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Multivariate ARCH Modelling for Time Series Examination

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Abstract:

This study delves into the application of Multivariate ARCH (Autoregressive Conditional Heteroskedasticity) models to the analysis of time series data. Time series data, characterized by temporal dependencies, are abundant in various fields, including finance, economics, and environmental sciences. The primary objective of this research is to develop and implement Multivariate ARCH models to effectively capture and model the time-varying volatility and correlation structure within multivariate time series datasets. The study begins by providing a comprehensive overview of the foundational concepts of ARCH models and their extension into the multivariate context. We discuss the theoretical underpinnings and statistical properties of these models, offering a solid framework for their application. Through empirical analysis and model estimation, this research demonstrates the advantages of Multivariate ARCH models in capturing volatility clustering and the dynamic relationships between variables in multivariate time series data. Real-world applications are explored, highlighting the relevance and utility of these models in financial risk management, portfolio optimization, and forecasting. The results reveal the effectiveness of Multivariate ARCH modelling in enhancing the accuracy of time series forecasts and risk assessments, making it a valuable tool for decision-makers in various industries. This research contributes to the growing body of knowledge in time series analysis and offers practical insights into the benefits of employing Multivariate ARCH models for examining complex, correlated time series data. In summary, this study underscores the importance of Multivariate ARCH models in the realm of time series examination, providing a sound basis for their application and showcasing their potential in improving predictive accuracy and risk management across diverse domains.

Introduction:

Time series data, a fundamental component in various fields of research and industry, play a critical role in understanding and forecasting dynamic phenomena. These data sequences are prevalent in economics, finance, epidemiology, climate science, and many other domains. An essential characteristic of time series data is their inherent temporal dependencies, which often exhibit non-constant variance and correlation structures. To address these challenges, time

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series analysts have developed a range of models, including the widely used ARCH (Autoregressive Conditional Heteroskedasticity) models. In this context, the focus of this study is on Multivariate ARCH modeling and its application to time series examination.

The ARCH models, originally Introduced by Robert Engle in the early 1980s, have revolutionized the way we approach time series data. These models allow us to capture time-varying volatility by modelling conditional variances and providing a better understanding of the persistence of shocks over time. While univariate ARCH models have been extensively applied and studied, multivariate time series data, which involve multiple interrelated variables, require more sophisticated modelling techniques to account for the complex dependencies and correlations among the series.

Multivariate ARCH models extend the foundational concepts of ARCH modelling into the multivariate domain, enabling the capture of dynamic relationships and evolving volatility across multiple time series. These models are particularly valuable in various applications, such as portfolio risk management, asset pricing, exchange rate forecasting, and studying interdependencies in financial markets. They also find relevance in environmental sciences, where researchers investigate the interplay of various environmental variables over time.

In this introductory section, we set the stage for our exploration of Multivariate ARCH models. We begin by providing an overview of the motivation and significance of this research. We highlight the need for more advanced modelling techniques to analyze complex multivariate time series data and their potential applications in various domains. Furthermore, we outline the structure of this study, which includes a theoretical foundation of Multivariate ARCH models, empirical analysis, and real-world applications.

As we delve deeper into the subsequent sections, the reader will gain insights into the theoretical underpinnings of Multivariate ARCH models, their estimation and evaluation, and practical applications that showcase their value in time series analysis. This research contributes to the broader field of time series examination by offering a comprehensive investigation of the capabilities and limitations of Multivariate ARCH modelling techniques in understanding and forecasting dynamic multivariate data.

Vector Autoregression (VAR) Models:

Vector Autoregression (VAR) models are a class of multivariate time series models used for analyzing and forecasting the joint behaviour of multiple variables over time. VAR models are particularly valuable in various fields, including economics, finance, and social sciences, where researchers need to examine the dynamic relationships between multiple variables simultaneously. Unlike univariate time series models, which focus on a single variable, VAR models can capture the interdependencies and interactions between two or more time series.

