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Research Paper

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A STUDY ON GRAPH LABELING IN GRAPH THEORY

*1 .Hari Priya A
M.Phil. Scholar, Department of Mathematics, Bharath Institute of Higher Education and Research, Chennai-72
*2 Dr. V. Vetrivel
Associate Professor, Department of Mathematics, Bharath Institute of Higher Education and Research, Chennai-72.
haripriyaprabhu@yahoo.com . vetrivel.maths@bharathuniv

Address for Correspondence

*1 .Hari Priya A
M.Phil. Scholar, Department of Mathematics, Bharath Institute of Higher Education and Research, Chennai-72
*2 Dr. V. Vetrivel
Associate Professor, Department of Mathematics, Bharath Institute of Higher Education and Research, Chennai-72.
haripriyaprabhu@yahoo.com . vetrivel.maths@bharathuniv

Abstract

A graph labelling trace their origin to one introduced by Rosa. A graph labelling is an assignment of integers to the vertices or the edges, or both, subject to certain conditions. If the domain is the set of vertices we speak about the vertex labelling. If the domain is the set of edges, then the labelling is called the edge labelling. If the labels are assigned to the vertices and also the edge of a graph such a labelling is called total.

A pair sum labeling of a graph G(V,E) is defined as an injective map g from V(G) to $\{\pm1,\pm2\ldots\pm p\}$ is such that $g_e:E(G)\rightarrow Z-\{0\}$ defined by $g_e(uv)=g(u)+g(v)$ is one-one and $g_e(E(G))$ is either of the form $\{\pm k_1\ \pm k_2\ldots\ldots\pm k_{q/2}\}$ or $\{\pm k_1\ \pm k_2\ldots\ldots k_{q-1/2}\}\cup\{k_{q+1/2}\}$ accordingly q is even or odd.

The graph considered here will be finite, undirected and simple.

Keywords:

Labeling, graph, edge, definition, injective, map, etc.,

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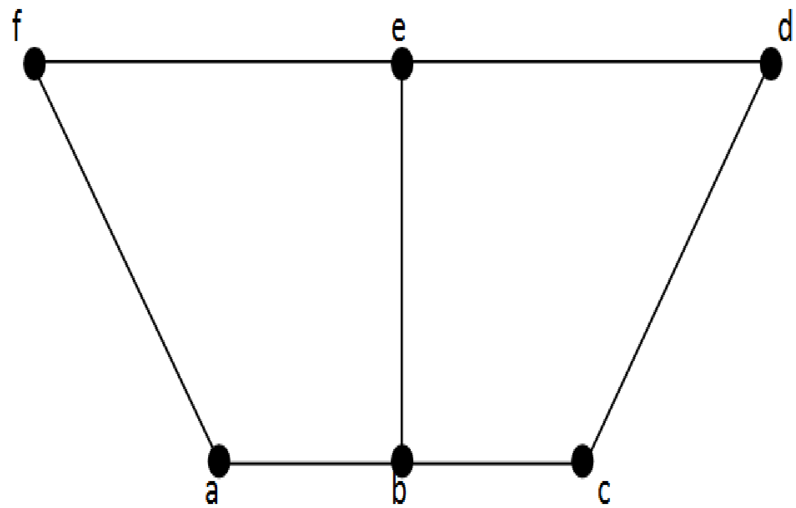
INTRODUCTION

Planar graph

In graph theory, a Planar graph is a graph that can be embedded in the plane, i.e., It can be drawn on the plane in such a way that its edges intersect only at their endpoints. In other words, it's can be drawn in such a way that no edges cross each other.

The graph can be down in the plane without crossing edges such graph are known as planar graph.



Figure

Definition

Complete graph

A Complete graph is a simple undirected graph in which every pair of distinct vertices is connected by a unique edge.

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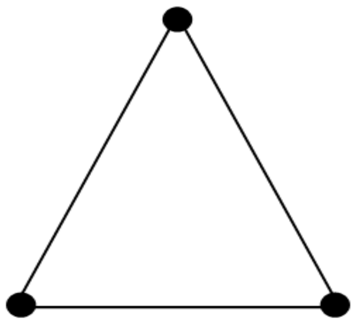
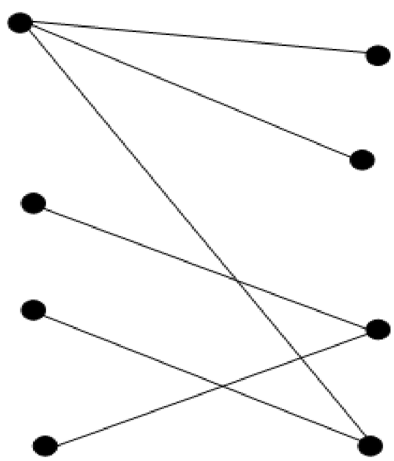


Figure

Definition

Bipartite graph

A graph G is called bi –Partite if it vertex set V(G) can be ÷partitioned into two non-empty disjoint subsets V_1 and V_2 in such a way that each edge e∈E (G) has it one end point in V_1 and other end point in V_2, the partition $V=V_1 \cup V_2$ called the bi partition of G.



