

Evaluation the suitable probability distribution for the occurrence pattern of insect pest on green gram (*Vigna radiata*).

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Key Word- chi-square, green gram, passion distribution, Polya-Aeppli distribution, probability distribution.

Abstract- This study based on Evaluation the suitable probability distribution for the occurrence pattern of insect pest on green gram (*Vigna radiata*), the numbers of larvae insects were counted weekly on each plant. The data were taken from an experiment which was conducted by Entomology Department at Breeding Seed Production unit of Plant Breeding and Department, JNKVV Jabalpur M.P. In the experiment there are 3 replications each consist 13 treatments, from each treatment 10 plants were selected randomly and tagged for the recording of the number of major insect pests consists larvae insects (whitefly, leaf hopper, aphid, *spodoptera litura* and blue butterfly). Thus overall, 390 plants were selected and the data were recorded once in a standard week. Treatment details T₁-TM-37, T₂- TJM-160, T₃- TJM-196, T₄- Sikha, T₅- PDM-139, T₆- TJM-140, T₇- Virat, T₈- TM-115, T₉- TJM-141, T₁₀-TJM-136, T₁₁-TJM-111, T₁₂- TJM-155, T₁₃- TJM-137. The method of proportion of zero and method of moments were found to be suitable for estimating the parameters in the concerned distributions.

Introduction- The green gram (*Vigna radiata*), alternatively known as the mung bean, maash, or moong is a plant species in the legume family. The mung bean is mainly cultivated in India, Pakistan, Bangladesh, Nepal, Sri Lanka, China, Taiwan, Korea, South Asia and Southeast Asia. It is used as an ingredient in both savory and sweet dishes. It is a good source of proteins, carbohydrates and vitamins for the human race all over the world. Mung bean (*Vigna radiata* L Wilczek) belongs to the family leguminoceae and sub family papilionaceae, is being grown as one of the principal crops since ages in our state as well as in the country. The annual world production area of mungbean is about 5.5 million hectares. India is the primary green gram producer and contributes about 75% of the world's production (Taunk *et al.*, 2012).

It is one of the major *Kharif* pulse crops in India covering 34.4 lakh hectare of area in the country with total production of 14 lakh tonnes and productivity of 415.70 kilogram per hectare (Anonymous, 2015). It is highly nutritious pulse crop having nearly 24 to 25% protein in seed. It is commonly grown in rainy and summer seasons in central India.

Material Methods- This study the occurrence pattern of major insect pest in green gram and seasonal fluctuations in relation to yield, the numbers of larvae insects were counted weekly on each plant. The data were taken from an experiment which was conducted by Entomology Department at Breeding Seed Production unit of Plant Breeding and Department, JNKVV Jabalpur M.P. and the weather report is taken from the Agro-Meteorological Department of year 2018. The sowing of green gram was done on 14th August 2018. The germination started on 18 August 2018, 2-4 leaves appeared on 25 August 2018, flowering started on 20-24 September 2018 and podding start on 28-30 September 2018. The green gram crop was harvested on 29 October 2018. The data on occurrence of major larvae insects were gathered since the incidence of pest. In the experiment there are 3 replications each consist 13 treatments, from each treatment 10 plants were selected randomly and tagged for the recording of the number of major insect pests consists larvae insects (whitefly, leaf hopper, aphid, *spodoptera litura* and blue butterfly). Thus overall, 390 plants were selected and the data were recorded once in a standard week.

In order to study the occurrence pattern of major insect pest of green gram and seasonal fluctuations in relation to yield, the numbers of larvae insects (whitefly, leaf hopper, aphid, *spodoptera litura* and blue butterfly) were counted weekly on each plant. The original counts were then summarized in the term of frequency distribution showing the number of plants containing $X=0,1,2,3,\dots$ insect pests of given species. To see the chance of containing the major insect-pests during the study period, the probable probability distributions would be Poisson and Polya-Aeppli distribution so these two distributions were preferred in this thesis.