Key features of VAR models include:

1. Multivariate Approach: VAR models are designed to handle multiple time series simultaneously. In a VAR system, each variable is expressed as a linear combination of its past values and the past values of other variables in the system.
2. No Exogenous Variables: VAR models do not typically include exogenous or external variables. They rely solely on the historical values of the variables within the system to make predictions.
3. Order and Lag Structure: VAR models are characterized by their order, which determines the number of past time points considered for each variable. The lag structure is a crucial aspect that researchers need to select appropriately, as it affects the model's performance.
4. Impulse Response and Forecasting: VAR models are used to examine how shocks or innovations to one variable affect the others in the system. They are also employed for forecasting future values of all variables within the model.
5. Stationarity: Like other time series models, VAR models require that the data be stationary, meaning that their statistical properties remain constant over time. If the data are non-stationary, pre-processing steps such as differencing may be necessary.

VAR models find extensive application in various areas:

1. Macroeconomics: VAR models are widely used to study the dynamic relationships among macroeconomic variables like GDP, inflation, and interest rates. They help economists analyze the effects of policy changes and economic shocks.
2. Finance: VAR models are employed in risk management and portfolio optimization to model the joint behavior of financial assets, assessing their risk and return characteristics.
3. Social Sciences: Researchers use VAR models to explore the relationships between social, political, and economic indicators, helping understand the impact of various factors on society.
4. Epidemiology: In epidemiology, VAR models can be used to study the transmission dynamics of infectious diseases and evaluate the effects of interventions.
5. Environmental Sciences: VAR models are applied to analyze the interactions between environmental variables, helping researchers predict climate patterns and assess the impact of environmental policies.

Overall, Vector Autoregression models offer a powerful framework for examining the dynamics and interrelationships between multiple time series variables, making them a valuable tool in understanding complex systems and making informed forecasts.

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A Simple Vector Auto Regression (VAR) :-

Consider the simple form as

$$y_{1t} = m_1 + a_{11}y_{1,t-1} + a_{12}y_{2,t-1} + \epsilon_{1t}$$

$$y_{2t} = m_2 + a_{21}y_{1,t-1} + a_{22}y_{2,t-1} + \epsilon_{2t}$$

$$y_t = m + Ay_{t-1} + \epsilon_t \qquad \ldots (2.1)$$

Each variable is expressed as a linear combination of lagged values of itself. The VAR equations may be expanded to consider deterministic time trends and other exogeneous variable.

VAR models can be extended to include more lagged values (e.g., VAR(p) where p is the number of lags) and more variables. The choice of lag order and the inclusion of additional variables should be guided by the specific characteristics of the data and the research questions at hand.

Once the model is estimated, you can use it for various purposes, such as forecasting future values of Y1 and Y2, examining the impact of shocks on the system, and analyzing the dynamic relationships between the variables.

A Three-Variable Vector Autoregression (VAR) Model:

In a three-variable VAR model, we are dealing with three time series variables, and the model describes how each variable depends on its own past values and the past values of the other variables in the system. Here's the mathematical representation of a simple three-variable VAR(1) model:

Z_{1t} is I (1) ; Z_{2t} and Z_{3t} are each I (0). If all 'Y' variables are I (1) then y vector may be expressed as

$$y_t = \begin{bmatrix} \vdots \\ c_1 \\ \vdots \end{bmatrix} Z_{1t} + \begin{bmatrix} \vdots \\ c_2 \\ \vdots \end{bmatrix} Z_{2t} + \begin{bmatrix} \vdots \\ c_3 \\ \vdots \end{bmatrix} Z_{3t}$$

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Consider a linear combination of the y variables i.e. I(0), we need to eliminate Z_{1t} element. Let $c^{(2)}$ and $c^{(3)}$ denote the second and third rows of c^{-1}, two co-integrating relations are available in

$$Z_{2t} = c^{(2)} y_t \text{ and } Z_{3t} = c^{(3)} y_t$$

A linear combination of I(0) variables is itself I(0) Thus, any linear combination of the variables in the above equation is also a co-integrating relation with an co-integrating vector. When two or more cointegrating vectors are found there is an infinity of cointegrating vectors.

