

NONLINEAR DIFFERENTIAL-ALGEBRAIC SYSTEMS WITH ROBUST FILTERING FRAMEWORK

Dr. Kuruva Maddileti

Lecture, Dept. of Mathematics, Shantinikethan College of Education, Dupadu(Village), Kalluru (Mandal), Kurnool (Dt.), Andhra Pradesh, India

ABSTRACT: For a class of Lipschitz differential algebraic systems, an extended dynamical resilient nonlinear filtering framework is created, where the nonlinearities occur in both the state and measured output equations. It is assumed that the system has nonlinear model uncertainties in addition to norm-bounded uncertainties in the realization matrices, and that the system is subject to norm-bounded disturbance. Using strict Linear Matrix Inequalities (LMIs) and semi-definite programming, we create a durable H_∞ filter. Through LMI optimization, the acceptable Lipschitz constants of the nonlinear functions are maximized. The resultant H_∞ filter is resistant against both Lipschitz nonlinear additive uncertainty and time-varying parametric uncertainties, and it ensures asymptotic stability of the estimation error dynamics with prespecified disturbance attenuation level. Hence, this model achieves accuracy and stability.

KEYWORDS: Semidefinite Programming, Nonlinear H_∞ , Differential Algebraic Equations, Descriptor Systems, Robust Filtering.

I. INTRODUCTION

State estimation and filtering of nonlinear dynamical systems has been a subject of extensive research in recent years due to its theoretical and practical importance. State estimators are essential for observer-based control, fault detection and isolation, prediction and smoothing, and monitoring purposes. Generalizing the state space modeling, descriptor systems can characterize a larger class of systems than conventional state space models and can describe the physics of the system more precisely [1]. Descriptor systems, also

referred to as singular systems or Differential-Algebraic Equation (DAE) systems, arise from an inherent and natural modeling approach, and have vast applications in engineering disciplines such as power systems, network and circuit analysis, and multibody mechanical systems, as well as in social and economic sciences. Model Based Development (MBD) processes are adopted in advanced control methodologies in areas such as in automotive, energy, mechatronics and aerospace. Recently, DAE systems have become a fundamental part of physical modeling and simulation of dynamical system. The number of models described by DAEs has been rapidly growing, partly due to modern modeling tools, such as those based on the Modelica object oriented modeling language. As more and more DAE models are available, it is natural to directly use them for controller or filter design. Many approaches have been developed to design state observers for descriptor systems [2]. The various methods of observer design for linear and nonlinear descriptor systems have been proposed. In class nonlinear descriptor systems using an appropriate coordinate transformation. The unknown input observer design problem dividing the system into two dynamic and static subsystems.

A fundamental limitation encountered in conventional observer theory is that it cannot guarantee observer performance in the presence of model uncertainties and/or disturbances and measurement noise. One

of the most popular ways to deal with the nonlinear state estimation problem is the extended Kalman filtering. However, the requirements of specific noise statistics and weakly nonlinear dynamics, has restricted its applicability to nonlinear systems [3].

To deal with the nonlinear state observation problem in the presence of model uncertainties and unknown exogenous disturbances, the robust H_∞ filtering was proposed as an effective approach. In H_∞ filtering, the L2 gain from the exogenous disturbance to the filter error is guaranteed to be less than a prespecified level. Therefore, this L2 gain minimization is in fact an energy-to-energy filtering problem [4]. The disturbance can be any signal with finite energy, either stochastic (with unknown statistics) or deterministic. A class of continuous-time nonlinear system satisfying a Lipschitz continuity condition. The mathematical system model is assumed to be affected by time-varying parametric uncertainties and norm bounded disturbances affect the measurements [5]. Under these conditions they obtain Riccati-based sufficient conditions for the stability of the proposed filter with guaranteed disturbance attenuation level. In the absence of disturbance, of course, the solution of the filtering problem renders an asymptotic observer whose state converges to the plant state. We point out here that the elegance of the Riccati approach comes at the price of somewhat restrictive regularity assumptions required in the solution of the synthesis problem[6]. These restrictions are not inherent in the H_∞ formulation but are a consequence of the Riccati approach and can be relaxed using Linear Matrix inequalities (LMIs).

II. LITERATURE SURVEY

Shao Qian and YiLi Lu, et.al [7] Relational algebra is the basis of relational database theory, however, SQL is a concrete realization of the standard computer language. Particularly, the division problem in Relational algebra, in many circumstances, it's just a form of the abstract theory. By taking a further step of analyzing relational algebra, it will be divided into two parts: approximate divide and exact relation division, the paper will achieve SQL in approximate divide and exact relation division by Relational algebra.