Poisson Distribution

Poisson distribution is a discrete probability distribution and is very widely used in statistical analysis. It was developed by a French mathematician Simeon Denis Poisson (1781-1840) in 1837 and he approached the distribution by considering limiting form of the binomial distribution. Poisson distribution may be expected in cases where the chance of any individual event being a success is small. It is also called the probability distribution of rare events. In recent years the Poisson distribution has been used to way markedly increasing number of applications in the great majority of cases. Poisson distribution often serves as a standard form.

A random variable X is said to follow a Poisson distribution if it assumes only non-negative values and its probability mass function is given by:

$$p(x, \lambda) = P(X = x) = \begin{cases} \frac{e^{-\lambda} \lambda^x}{x!}; & x = 0, 1, 2, \dots; \lambda > 0 \\ 0, & \text{otherwise} \end{cases}$$

Estimation

The distribution contains one parameter λ . It is estimated by following methods given below:

i. Methods of proportion of Zero Cell (method-I)

It is estimated by equating proportion of zero cell with their corresponding theoretical values.

$$e^{-\lambda} = \frac{n_0}{N}$$

$$\lambda = -\ln (n_0/N)$$

ii. Method of Moments (method-II)

The parameter λ in Poisson distribution is estimated by method of moments which is given below:

$$\lambda = m'_1$$

3.2 Polya-Aeppli Distribution

Polya-Aeppli process was introduced by Minkova (2004) as a compound Poisson process with geometric compounding distribution. It is a generalization of homogeneous Poisson process and is used to model over-dispersed count data. In order to allow for lack of homogeneity, some random variation is introduced in the parameter λ (see Minkova (2013) of Polya-Aeppli process. This distribution is one of the important contagious distributions and is useful for the situation where events (which are to be counted) occur in clusters and the number of clusters follows a Poisson distribution with expectation θ , and the number of individuals per clusters follows a geometric distribution with parameter q .

For completeness, this distribution is defined by

$$P_0 = P[X = 0] = e^{-\theta}$$

$$P_k = P[X = k] = e^{-\theta} p^k \sum_{j=1}^k \binom{k-1}{j-1} \left(\frac{\theta q}{p} \right)^j$$

$$(k \geq 1); q = 1 - p$$

Polya-Aeppli distribution was described by Polya (1931). He ascribed the derivation of the distribution to Aeppli (1924) in a thesis.

Estimation

This distribution consists of two parameters θ and q and these are estimated by following methods given below:

i. Methods of proportion of Zero Cell (method- I)

In this method, the observed proportion of zeroes (n_0/N) and sample mean (m'_1) are equated to their corresponding theoretical values. It is given below:

$$e^{-\theta} = \frac{n_0}{N}$$

and

$$m'_1 = \frac{\theta}{q}$$

The parameters θ and q were estimated from the above relationships.

ii. Method of Moments (method-II)

In this method, these two parameters were estimated by equating the observed mean and observed variance with their corresponding theoretical values. It is given below:

$$m'_1 = \frac{\theta}{q}$$

$$m_2 = \frac{\theta(1+p)}{q^2}$$

where, m'_1 and m_2 are the sample mean and sample variance of the observed data respectively (Johnson and Kotz, 1969).

Chi-square test

In order to test the adequacy of the model we use Chi-square test for testing the significance of the discrepancy between theory and experiment. It was given by Prof. Karl Pearson in 1900 and is known as "Chi-square test of goodness of fit".

If f_i ($i=1, 2, \dots, n$) is a set of observed (experimental) frequencies and e_i ($i=1, 2, \dots, n$) is the corresponding set of expected (theoretical or hypothetical) frequencies, then Karl Pearson's chi-square, given by:

$$\chi^2 = \sum_{i=1}^n \left[\frac{(f_i - e_i)^2}{e_i} \right]$$

follows chi-square distribution with $(n-1)$ d.f.

