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Stability of Ulam's Type and the Existence of Sylvester Matrix Impulsive Volterra Integro-Dynamic System on Time Scale

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Abstract In this paper, the existence and Ulam's type stability for the Sylvester matrix impulsive Volterra integro-dynamic system on a time scale calculus are developed. These results are established using the Banach fixed point theorem.

INTRODUCTION

Integro-differential equations with impulsive matrix dynamical systems have been considered important in various applications, such as physics, biological systems like heartbeats, economics, mechanical systems with impacts, control theory, and so on. The monograph provided by [1] showcases the significance of these systems. The consequence of a comparative system with Q(t)=0 was established by [2], while later, Murty et.al. [3] delved into this aspect further [4].

There are numerous physical problems characterized by unexpected changes in their states, which are referred to as impulsive effects within the system [5]. In the current literature, two types of impulsive dynamical systems are recognized: linear impulsive dynamical systems and nonlinear dynamical systems [6]. Within the context of these unexpected changes, the linear impulsive dynamical system holds significance [7].

Very little attention is given to examinations involving the term of a complete advancement measure, such as shocks and natural disasters, in non-linear impulsive dynamical systems. In the case of these progressions, the span of changes proceeds over a finite time interval and is referenced by [8]. The concept of Hyers-Ulams type stability was introduced in the 19th century and has since gained substantial attention through numerous articles [9].

The qualitative principal focusing on enhancement and the mathematical factor of view is dedicated to the stability analysis of solution to differential equations [10]. The stability of the results of differential equations has been conveyed in parts of articles, as seen in references [11].

Currently, attention is directed towards the study of delta differentiable existence, uniqueness, and Ulam's type stability of the Volterra integro-dynamical systems with Sylvester matrix impulsive on time scales [12].

In sections 2 and 3, the basic techniques of time scales are analyzed,[13] and fundamental concepts for converting a given matrix-valued system into a Kronecker product system are derived using the variation of parameters[14]. The existence and uniqueness stability of the Volterra integro-dynamical system with Sylvester matrix impulsive on the time scale are developed[15].

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$$(1.1) \quad \begin{cases} x^{\Delta}(t) = P(t)x(t) + x(t)Q(t) + \mu(t)A(t)x(t)B(t) \\ \quad + \int_{t_0}^{t}(L_1(t,s)x(s) + x(s)L_2(t,s)) \\ \quad + F(t, X(t)), t \in \mathbb{T}_0 \backslash \{t_k\}_{k=1}^{\infty} \\ x(t_k^+) = (I + D_k)x(t_k), k = 1, 2, \ldots \\ x(t_0) = x_0 \end{cases},$$

where $\mathbb{T}$ has the property unbounded above time scale with bounded graininess, $\mathbb{T}_0 := [t_0, \infty) \cap \mathbb{T}, t_k \in \mathbb{T}_0$ are right dense, $0 \leq t_0 \leq t_1 \leq, \ldots, \leq t_k \leq \ldots, \lim_{k\to\infty} t_k = \infty, x(t_k^-) = \lim_{h\to 0^-} x(t_k - h) and x(t_k^+) = \lim_{h\to 0^+} x(t_k + h), D_k \in M_n(\mathbb{R})$,$x(t) \in M_{n\times n}(\mathbb{R})$ is a state varuable, $F(t, X(t))$ is an $n \times n$ function and $P(t) \in C_{rd}\mathcal{R}\mathbb{M}_{n\times n}(\mathbb{R})$, $Q(t) \in C_{rd}\mathcal{R}\mathbb{M}_{n\times n}(\mathbb{R})$, $L_1(t) \in C_{rd}\mathcal{R}\mathbb{M}_{n\times n}(\mathbb{R})$,and $L_2(t) \in C_{rd}\mathcal{R}\mathbb{M}_{n\times n}(\mathbb{R})$ respectively, $X^{\Delta}(t)$ is the generalized delta derivative of X and $\mu(t)$ is a graininess function.

1. PRELIMINARIES

In 1988, the introduction of time scales calculus was made by Stefan Hilger in his Ph.D. thesis, thereby binding together[16] the analysis of the system in both continuous and discrete domains. In this paper,[17] the notation 'T' is used to denote the time scales calculus. For more detailed information, reference can be made to the textbooks [18] and the research paper [19].

Some fundamental definitions, notations, and useful lemmas are recalled. The Banach space encompassing all continuous functions[20].

norm $||f||_c = sup_{t\in I ||f(t)||}$is denoted by $||.||C(I, \mathbb{R}^n)$ and let $\mathbb{R}^n$be the space of n-dimensional column vectors $x(t) = col(x_1, x_2, \ldots, x_n)$. denotes the Banach space of Lebasque integrable functions from I into $\mathbb{R}^n$ is denoted by $L^1(I, \mathbb{R}^n)$. The Banach space of piecewise continuous functions as $PC(I, \mathbb{R}^n) = x : I \to \mathbb{R}^n : x \in C((t_k, t_k + 1], \mathbb{R}^n), k = 0, 1, 2, \ldots,$ and for some $x(t_k^-)$ and $x(t_k^+)$ For our convenience notation $PC(I, \mathbb{R}^n)$ is $||x||_{PC} = sup_{t\in[a,b]} \frac{||x(t)||}{e_{\Omega}(t,a)}$, for some $\Omega \in \mathbb{R}^+$ Next, we define $PC_{rd}(I, \mathbb{R}^n) = \{x \in PC(I, \mathbb{R}^n)\}$ $PC_{rd}(I, \mathbb{R}^n)$ from a space the supermom norm $||x||_1 = max\{||x||_{PC}, ||x_{\Delta}||_P C\}$.