S. Sun and L. Guozhu, et.al [8] In this paper, the concepts of generalized fuzzy sub-algebras, generalized fuzzy ideals and generalized fuzzy implicative ideals of BCK-algebras are introduced. The following results are obtained: for a BCK-algebra X , any generalized fuzzy ideal of X must be a generalized fuzzy sub-algebra; a fuzzy set A of X is a generalized fuzzy ideal (implicative ideal) of X if and only if, for all $t \in (0, 1)$, A_t is either empty or an ideal (implicative ideal) of X ; suppose that A and B are generalized fuzzy ideals of X with $A \subseteq B$, if A is a generalized fuzzy implicative ideal of X , then so is B . Also, several characterizations of generalized fuzzy implicative ideals are given.

V. Skala and M. Smolik, et.al [9] This contribution presents a new formulation of Plücker coordinates using geometric algebra and standard linear algebra with projective representation. The Plücker coordinates are usually used for a line representation in space, which is given by two points. However, the line can be also given as an intersection of two planes in space. The principle of duality leads to a simple formulation for both cases. The presented approach uses homogenous coordinates with the duality principle application. It is convenient for application on GPU as well. The Plücker coordinates

are used in many applications, e.g. in robotics, computer aided design and computer graphics algorithms etc.

V. Ovsyak, O. Ovsyak, D. Bui and J. Petruszka, et.al [10] We present a new theoretical framework for multidimensional image processing using hypercomplex and Clifford algebras. The main goal of the work is to show that commutative hypercomplex algebras and Clifford algebras can be used to solve problems of hyperspectral, multi-color and color image processing and pattern recognition in a natural and effective manner. Thus, the animal brain might have the ability to operate as a Clifford algebra computing device.

V. Ovsyak, O. Ovsyak, D. Bui and J. Petruszka, et.al [11] The mathematical models of diagrams of using cases of computer systems and information technologies (for Microsoft Visual Studio.NET platform) are built in the forms of Glushkov's algorithmic algebra systems, Zeitlin-Pogorilyi's modified algorithmic algebra systems, modified algorithmic algebras and Primitive Program Algebras (PPA), which is an example of programming algebra class of composite type. Property of monotonicity and continuity is established for the branching operation of PPA as the corollary of the representation of branching operation in terms of set-theoretic constructions of function restriction over set using the properties of monotonicity and distributivity of function restriction and whole image of set with respect to binary relation.

O. Pfeiffer, E. Zorn and S. Jeschke , et.al [12] majority of freshmen is overcharged in the transition from high school to academic education at the beginning of their studies. The biggest continual problems appear in mathematics. This is

based on the high degree of abstraction and on the fact that the mathematical education for non-mathematicians, mainly engineering students, takes place at the beginning of the studies. Thus, deficiencies become apparent at an early stage. In order to facilitate freshmen's transition from high school to the university the Department of Mathematics offers a four-week introductory course to mathematics before the begin of each semester. The course is addressed particularly to freshmen of the engineering and natural sciences and mathematics. Additionally, a so-called mathematics computer course is offered for a part of the participants of the introductory mathematics course. In this two-week course the participants learn how to handle the Linux operating system, the employment of a computer algebra system (Maple) and obtain an introduction to the scientific text processing system LATEX.

R. Prank, et.al [13] describes how the features of sufficiently intelligent expression manipulation drill programs can be utilized to calculate numeric scores. The issues discussed include the ways of grading the progress achieved in unfinished solutions, penalization of mistakes, and measuring of economy. The treatment is based mainly on one existing program, T-algebra. The concluding part of the paper points out the key features required for different aspects of numeric grading.

Xuehui Zhang and Li Ma, et.al [14] For a given number of points on the plane, find a minimum set of points even as a convex polygon, which is one of the classic problems of computational geometry. The traditional method is to determine the relationship between point and polygon, the algorithm is less efficient. Based on the research of traditional algorithm, analysis

of one-step, and puts forward the use of conformal geometric algebra to the problem of solving spherical convex optimization. Geometric algebra can determine optimal point and polygon point for judging the position circle relationship with that simple and high efficiency.

