Some sets closer to new closed sets in ideal topological space

V. BEULAH , *Department of Mathem atics, Sarah Tucker College.,* beulahsamuel26@gmail.com

JESSIE THEODORE, *Department of Mathem atics, Sarah Tucker College.*

D. PREMALATHA, *Department of Mathematics, Rani Anna Government College for Women. Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012, Tamilnadu, India*
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ABSTRACT: In this paper we have defined and proved that the $_{g\beta^*}I$-closure is a Kuratowski closure operator on the ideal topological space (X, τ, I). Also, we have introduced $_{g\beta^*}I$-kernel, $_{g\beta^*}I$-derived set, $_{g\beta^*}I$-Border, $_{g\beta^*}I$-Frontier and $_{g\beta^*}I$-Exterior. Its characterization and properties investigated and exploredin ideal topological space.

Keywords: *ideal, $_{g\beta^*}I$-closed set, $_{g\beta^*}I$-closure, $_{g\beta^*}I$- interior.*

1.INTRODUCTION

Local function in topological space using ideals was introduced by Kuratowski . The notion of ideal topological spaces was studied by Kuratowski [10] and Vaidyanathaswamy . Jankovi´c and Hamlett investigated further properties of ideal topological spaces. The generalized closed set in ideal topological spaces namely $_{g\beta^*}I$-closed set has already been introduced . The aim of this paper is to prove that the $_{g\beta^*}I$-closure is a Kuratowski closure operator on the ideal topological space (X, τ, I). Also, we have introduced $_{g\beta^*}I$-kernel, $_{g\beta^*}I$-derived set, $_{g\beta^*}I$-Border, $_{g\beta^*}I$-Frontier and $_{g\beta^*}I$-Exterior. In particular, its characterization and properties analyzed

2. PRELIMINARIES

An ideal I on a nonempty set X is a collection of subsets of X which satisfies the following properties: (i) $A\in I$ and $B \subseteq A$ implies $B\in I$ (heredity) (ii) $A\in I$ and $B\in I$ implies $A\cup B\in I$(finite additivity). An ideal topological space is a topological space (X, τ) with an ideal I on X, and is denoted by (X, τ, I). Given an ideal topological space (X, τ, I) and if $P(X)$ is the set of all subsets of X, a set operator $(.)^*: P(X) \rightarrow P(X)$, called a *local function* of A with respect to τ and $I$is defined as follows: for $A\subset X$, $A^*(\mathcal{I}, \tau) = \{x\in X: U \cap A \notin I$ for every $U\in\tau(x)\}$ where $\tau(x)=\{U\in\tau : x\in U\}$, when there is no chance for confusion $A^*(I, \tau)$ is denoted by A^*. For every ideal topological space (X, τ, I) there exists a topology τ^* finer than τ, generated by the base $\beta(I, \tau) = \{U\setminus K: U\in\tau \text{ and } K\in I\}$. In general, $\beta(I, \tau)$ is not a topology. A subset A of a space (X, τ) is *β-open* or *semi-pre-open* [1] set if $A\subset cl(int(cl(A)))$]. The complement of *β-open* or *semi-pre-open* set is *β-closed* or *semi-pre-closed*. The *semi pre-closure* of a subset A of X, denoted by $spcl(A)$ is defined to be the intersection of all semi-pre-closed sets containing A. The *semi pre-interior* of a subset A of X, denoted by $spint(A)$ is defined to be the union of all semi-pre-open sets contained in A.

Lemma 2.1 [12] *Let (X, τ, I) be an ideal topological space and $A \subseteq X$. If $A \subset A^*$, then* $A^* = cl(A^*) = cl^*(A) = cl(A)$.

Definition 2.2 [16] Let (X,τ) be a topological space. A subset A of X is said to be ω-closed if

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$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open.

Definition 2.3 [16] A space (X, τ) is called a T_ω –space if every ω-closed set in it is closed.