Definition 2.1. [6] *A nonempty closed subset of* $\mathbb{R}$ *is called a time scale. It is denoted by* $\mathbb{T}$*. We define a T interval as* $[a, b]_{\mathbb{T}} = \{t \in \mathbb{T} : a \leq t \leq b\}$ *accordingly, we define* $(a, b)_{\mathbb{T}}, [a, b)_{\mathbb{T}}, (a, b]_{\mathbb{T}}$ *and so on. Also, we define* $\mathbb{T}^k = \mathbb{T}\{max\mathbb{T}\}$ *if* $max\mathbb{T}$ *exists, otherwise the forward jump operator* $\sigma : \mathbb{T} \to \mathbb{T}$ *is defined by* $\sigma(t) = inf\{s \in \mathbb{T} : s > t\} \in \mathbb{T}$ *with the substitution* $inf\{\emptyset\} = sup\mathbb{T}$ *and The graininess function* $\mu(t) : \mathbb{T} \to [0, \infty) by \mu(t) = \sigma(t) - t, \forall t \in \mathbb{T}$.

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Definition 2.2. [6] *The mapping x from $\mathbb{T} \to \mathbb{R}$ (when τ=sup$\mathbb{T}$, choose τ is not left scattered). The generalized delta derivative of x(t), represented by x^{Δ} (τ), having the nature that, for any $\varepsilon < 0$. There exists a nbd U(τ) implies*

$$|[x(\sigma(\tau)) - x(s)] - x^{\Delta}(\tau)[\sigma(\tau) - s]| \in \varepsilon|\sigma(\tau) - s|,$$

each s$\in U$.

Here x is delta derivative for every $\tau \in \mathbb{T}$; then mapping x from $\mathbb{T}$ to $\mathbb{R}$ is called as generalized derivative on time scales calculus.

Definition 2.3. [6] *The mapping H from $\mathbb{T}^k$ to $\mathbb{R}$ is know as anti-derivative of h from $\mathbb{T}^k$ to $\mathbb{R}$ only if $h^{\Delta}(\tau) = H(\tau)$ fulfilled, for all $\tau \in \mathbb{T}^k$. Then*

$$\int_a^t h(s)\Delta s = H(t) - H(a).$$

2. EXISTENCE AND ULAM'S TYPE STABILITY

Now, the existence and Ulam's type stability for the system (2.1) have been developed by using Banach's fixed point theorem.

Theorem 3.1 states that if the conditions (C1)-(C2) are satisfied, a unique solution for system (2.1) is ensured.

Proof. Let$\mathcal{D} \subseteq PC$ such that $\mathcal{D} = \{z \in PC(I, \mathbb{R}^{n^2}) : ||z||_{PC} \leq \gamma\}$, where $\gamma = \max(LL_g + LL_fT, L||z_0|| + LL_fT, L_g)$.

Now, the operator $\mathcal{G} : \mathcal{D} \to \mathcal{D}$, we have

$$(3.1) \qquad (\mathcal{G}z)(t) = \emptyset(t,0)z_0 + \int_0^t \emptyset(t,\sigma(\tau))f(\tau, z(\tau))\Delta\tau, \forall t \in [0, t_1].$$

$$(3.2) \quad (\mathcal{G}z)(t) = g_k(t, \emptyset(t_k, s_{k-1})([I_n \otimes R_k]z(t_k) + \int_{s_{k-1}}^t \emptyset(t_k, \sigma(\tau))f(\tau, z(\tau))\Delta\tau),$$

$\forall t \in (t_k, s_k]_{\mathbb{T}}, k = 1, 2, \ldots, m.$

$$(3.3) \quad (\mathcal{G}z)(t) = \emptyset(t, s_k)([I_n \otimes R_k]z(t_k) + \int_{s_k}^t \emptyset(t, \sigma(\tau))f(\tau, z(\tau))\Delta\tau), \forall t \in (t_k, s_k]_{\mathbb{T}},$$

$k = 1, 2, \ldots, m.$

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holds. Thus, from Theorem 3.1 has a Ulam Hyer's stable solution which is unique.

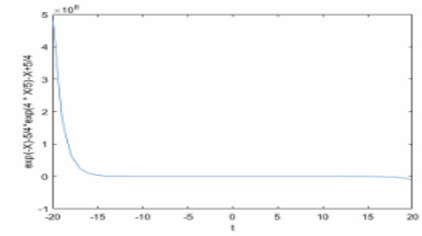

FIGURE 1. Ulam hyer's stability

3. CONCLUSION

Positive non-linear functional analysis was explored, and furthermore, the existence and Ulam's type stability for a Sylvester matrix impulsive Volterra integro-dynamical system (2.1) have been successfully developed using Banach's fixed point theorem on T.

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