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Research Paper

PRIME AVERAGE HARMONIOUS LABELING OF GRAPHS

G. S. GANESHWARI, Research Scholar (Reg No: 20123112092025), Department of Mathematics, Nesamony Memorial Christian College, Marthandam 629 165, Tamil Nadu, India

email:ganeswari247.gs@gmail.com

G. SUDHANA, Assistant Professor, Department of Mathematics, Nesamony Memorial Christian College, Marthandam – 629165, Tamil Nadu, India'
email:sudhanaarun1985@gmail.com

ABSTRACT- In this article study, we have presented innovative harmonious labeling as prime Average Harmonious Labeling of the graph . A function L is called prime average harmonious labeling of graph G with n vertices and m edges if $L : V \to \{0,1,2,3 \dots \dots , 2n-1\}$ is injective and the induced function $L^* : E(G) \to \{0,1,2, \dots .2m\}$ is defined by $L^*(uv) =$

(i) $GCD\left(L(u), L(v)\right) = 1$, (ii) $L(uv) = \frac{L(u)+L(v)+1}{2} (mod\ m)\ if\ L(u) + L(v)\ is\ odd$, (iii)$L(uv) = \frac{L(u)+L(v)}{2} (mod\ m)\ if\ L(u) + L(v)\ is\ even$.A graph that satisfies the above three conditionsare called a prime average harmonious labeling. A graph that admits a prime average harmonious labeling is called a prime average harmonious graph.

Keywords: Average Harmonious, Prime Harmonious, Prime average harmonious,Bijective .

AMS Subject Classification: 05C78

1 INTRODUCTION

Harmonious labeling were defined by Graham and Sloane [3] as part of theirstudy of additive basis and are applicable to error – correcting codes. Since m their introduction in 1980, harmonious labeling has piqued the interest of many researchers, and they are now among the most widely used labeling in the literature. Sarasija and Binthiya introduced an even harmonious labeling. AdalinBeatress and Sarasija [2] introduced average harmonious graph in 2015. The concept of prime harmonious labeling was introduced and studied by P.Deepa and K .Indirani in 2016. The concept of prime harmonious labeling of Some Graphs was extended and inroduced by Dr. M. Tamilselvi, K. Akilandeswari , S. Mahalakshmi 2018.On the basis of this work we introduced a new concept Prime Average harmonious Graphs.

II DEFINITIONS

Definition 2.1.A function L is called prime harmonious labeling of graph G with n vertices and m edges if $L : V \to \{0,1,2,3 \dots \dots , 2n-1\}$ is injective and the induced function $L^* : E(G) \to \{0,1,2, \dots .2m\}$ is defined by $L^*(uv) = \begin{cases} GCD\left(L(u), L(v)\right) = 1 \\ L(uv) = L(u) + L(v)(mod\ m) \end{cases}$

A graph which satisfies the conditions of prime labeling and harmonious labeling is called a prime harmonious labeling. A graph that admits a prime harmonious labeling is called a prime harmonious graph.

Definition 2.2: A quadrilateral snake Q_n is obtained from a pathu_1,u_2,u_3, . . . ,$u_n$by joining u_i and $u_{i+1}$to a new vertices v_i and w_i respectively and then joiningv_i and w_i for $1 \le i \le n - 1$. That is, every edge of a path is replaced bya cycle C_4.

III PRIME AVERAGE HARMONIOUS OF SOME GRAPHS

Definition : A function L is called prime average harmonious labeling of graph G with n vertices and m edges if $L : V \to \{0,1,2,3 \dots \dots, 2n-1\}$ is injective and the induced function $L^* : E(G) \to \{0,1,2, \dots .2m\}$ is defined by

$L^*(uv) =$ (i)$GCD\left(L(u), L(v)\right) = 1$,; (ii)$L(uv) = \frac{L(u)+L(v)+1}{2} (mod\ m);\ if\ L(u) + L(v)\ is\ odd,$

(iii)$L(uv) = \frac{L(u)+L(v)}{2} (mod\ m);\ if\ L(u) + L(v)\ is\ even.$

A graph that satisfies the conditions of prime labeling and harmonious labeling is called a prime average harmonious labeling. A graph that admits a prime average harmonious labeling is called a prime average harmonious graph.

Theorem 3.1. An KC_4 , $k \ge 1$ snake graph is an Prime average harmonious graph.