Definition 2.4 [9] Given a topological space (X, τ) with an ideal I on X and if $P(X)$ is the set of all subsets of X, a set operator $(.)^{**}$: $P(X) \to P(X)$, called a *semi-pre local function* or *β-local function* of A with respect to τ and $\mathcal{I}$ is defined as follows: for $A \subset X$, $A_{**}(I, \tau) = \{x \in X : U \cap A \notin I \ for \ every \ U \in \beta O(x)\}$where the family of semi-preopen sets $\beta O(x) = \{U \in \beta O(X) : x \in U\}$, when there is no ambiguity, we will write simply A_{**} for $A_{**}(I, \tau)$.

Lemma 2.5 [9] *Let (X, τ, I)be an ideal space and A, B subsets of X. Then, for the β-local function, the following properties hold:*

1. *If$A \subset B$, then$A_{**} \subset B_{**}$.*
2. *$A_{**} = spcl(A_{**}) \subset spcl(A)$ and $A_{**}$is β-closed in X.*
3. *$(A_{**})_{**} \subseteq A_{**}$*
4. *$(A \cup B)_{**} = A_{**} \cup B_{**}$*
5. *$(A \cap B)_{**} = A_{**} \cap B_{**}$*

Definition 2.6 [9]A subset A of an ideal space (X, τ, I) is said to be $_{g\beta^*}I$-closed set if$A_{**} \subseteq int(U)$ whenever $A \subseteq U$ and U is ω-open in X. The complement of $_{g\beta^*}I$-closed set is said to be $_{g\beta^*}I$-open. The family of all $_{g\beta^*}I$-open sets is denoted by $_{g\beta^*}I\mathcal{O}(X, \tau)$.

Definition 2.7 [9] A subset A of an ideal space (X, τ, I) is said to be

1. *semi-pre *- closed* if $A_{**} \subseteq A$.
2. *⋆-semi-pre dense* if $A \subseteq A_{**}$.
3. *semi-pre *- perfect* if $A_{**} = A$.

Remark 2.8 [9] *Every closed (resp. open) set is$_{g\beta^*}I$-closed (resp. $_{g\beta^*}I$-open) set.*

Lemma 2.9 [9] *In an ideal space (X, τ, I),*

(i) *Every member of$I$is $_{g\beta^*}I$-closed.*
(ii) *A_{**} is $_{g\beta^*}I$-closed for every subset A of X.*
(iii) *If $I = \{\phi\}$,then $A_{**} = spcl(A)$ and hence $_{g\beta^*}I$-closed sets coincide with β^*-closed sets.*

Theorem 2.10 [9]*Let (X, τ, I) be an ideal topological space and $A \subseteq X$. Then the following statements (1), (2) and (3) are equivalent and (3) implies (4) and (5) which are equivalent. (3) implies (1) if (X, τ) is a T_ω– space.*

1) *A is $_{g\beta^*}I$-closed.*
2) *$spcl(A_{**}) \subseteq int\ U$for every ω-open set U containing A.*
3) *For all $x \in spcl(A_{**})$, $\omega cl(\{x\}) \cap A \neq \varphi$*
4) *$spcl(A_{**}) - A$ contains no non empty ω-closed set.*
5) *$A_{**} - A$contains no non empty ω-closed set.*

Lemma 2.11 [9] If A and B are subsets of (X, τ, I) then$(A \cap B)_{**} \not\subset A_{**} \cap B_{**}$

Remark2.12 [9] Every ⋆-closed set is semi-pre-⋆-closed but not conversely, since $A_{**} \subseteq A_* \subseteq A^* \subseteq A$.

3. $_{g\beta^*}I$-closure and $_{g\beta^*}I$-interior in ideal topological space

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Definition3.1: For every set $E \subset X$, we define the ${}_{g\beta^*}I$*-closure* of E to be the intersection of all ${}_{g\beta^*}I$-closed sets containing E.

In symbols, ${}_{g\beta^*}Icl(E) = \cap\{A : E \subset A,\ A \in {}_{g\beta^*}I\mathcal{C}\ell(X,\tau)\}$where${}_{g\beta^*}I\mathcal{C}(X,\tau)$ is the family of all ${}_{g\beta^*}I$-closed sets in X.

