, kt), N^2(ABx_{2n}, STz, STz, \theta, t), \\ N^2(Px_{2n}, ABx_{2n}, ABx_{2n}\ \theta, t), N^2(Qz, STz, STz, \theta, t), \\ N^2(Px_{2n}, STz, STz, \theta, t), N^2\ (ABx_{2n}, Qz, Qz, \theta, t) \end{pmatrix} < 1$$

$M^2(z, Qz, Qz, \theta, kt) \geq M^2(z, Qz, Qz, \theta, t)$, $N^2(z, Qz, Qz, \theta, kt) \leq N^2(z, Qz, Qz, \theta, t)$

and hence

$M(z, Qz, Qz, \theta, kt) \geq M(z, Qz, Qz, \theta, t)$, $N(z, Qz, Qz, \theta, kt) \leq N(z, Qz, Qz, \theta, t)$

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We get Qz = z. Put x = x_{2n} and y = Tz in 3.2 (c) weget

$$F\begin{pmatrix} M^2(Px_{2n}, QTz, QTz, \theta, kt), M^2(ABx_{2n}, STTz, STTz, \theta, t), \\ M^2(Px_{2n}, ABx_{2n}, ABx_{2n}\ \theta, t), M^2(QTz, STTz, STTz, \theta, t), \\ M^2(Px_{2n}, STTz, STTz, \theta, t), M^2\ (ABx_{2n}, QTz, QTz, \theta, t) \end{pmatrix} > 1$$

$$G\begin{pmatrix} N^2(Px_{2n}, QTz, QTz, \theta, kt), N^2(ABx_{2n}, STTz, STTz, \theta, t), \\ N^2(Px_{2n}, ABx_{2n}, ABx_{2n}\ \theta, t), N^2(Qz, STTz, STTz, \theta, t), \\ N^2(Px_{2n}, STTz, STTz, \theta, t), N^2\ (ABx_{2n}, QTz, QTz, \theta, t) \end{pmatrix} < 1$$

QT = TQ and ST = TS, we have QTz = TQz = Tz and ST(Tz) = T(STz) = TQz = Tz.

Taking n → ∞ we get

$$F\begin{pmatrix} M^2(z, Tu, Tu, \theta, kt), M^2(z, Tz, Tz, \theta, t), \\ M^2(z, z, z, \theta, t), M^2(Tz, Tz, Tz, \theta, t), \\ M^2(z, Tz, Tz, \theta, t), M^2\ (z, Tz, Tz, \theta, t) \end{pmatrix} > 1$$

$$G\begin{pmatrix} N^2(z, Tz, Tz, \theta, kt), N^2(z, Tz, Tz, \theta, t), \\ N^2(z, z, z, \theta, t), N^2(Tz, Tz, Tz, \theta, t), \\ N^2(z, Tz, Tz, \theta, t), N^2\ (z, Tz, Tz, \theta, t) \end{pmatrix} < 1$$

$M^2(z, Tz, Tz, \theta, kt) \geq M^2(z, Tz, Tz, \theta, t)$, $N^2(z, Tz, Tz, \theta, kt) \leq \ N^2(z, Tz, Tz, \theta, t)$.

Therefore

$M(z, Tz, Tz, \theta, kt) \geq\ M(z, Tz, Tuz, \theta, t)$,$N(z, Tz, Tz, \theta, kt) \leq\ N(z, Tz, Tz, \theta, t)$

we have Tz = z. Now STz = Tz = z implies Sz = z. Hence Sz = Tz = Qz = z.

we have Az = Bz = Pz = Sz = Tz = Qz = z.

Hence z is the common fixed point of A, B, S, T, P and Q.

Uniqueness : Let u be another common fixed point of A, B, S, T, P and Q.

Then Au = Bu = Pu = Su = Tu = Qu = u.

Putting x = u and y = z in (3.1.3) then weget

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$$F\begin{pmatrix} M^2(Pu,Qz,Qz,\theta,kt), M^2(ABu,STz,STz,\theta,t), \\ M^2(Pu,ABu,ABu,\theta,t), M^2(Qz,STz,STz,\theta,t), \\ M^2(Pu,S\,Tz,STz,\theta,t), M^2\ (ABu,Qz,Qz,\theta,t) \end{pmatrix} > 1$$

$$G\begin{pmatrix} N^2(Pu,Qz,Qz,\theta,kt), N^2(ABu,STz,STz,\theta,t), \\ N^2(Pu,ABu,ABu,\theta,t), N^2(Qz,STz,STz,\theta,t), \\ N^2(Pu,S\,Tz,STz,\theta,t), N^2\ (ABu,Qz,Qz,\theta,t) \end{pmatrix} < 1$$

Taking limit both side, then weget

$$F\begin{pmatrix} M^2(u,z,z,\theta,kt), M^2(u,z,z,\theta,t), \\ M^2(u,u,u,\theta,t), M^2(z,z,z,\theta,t), \\ M^2(u,z,z,\theta,t), M^2\ (u,z,z,\theta,t) \end{pmatrix} > 1 \text{ and}$$

$$G\begin{pmatrix} N^2(u,z,z,\theta,kt), N^2(u,z,z,\theta,t), \\ N^2(u,u,u,\theta,t), N^2(z,z,z,\theta,t), \\ N^2(u,z,z,\theta,t), N^2\ (u,z,z,\theta,t) \end{pmatrix} < 1$$

$M^2(u,z,z,\theta,kt) \geq M^2(u,z,z,\theta,t), N^2(u,z,z,\theta,kt) \leq\ N^2(u,z,z,\theta,t).$

Hence $M(u,z,z,\theta,kt) \geq\ M(u,z,z,\theta,t), N(u,z,z,\theta,kt) \leq\ N(u,z,z,\theta,t).$

We get z = u. That is z is a unique common fixed point of A, B, S, T, P and Q in X.

Remark 3.2.:If we take B = T = I identity map on X in theorem then we get following Corollary.

Corollary 3.3. : Let (X, M, N, *, ◊) be a complete intuitionistic generalized fuzzy 2 metric space and let A, B, S, T, P and Q be mapping from X into itself such that the following conditions are satisfied:

(3.3.1) P(X) ⊂ S(X), Q(X) ⊂ A(X),

(3.3.2) (P, A) is compatible of type (β) and (Q, S) is weakly compatible,

(3.3.3) there exists k ∈ (0, 1) such that for every x, y ∈ X, t > 0

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$$F\begin{pmatrix} M^2(Px,Qy,Qy,\theta,kt), M^2(Ax,Sy,Sy,\theta,t), \\ M^2(Px,Ax,Ax,\theta,t), M^2(Qy,Sy,Sy,\theta,t), \\ M^2(Px,Sy,Sy,\theta,t), M^2\ (Ax,Qy,Qy,\theta,t) \end{pmatrix} > 1$$

$$G\begin{pmatrix} N^2(Px,Qy,Qy,\theta,kt), N^2(Ax,Sy,Sy,\theta,t), \\ N^2(Px,Ax,Ax,\theta,t), N^2(Qy,Sy,Sy,\theta,t), \\ N^2(Px,Sy,Sy,\theta,t), N^2\ (Ax,Qy,Qy,\theta,t) \end{pmatrix} < 1$$

Then A, S, P and Q have a unique common fixed point in X.

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