Figure

Definition:

Complete bipartite graph

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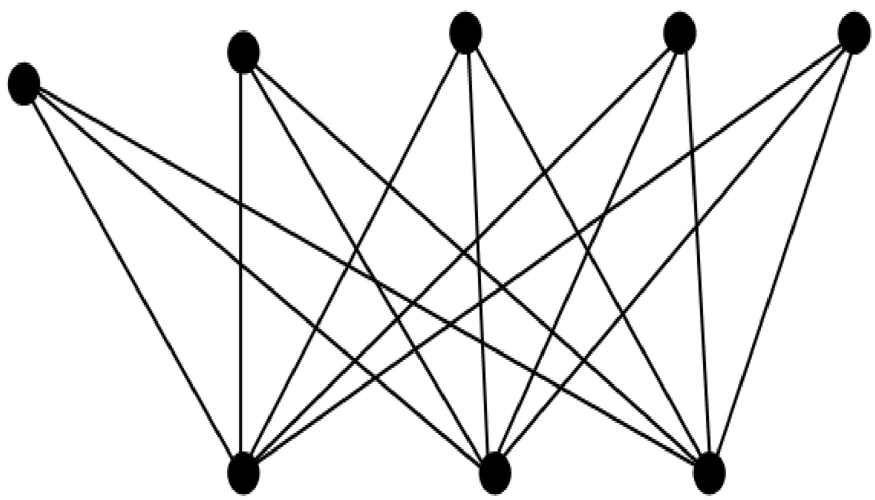
A Complete bi partite graph is a special kind of bi partite graph where every vertex of the first set is connected to the every vertex of the second set.



Figure

Definition

Sub graph

A sub graph S of a graph G is a graph whose set of vertices and set of edges are all subsets of G.

Definition

Degree of vertex

The degree of vertex V in a graph G, written $d_G(v)$ or d(v) is the number of edges incident to V_1 expect that each loop at v counts twice . The maximum degree is A (G) the minimum degree is S(G).

Definition

Regular graph

A graph is said to be regular if all the vertices are of some degree.

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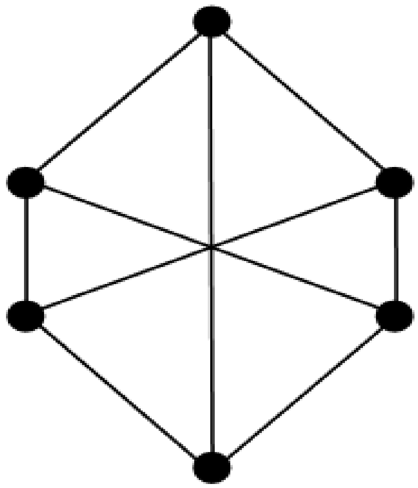

Figure 2.12

Definition

Walk, trail, path and cycle

A **Walk** is an alternating list $v_0,e_1,v_1,e_2.....e_k,v_k$ of vertices and edges such that for $1 \leq i \leq k$, the edge e_i has end points v_{i-1} and v_i.

A **trial** is a walk with no repeated edge.

A **path** is a walk with no repeated vertex.

A closed trial is a circuit. A circuit with no repeated vertex is called a **cycle**

Definition:

Connected graph

A graph in which there is a route of edges and notes between each two nodes.

Definition

Directed graph

A directed graph G consists of a set of directed edges E. Each edges e of E is specified by an ordered pair of vertices u_1, v∈V. A directed graph is simple if it has no loops and no multiple edges.

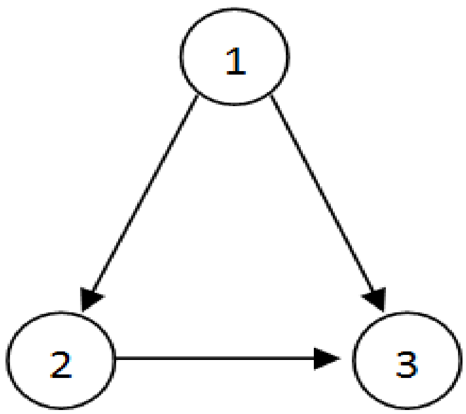

Figure 2.13

Definition

Graph labeling

A graph labeling is the assignment of labels, represented by integers to the edges or vertices or both subject to contain conditions

Definition

Vertex labeling

A graph G, a vertex labeling is a function of V to a set of labels A graph with such a function defined is called a vertex – labeled graph.