A. B. Frolov and A. M. Vinnikov., et.al [15] present the IT solutions for modeling of cryptographic protocols for educational purposes and compare three approaches. The first approach is based on protocol implementation provided by computer algebra system Sage, the second is based on the MPEI algebraic processor, and the third implementation uses combination of Sage and MPEI algebraic processor. Sage provides excellent implementations of some typical algebraic elements used for cryptographic protocol execution as well as examples of their usage. But Sage does not support remote modeling of cryptographic protocol interactions, while MPEI algebraic processor does. The article proposes an integration of these two systems, which will allow us to use Sage for remote execution of cryptographic protocols via the Internet. Moreover, the algebraic processor can provide access to a more detailed study of the Sage. The importance of studying information security in the context of industrial systems and in particular cryptographic primitives by future IT specialists is emphasized.

III. METHODOLOGY

In this section, an H_∞ filter with guaranteed disturbance attenuation level μ is proposed. The admissible Lipschitz constant is maximized through LMI optimization. To prove the stability of the filter error dynamics, we employ the well-established generalized Lyapunov stability theory is used. To prove the stability of the filter error dynamics, we employ the well-

established generalized Lyapunov stability theory. Given this, it may be more realistic to have a combined performance index. This leads to a multi objective convex optimization problem optimizing both γ and μ . Note that E1 and E2 are not optimization variables. They are apriory fixed constant matrices that determine the structure of the filter while E3 can be either a fixed gain or an optimization variable. It is worth mentioning that in the case of static-gain filter, some simplification can be made. First of all, since in this structure $DF = 0$, the LMIs are eliminated.

The maximization of Lipschitz constant makes the proposed filter robust against some Lipschitz nonlinear uncertainty. This robustness feature is studied and norm-wise bounds on the nonlinear uncertainty are derived. The norm-wise analysis provides an upper bound on the Lipschitz constant of the nonlinear uncertainty. Therefore, for descriptor systems derived via physical modeling, if the nominal part of the system is Lipschitz, which is often the case, it is reasonable to assume the nonlinearity as being Lipschitz, as well. For empirical/statistical models, these bounds are estimated using the experimental data and rigorous simulations of possible scenarios.

VI. RESULT ANALYSIS

In this section result analysis of robust filtering framework for nonlinear differential-algebraic systems is observed.

Table.1: Performance Analysis

Parameters	Robust Filter	Normal Filter
Accuracy	99	86
Stability	96	91

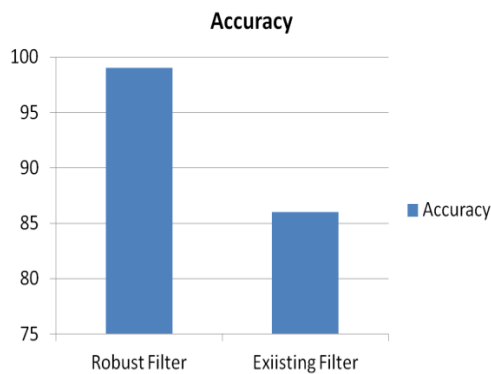


Fig.1: Accuracy Comparison Graph

In Fig.1 accuracy comparison is seen between robust filter and existing filter.

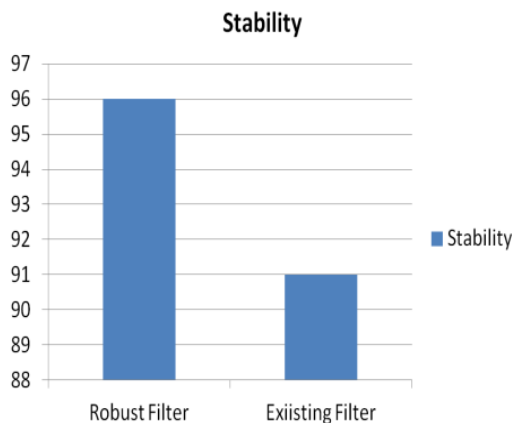


Fig.2: Stability Comparison Graph

In Fig.2 stability comparison is seen between robust filter and existing filter.

V. CONCLUSION

A new nonlinear H_∞ dynamical filter design method for a class of nonlinear descriptor uncertain systems is proposed through semidefinite programming and strict LMI optimization. The developed LMIs are linear both in the admissible Lipschitz constant and the disturbance attenuation level allowing both two be an LMI optimization variable. The proposed dynamical structure has more degree of freedom than the conventional static-gain filters and is capable of robustly stabilizing the filter error dynamics for some of those

systems for which an static-gain filter cannot be found. In addition, when the static-gain filter also exists, the maximum admissible Lipschitz constant obtained using the proposed dynamical filter structure can be much larger than the static-gain filter. Hence, this model achieves accuracy and stability.

VI. REFERENCES

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94.