Proof. Let G be the KC_4 snake graph. Let $V = \{u_0, u_h, v_h, w_h : 1 \le h \le k\}$ be the vertices of KC_4 snake graph and thus the edges of the KC_4snake graph $E = \{u_h v_{h+1}, u_h w_{h+1} : 0 \le h \le k-1\} \cup \{\{u_h v_h, u_h w_h\ where\ 1 \le h \le k$; There are 3k + 1 vertices and 4k edges. To define $L : V(G) \to \{0, 1, 2, \dots, (8k-1)\}$ by $L(u_0) = 1, L(v_h) = 4h - 1, for\ 1 \le h \le k, L(w_h) = 4h, for\ 1 \le h \le k, L(u_h) = 4h + 1, for\ 1 \le h \le k$. Then L induced a bijective function $L^* : V \to \{0,1,2,3 \dots \dots, 2m\}$defined by the edge functions are as follows :

$L^*(u_0 v_1) = 2,\ L^*(u_0 w_1) = 2$, $L^*(u_h v_h) = 4h$; $1 \le h \le k$; $L^*(w_h v_h) = 4h + 1$; $1 \le h \le k$, $L^*(u_h v_{h+1}) = 4h + 2$; $1 \le h \le k$; $L^*(u_h w_{h+1}) = 4h + 3$; $1 \le h \le k$

Thus L admits an PAHL for the KC_4snake graph. Hence the KC_4snake graph is an PAHL graph.

Theorem 3.2. The quadrilateral snake $Q_n$is a graph with a prime average

harmonious graph.

Proof: $V[Q_n] = \{u_h, 1 \le h \le n\ and\ v_h,\ w_h, 1 \le h \le n-1\}$ be the vertices

and$u_0, u_h, v_h, w_h : 1 \le h \le k\}$

$E[Q_n] = \{u_h u_{h+1}, u_h v_h, u_{h+1} w_h, v_h, w_h;\ 1 \le h \le n-1\}$ be the edges.

There are 2n + 4 vertices and 3n + 2 edges.

Then L induced a bijective function $L^* : V \to \{0, 1,2,3 \dots \dots, p+3-1\}$defined by the edge functions are as follows :$L^*(u_h v_h) = 4h - 2$; $1 \le h \le n-2$, $L^*(w_h u_{h+1}) = 4h + 1$; $1 \le h \le n-2$, $L^*(u_h u_{h+1}) = 4h - 1$; $1 \le h \le n-1$, $L^*(v_h w_{h+1}) = 4h$; $1 \le h \le n-2$

$L^*(u_n v_n) = 0$, $L^*(u_{n+1} v_n) = 1$**.** Then L is a prime average harmonious then both vertex and edge label are distinct. Hence Q_n is a prime average harmonious graph.

Theorem 3.4. The Bistar graph $B_{p,q}, (p, q \geq 1)$ is a prime average harmonious.

Proof: The vertices of K_2 in $B_{p,q}$ are u, v .Let $U = \{u_h, 1 \leq h \leq p\}$ *and* $V = \{v_h, 1 \leq h \leq q\}$ are the neighboring vertices to u and v.

$E[B_{p,q}] = \{uv, uu_h, 1 \leq h \leq p; \quad vv_h, 1 \leq h \leq q\}$

There are $n = p + q + 2$ vertices and $m = (p + q + 1)$ edges and

the modulo $(p + q + 1)$ in the graph $B_{p,q}$

Define $L : V(P_r \odot K_1) \rightarrow \{1,2,3 \dots \dots, 2n - 1\}$ *by*

$L(u) = 1, L(v) = 2p + 2q + 2, ; L(u_h) = 2q + 2h, \ 1 \leq h \leq p$ *and*

$L(v_h) = 2h + 1, \quad 1 \leq b \leq q;$ and a bijective function $L^* : E(B_{p,q}) \rightarrow \{0, 1, 2, 3, \dots, (p + q)\}$ the edge label are as follows:

$L^*(uv) = 1, L^*(vv_h) = (h + 1), 1 \leq h \leq q, L * (uu_h) = (q + h + 1), 1 \leq h \leq p$ .The edge labels and vertex labels are distinct. Thus the bistar$B_{p,q}$ was also a prime average harmonious .

IV CONCLUSION

In this paper, we investigated the Prime average harmonious labeling which is oneof the most important labeling techniques. As all the graphs are not prime averageharmonious, it is very interesting to investigate the different types of graphs which admit the prime average harmonious. Using some mathematical derivations, wehave reported the Prime average harmonious labeling of various graphs.

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