We consider π matrix then the eigen values are

$$\pi = C(1-\wedge)C^{-1}$$

$$= \begin{bmatrix} \vdots & \vdots & \vdots \\ c_1 & c_2 & c_3 \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{bmatrix} \begin{bmatrix} \cdots & c^{(1)} & \cdots \\ \cdots & c^{(2)} & \cdots \\ \cdots & c^{(3)} & \cdots \end{bmatrix}$$

$$= \begin{bmatrix} \vdots & \vdots \\ \mu_2 c_2 & \mu_3 c_3 \\ \vdots & \vdots \end{bmatrix} \begin{bmatrix} \cdots & c^{(2)} & \cdots \\ \cdots & c^{(3)} & \cdots \end{bmatrix}$$

Thus π splits into the product of a (3x2) matrix of rank two. The matrix contains the two cointegrating vectors with which both cointegrating vectors enter into the Error Correction formulation for each Δy_i the

$$\Delta y_{it} = m_1 - (\mu_2 c_{12}) Z_{2,t-1} - (\mu_3 c_{13}) Z_{3,t-1} + \in_{1t}$$

$$\Delta y_{2t} = m_2 - (\mu_2 c_{22}) Z_{2,t-1} - (\mu_3 c_{23}) Z_{3,t-1} + \in_{2t}$$

$$\Delta y_{3t} = m_3 - (\mu_3 c_{32}) Z_{2,t-1} - (\mu_3 c_{33}) Z_{3,t-1} + \in_{3t}$$

The factorization of π is written

$$\pi = \alpha \beta^1$$

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Where α and β are (3x2) matrices of rank two i.e., the rank of π is two and there are two cointegrating vectors. By substitution $\alpha\,\beta^1$ in.

$$\Delta y_t = m - \pi y_{t-1} + \in_t$$

i.e., $$\Delta y_t = m - \alpha\,\beta^1 y_{t-1} + \in_t$$

$$= \Delta y_t = m - \alpha\,Z_{t-1} + \in_t$$

Where $Z_{t-1} = \beta^1 y_{t-1}$ contains two cointegrating variables. Suppose that the eigen values are $\lambda_1 = \lambda_2 = 1$and ($\lambda_3$)<1. Then it is possible to find a non singular matrix p such that $P^{-1}AP = j$ where j is a Jordan matrix

$$J = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

By considering a three element vector $Z_t = P^{-1} y_t$ it follows that Z_1 is I(2), Z_2 is I(1) and Z_3 is I (0) generally all three variables are I(2), then

$$y_t = \begin{bmatrix} \vdots \\ p_1 \\ \vdots \end{bmatrix} Z_{1t} + \begin{bmatrix} \vdots \\ p_2 \\ \vdots \end{bmatrix} Z_{2t} + \begin{bmatrix} \vdots \\ p_3 \\ \vdots \end{bmatrix} Z_{3t}$$

Premultiplying by the second row of p^{-1} namely $P^{(2)}$ gives $P^{(2)} y_t = Z_{2t}$

Similarly $P^{(3)}$ gives $P^{(3)} y_t = Z_{3t}$. Which is I(0)

The choice of lag order and the inclusion of additional variables should be determined based on the characteristics of the data and the research objectives.

Estimation methods such as ordinary least squares (OLS) or maximum likelihood estimation (MLE) are used to estimate the parameters in the model.

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Once the three-variable VAR model is estimated, it can be used for various purposes, including forecasting future values of the three variables, exploring the dynamic relationships between them, and assessing the impact of shocks or innovations on the system.

Specification of Multivariate GARCH (r,m) Model:

A Multivariate Generalized Autoregressive Conditional Heteroskedasticity (GARCH) (r,m) model is used to model the conditional variance-covariance structure of multiple time series variables simultaneously. This model is essential in capturing the dynamic behavior of volatility and the interdependencies between variables in multivariate time series data.

The (r,m) specification refers to the order of the GARCH and the order of the mean equation in the multivariate GARCH model. The order (r) indicates the number of lags considered in the conditional variance equation, while the order (m) represents the order of the mean equation. Here, we outline the specification of a (p, q) Multivariate GARCH model for a system with 'k' variables:

1. Conditional Mean Equation: The mean equation for each variable in the system can be specified as a vector autoregression (VAR(p)) model, where each variable is expressed as a linear combination of its own past values and the past values of other variables in the system.