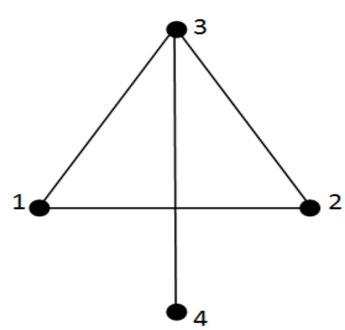

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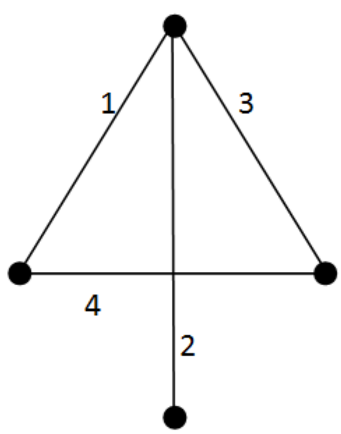
Figure

Definition

Edge Labeling

A edge labeling is a function of E to a set of labels. In this case, the graph is called an edge – labeled graph.



Figure

Theorem

If G is the tree with $V(G)= V(B_{n,m}) \cup \{z_j:1\leq j\leq 3\}$ and

$$E(G)= E(B_{n,m})\cup\{xz_1, z_1 z_2, z_2 z_3, z_3 y\}/\{xy\}.$$

Then G is a pair sum tree.

Proof

Define a function $g:V(G)\rightarrow\{\pm1,\pm2,\ldots,\pm(n+m+5)\}$ βψ

$g(x)= -1, g(y)=3, g(z_1)= -4,$

$g(z_2)=1, g(z_3)=2.$

Case 1): $n=m$.

$g(x_j)= -5-j,\ 1\leq j\leq m$ and

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g $(y_i)=3+j$, $1\leq j\leq m$.

case 2): n>m

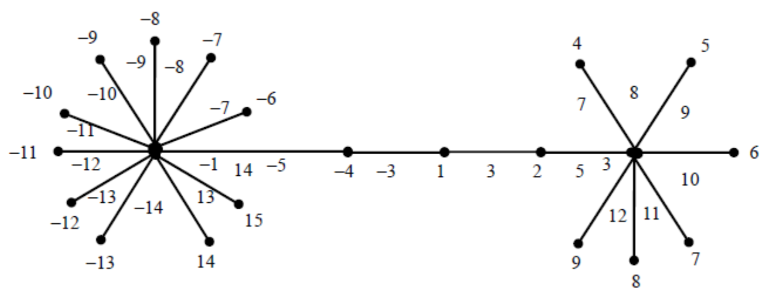
Assign the label to x_j, $y_j(1\leq j\leq m)$ as in case 1.

Define $g(x_{m+j})=-5-m-j$, $1\leq j\leq \lceil \frac{n-m}{2} \rceil$

and $g(x_{\lfloor (n+m)/2 \rfloor + j})=7+m+j$, $1\leq j\leq \lfloor \frac{n-m}{2} \rfloor$.

Then G is a pair sum graph

Illustration of theorem is shown in figure



Figure

Theorem

Let G be the tree with $V(G)=V(B_{n,m})\cup\{z_j:1\leq j\leq 4\}$and

$E(G)=E(B_{n,m})\cup\{xz_1, z_1z_2, z_2y, yz_3, z_3z_4\}/\{xy\}$.Then G is a pair sum graph.

Proof

Define $g: V(G)\rightarrow\{\pm1, \pm2,\ldots, \pm(n+m+6)\}$

by $g(x)=-1$, $g(y)=2$,$g(z_1)=-4$,

$g(z_2)=1$,$g(z_3)=3$,$g(z_4)=4$.

Case 1): $n=m$.

$g(x_1)=-6$,

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$g(x_{j+1}) = -6 - j$, $1 \le j \le m - 1$

and $g(y_j) = 5 + j$, $1 \le j \le m$.

case 2): n>m

Assign the label to x_j, $y_j (1 \le j \le m)$ as in case 1.

Define $g(x_{m+j}) = -5 - m - j$, $1 \le j \le \lceil \frac{n-m}{2} \rceil$ and

$g(x_{\lceil (n+m)/2 \rceil + j}) = 8 + m + j$, $1 \le j \le \lfloor \frac{n-m}{2} \rfloor$.

case 3): n<m

Assign the label to x_j, $y_j (1 \le j \le m)$ as in case 1.

Define $g(y_{m+j}) = -8 - n - j$, $1 \le j \le \lceil \frac{m-n}{2} \rceil$ and

$g(y_{\lceil (n+m)/2 \rceil + j}) = 5 + n + j$, $1 \le j \le \lfloor \frac{m-n}{2} \rfloor$.

Then G is a pair sum graph.

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