This is an approximate test for large values of n . For the validity of chi-square test of 'goodness of fit' between theory and experiment, the following conditions must be satisfied:

- i. The sample observations should be independent.
- ii. Constraints on the cell frequencies, if any, should be linear,
e.g. $\sum O_i = \sum E_i$
- iii. N , the total frequency should be reasonably large, say, greater than 50.
- iv. No theoretical cell frequency should be less than 5.

If any theoretical cell frequency is less than 5, then for the application of chi-square test, it is pooled with the preceding or succeeding frequency so that the pooled frequency is more than 5 and finally adjusts for the d.f. lost in pooling.

The goodness of fit test uses the chi-square distribution to determine if a hypothesized probability distribution for a population provides a good fit. Acceptance or rejection of the hypothesized population distribution is based upon differences between observed frequencies (f_i 's) in a sample and the expected frequencies (e_i 's) obtained under null hypothesis H_0 .

Result and Discussion- In order to study the occurrence pattern of major insect pest of green gram and seasonal fluctuations in relation to yield, the numbers of larvae insects (whitefly, leaf hopper, aphid, *spodoptera litura* and blue butterfly) were counted weekly on each plant. The original counts were then summarized in the term of frequency distribution showing the number of plants containing $X=0,1,2,3,\dots$ insect pests of given species. To see the chance of containing the major insect-pests during the study period, the probable probability distributions would be Poisson and Polya-Aeppli distribution so these two distributions were preferred in this thesis.

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Result and Discussion-

Population dynamics of insect pest on green gram

In order to describe the behaviour pattern of insect pest according to different DAS on green gram, the total insects were collected as per the treatments which are given in the following tables.

Table No.- 1 Distribution of observed and expected number of green gram plants according to number of insect pest at 15 DAS of the treatment 1 (TM-37), treatment 2 (TJM-160), treatment 3 (TJM-196).

No. of major insect	Observed freq. treatment 1 (TM-37).	Poisson Distribution treatment 1 (TM-37).		Observed freq. treatment 2 (TJM-160)	Poisson Distribution treatment 2 (TJM-160)		Observed freq. treatment 3 (TJM-196)	Poisson Distribution treatment 3 (TJM-196)	
		MPZC	MM		MPZC	MM		MPZC	MM
0	82	82.00	82.32	76	76.00	75.24	77	77.00	79.89
1	48	49.53	49.39	48	51.67	51.91	54	51.35	50.33
2	18	14.95	14.82	23	17.56	17.91	17	17.12	15.86
3	2	3.51	3.46	3	4.75	4.93	2	4.54	3.93
Total	150	150.00	150.00	150	150.00	150.00	150	150.00	150.00
Mean	0.60	-	-	0.69	-	-	0.63	-	-
Variance	0.52	-	-	0.64	-	-	0.54	-	-

Estimate of parameter λ	0.60	0.60	Estimate of parameter λ	0.68	0.69	Estimate of parameter λ	0.67	0.63
χ^2	0.18	0.20	χ^2	0.87	0.74	χ^2	0.46	0.41
d.f.	1	1	d.f.	1	1	d.f.	1	1

Table-1 revealed that the insect pest related to green gram plants follows a Poisson distribution because during 15 DAS of the first treatment (TM-37) the mean and variance of the distribution were found to be same. The risk parameter λ indicated 0.60 in both MPZC and MM. The values of χ^2 were non-significant in both methods. Second treatment (TJM-160) the mean and variance of the distribution were found to be same. The risk parameter λ indicated 0.68 and 0.69 respectively in MPZC and MM. The values of χ^2 were non-significant in both methods. And third treatment (TJM-196) the mean and variance of the distribution were found to be same. The risk parameter λ indicated 0.67 and 0.63 respectively in MPZC and MM. The values of χ^2 were non-significant in both methods.

Table No.-2 Distribution of observed and expected number of green gram plants according to number of insect pest at 15 DAS of the treatment 4 (Shikha), treatment 5 (PDM-139), treatment 6 (TJM-140).