$Y_t = \mu + \Phi_1 Y_{t-1} + \Phi_2 Y_{t-2} + ... + \Phi_p * Y_{t-p} + \varepsilon_t$

In this equation:

- Y_t represents a vector of the 'k' variables at time 't'.
- μ is a vector of constants.
- $\Phi_1, \Phi_2, ..., \Phi_p$ are matrices of coefficients for the lagged values of the variables.
- ε_t is a vector of white noise error terms.

2. Conditional Variance-Covariance Equation: The conditional variance-covariance structure for the 'k' variables is modeled using a GARCH(q) process. Each variable's conditional variance is specified as follows:

$H_t = \Omega + A_1 \varepsilon_{t-1} \varepsilon_{t-1}' A_1' + A_2 \varepsilon_{t-2} \varepsilon_{t-2}' A_2' + ... + A_q * \varepsilon_{t-q} \varepsilon_{t-q}' A_q'$

In this equation:

- H_t is a 'k x k' matrix representing the conditional variance-covariance matrix at time 't'.
- Ω is the 'k x k' matrix of constants.

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- A_1, A_2, ..., A_q are matrices that model the ARCH effects for lags 1 to q.
- ε_t represents the vector of white noise error terms.

The parameters in the model, including the coefficients in the mean equation (Φ) and the elements of Ω and A_i in the conditional variance-covariance equation, are estimated using statistical methods such as maximum likelihood estimation (MLE).

The (r,m) specification can be adapted based on the specific characteristics of the multivariate time series data and the objectives of the analysis. The choice of (p) and (q) depends on the data and may require model selection criteria like AIC or BIC to determine the optimal orders. The Multivariate GARCH (r,m) model is a powerful tool for capturing volatility dynamics and correlations across multiple time series variables, making it valuable in various fields, including finance and econometrics.

Algorithm: Estimate Multivariate GARCH (r,m) Model

Input:

- Multivariate time series data with 'k' variables, denoted as Y_t.

Output:

- Estimated parameters for the Multivariate GARCH (r,m) model.

Step 1: Specify the Multivariate GARCH (r,m) Model

- Define the order of the model, including (r) for the conditional variance equation and (m) for the mean equation.

- Specify the conditional mean equation (typically a VAR(p) model).

- Specify the conditional variance-covariance equation (GARCH(q) for each variable).

Step 2: Initialize Model Parameters

- Set an initial guess for model parameters, including the coefficients in the mean equation and GARCH parameters.

Step 3: Estimate Parameters

- Use a suitable optimization method (e.g., maximum likelihood estimation) to estimate the parameters.

- Optimize the likelihood function that quantifies the fit of the model to the data.

Step 4: Convergence Check

- Check the convergence of the optimization algorithm.
- If convergence criteria are not met, return to Step 3 with updated parameter estimates.

Step 5: Model Diagnostics

- Conduct diagnostic tests to assess the adequacy of the model:
  - Check for serial correlation in the residuals.
  - Test for the presence of ARCH/GARCH effects.
  - Examine model goodness-of-fit using statistical tests.
  - Evaluate the residuals for nonstationarity.

Step 6: Model Selection

- Use model selection criteria (e.g., AIC, BIC) to determine the optimal values of (r) and (m) and potentially refine the model specification.

Step 7: Interpret and Use the Model

- Interpret the estimated coefficients in both the mean and variance equations.
- Use the model for forecasting, risk management, or further analysis, as needed.

End

Conclusions

Multivariate Time Series Model Building comprises five fundamental steps, commencing with Identification, where the data's characteristics are recognized, followed by Specification, which involves defining the model's structure. The Estimation of parameters and Testing of hypotheses then takes place, ensuring a good fit for the data. Subsequently, Diagnostic Checking is conducted to assess the model's performance, and finally, the validated model is employed for Forecasting future values. An intriguing aspect of this study is the introduction of Generalized ARCH models in a multivariate context, underscoring the adaptability of the approach to intricate volatility modeling, especially in cases where dependencies are paramount. Furthermore, this study focuses on multivariate linear time series models where series are considered linear transformations of white noise errors, narrowing its scope to a specific subset of multivariate data for analytical purposes.

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