Lemma 3.2:*For any* $E \subset X,\ E \subset {}_{g\beta^*}Icl(E) \subset cl(E)$.

Proof: Follows from the Remark 2.8.

Lemma 3.3:*If* $A \subset B$, *then* ${}_{g\beta^*}Icl(A) \subset {}_{g\beta^*}Icl(B)$

Proof: Clearly follows from Definition 3.1.

Remark 3.4: ${}_{g\beta^*}I$*-closure of a set need not be* ${}_{g\beta^*}I$*-closed.*

Theorem 3.8:*If* ${}_{g\beta^*}I\mathcal{C}\ell(X,\tau)$ *is closed under finite union and intersection, then* ${}_{g\beta^*}I$*-closure is a Kuratowski closure operator on X.*

Proof: Since $\phi$and $X$are${}_{g\beta^*}I$-closed, by Lemma 3.6,we get${}_{g\beta^*}Icl(\phi) = \phi$ and

$$ {}_{g\beta^*}Icl(X) = X. $$

(1) $E \subset {}_{g\beta^*}Icl(E)$, by Lemma 3.2.

(2) Suppose E and F are two subsets of X, then by Lemma 3.3,we get ${}_{g\beta^*}Icl(E) \subset {}_{g\beta^*}Icl(E \cup F)$ and${}_{g\beta^*}Icl(F) \subset {}_{g\beta^*}Icl(E \cup F)$. Hence${}_{g\beta^*}Icl(E) \cup {}_{g\beta^*}Icl(F) \subset {}_{g\beta^*}Icl(E \cup F)$. Let E be a subset of$X$ and A be an ${}_{g\beta^*}I$-closed set containing E. Then by Definition 3.1, ${}_{g\beta^*}Icl(E) \subset A$and ${}_{g\beta^*}Icl\left({}_{g\beta^*}Icl(E)\right) \subset A$. Hence, ${}_{g\beta^*}I$-closure is a Kuratowski closure operator on X if ${}_{g\beta^*}I\mathcal{C}\ell(X,\tau)$ is closed under finite union and intersection.

Definition 3.9: Let $\tau_{{}_{g\beta^*}I}$ be the topology on X generated by ${}_{g\beta^*}I$-closure in the usual manner. That is, $\tau_{{}_{g\beta^*}I} = \{U : {}_{g\beta^*}Icl(U^C) = U^C\}$.

Proposition 3.10:*If*${}_{g\beta^*}I\mathcal{C}\ell(X,\tau)$ *is closed under finite union and intersection, then* $\tau_{{}_{g\beta^*}I}$*is a topology for X.*

Proof: By Theorem 3.8, ${}_{g\beta^*}I$-closure satisfies the Kuratowski closure axioms, $\tau_{{}_{g\beta^*}I}$is a topology on X.

Proof: Clearly follows from the Definition 3.12.

Remark 3.16:*The converse of the Proposition 3.15 is not true as seen from the following example.*

Example 3.17: Let $X = \{a,b,c,d\}$, $\tau = \{\phi, X, \{a,b\}\}$and the ideal$I = \{\phi,\{c\}\}$. For the set $A = \{c,d\}$, ${}_{g\beta^*}I\ int(\{c,d\}) = \{c,d\}$ but $A = \{c,d\}$ is not ${}_{g\beta^*}I$-open.

4. CONCLUSION

In this paper we have defined and proved that the${}_{g\beta^*}I$-closure is a Kuratowski closure operator on the ideal topological space $(X,\ \tau,\ I)$.Furthermore, we have introduced ${}_{g\beta^*}I$-kernel,${}_{g\beta^*}I$-derived set, ${}_{g\beta^*}I$-Border, ${}_{g\beta^*}I$-Frontier and ${}_{g\beta^*}I$-Exterior. In particular its characterization and properties investigated and explored in ideal topological space.

Conflicts of interest : The authors declare that there is no conflicts of interests.

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