No. of major insect	Observed freq. treatment 4 (Shikha)	Poisson Distribution treatment 4 (Shikha)		Observed freq. treatment 5 (PDM-139)	Poisson Distribution treatment 5 (PDM-139)		Observed freq. treatment 6 (TJM-140)	Poisson Distribution treatment 6 (TJM-140)	
		MPZC	MM		MPZC	MM		MPZC	MM
0	79	79.00	83.98	84	84.00	83.98	71	71.00	72.28
1	56	50.65	48.71	47	48.70	48.71	53	53.10	52.77
2	14	16.25	14.13	17	14.12	14.13	22	19.86	19.26
3	1	4.09	3.18	2	3.18	3.18	4	6.05	5.68
Total	150	150.00	150.00	150	150.00	150.00	150	150.00	150.00
Mean	0.58	-	-	0.58	-	-	0.73	-	-
Variance	0.47	-	-	0.55	-	-	0.65	-	-
Estimate of parameter λ		0.64	0.58	Estimate of parameter λ	0.58	0.58	Estimate of parameter λ	0.75	0.73
χ^2		1.97	1.69	χ^2	0.97	0.96	χ^2	0.01	0.07
d.f.		1	1	d.f.	1	1	d.f.	1	1

Table-2 focused that the insect pest related to green gram plants follows a Poisson distribution because during 15 DAS of the fourth treatment (Shikha) the mean and variance of the distribution were found to be same. The risk parameter λ indicated 0.64 and 0.58 respectively in MPZC and MM. The values of χ^2 were non-significant in both methods. Fifth treatment (PDM-139) the mean and variance of the distribution were found to be same. The risk parameter λ indicated 0.58 in both MPZC and MM in fifth treatment (PDM-139) during 15 DAS. The values of χ^2 were non-significant in both methods. And sixth treatment (TJM-140) the mean and variance of the distribution were found to be same. The risk parameter λ indicated 0.75 and 0.73 respectively in MPZC and MM. The values of χ^2 were non-significant in both methods.

Table No.-3 Distribution of observed and expected number of green gram plants according to number of insect pest at 15 DAS of the treatment⁷ (Virat), treatment 8 (TM-115), treatment 9 (TJM-141).

No. of major insect	Observed freq. treatment 7 (Virat)	Poisson Distribution treatment 7 (Virat)		Observed freq. treatment 8 (TM-115)	Poisson Distribution treatment 8 (TM-115)		Observed freq. treatment 9 (TJM-141)	Poisson Distribution treatment 9 (TJM-141)	
		MPZC	MM		MPZC	MM		MPZC	MM
0	83	83.00	82.32	75	75.00	75.24	78	78.00	78.32
1	46	49.11	49.39	50	51.99	51.92	49	51.00	50.91
2	19	14.54	14.82	22	18.02	17.91	20	16.68	16.55
3	2	3.36	3.46	3	4.99	4.94	3	4.32	4.23
Total	150	150.00	150.00	150	150.00	150.00	150	150.00	150.00
Mean	0.6	-	-	0.69	-	-	0.65	-	-
Variance	0.57	-	-	0.62	-	-	0.62	-	-
Estimate of parameter λ		0.59	0.60	Estimate of parameter λ	0.69	0.69	Estimate of parameter λ	0.65	0.65
χ^2		0.74	0.64	χ^2	0.25	0.27	χ^2	0.27	0.31
d.f.		1	1	d.f.	1	1	d.f.	1	1

Table-3 indicated that the insect pest related to green gram plants follows a Poisson distribution because during 15 DAS of the seventh treatment (virat) the mean and variance of the distribution were found to be same. The risk parameter λ indicated 0.59 and 0.60 respectively in MPZC and MM. The values of χ^2 were non-significant in both methods. Eighth treatment (TM-115) the mean and variance of the distribution were found to be same. The risk parameter λ indicated 0.69 in both MPZC and MM. The values of χ^2 were non-significant in both methods. And ninth treatment (TJM-141) the mean and variance of the distribution were found to be same. The risk

parameter λ indicated 0.65 in both MPZC and MM. The values of χ^2 were non-significant in both methods.

Table No.-4 Distribution of observed and expected number of green gram plants according to number of insect pest at 15 DAS of the treatment 10 (TJM-136), treatment 11 (TJM-111), treatment 12 (TJM-155), treatment 13 (TJM-137).

No. of major insect	Observed freq. treatment 10 (TJM-136)	Poisson Distribution treatment 10 (TJM-136)		Observed freq. treatment 11 (TJM-111)	Poisson Distribution treatment 11 (TJM-111)		Observed freq. treatment 12 (TJM-155)	Poisson Distribution treatment 12 (TJM-155)		Observed freq. treatment 13 (TJM-137)	Poisson Distribution treatment 13 (TJM-137)	
		MPZC	MM		MPZC	MM		MPZC	MM		MPZC	MM
0	80	80.00	82.32	82	82.00	83.99	70	70.00	70.86	83	82.99	81.51
1	52	50.28	49.39	51	49.53	48.70	50	53.36	53.15	45	49.11	49.73
2	16	15.81	14.82	15	14.96	14.13	27	20.34	19.94	19	14.54	15.16
3	2	3.92	3.47	2	3.51	3.18	3	6.30	6.06	3	3.36	3.60
Total	150	150.00	150.00	150	150.00	150.00	150	150.00	150.00	150	150.00	150.00
Mean	0.6	-	-	0.58	-	-	0.75	-	-	0.61	-	-
Variance	0.53	-	-	0.52	-	-	0.67	-	-	0.61	-	-
Estimate of parameter λ		0.62	0.60	Estimate of parameter λ	0.60	0.58	Estimate of parameter λ	0.76	0.75	Estimate of parameter λ	0.59	0.61

χ^2	0.21	0.20	χ^2	0.16	0.16	χ^2	1.47	0.81	χ^2	1.28	1.04
d.f.	1	1	d.f.	1	1	d.f.	1	1	d.f.	1	1

Table-4 focused that the insect pest related to green gram plants follows a Poisson distribution because during 15 DAS of the tenth treatment (TJM-136) the mean and variance of the distribution were found to be same. The risk parameter λ indicated 0.62 in MPZC and 0.60 in MM. The values of χ^2 were non-significant in both methods. Eleventh treatment (TJM-111) the mean and variance of the distribution were found to be same. The risk parameter λ indicated 0.60 and 0.58 respectively in MPZC and MM. The values of χ^2 were non-significant in both methods. Twelfth treatment (TJM-155) the mean and variance of the distribution were found to be same. The risk parameter λ indicated 0.76 and 0.75 respectively in MPZC and MM. The values of χ^2 were non-significant in both methods. And thirteenth treatment (TJM-137) the mean and variance of the distribution were found to be same. The risk parameter λ indicated 0.59 and 0.61 respectively in MPZC and MM. The values of χ^2 were non-significant in both methods.

Table No.- 5 Distribution of observed and expected number of green gram plants according to number of insect pest at 15 DAS of the treatment 1 (TM-37), treatment 2 (TJM-160), treatment 3 (TJM-196).

No. of major insect	Observed freq. treatment 1 (TM-37).	Poisson Distribution treatment 1 (TM-37).		Observed freq. treatment 2 (TJM-160)	Poisson Distribution treatment 2 (TJM-160)		Observed freq. treatment 3 (TJM-196)	Poisson Distribution treatment 3 (TJM-196)	
		MPZC	MM		MPZC	MM		MPZC	MM
0	65	65.00	64.50	62	62.00	55.95	64	64.00	69.21
1	37	38.76	39.00	36	37.50	43.50	37	38.93	34.80
2	20	22.80	22.50	22	22.80	26.10	22	23.04	20.93
3	15	12.00	12.00	18	12.75	13.38	15	12.44	11.85
4	8	6.00	6.00	7	6.00	5.93	7	6.24	6.41
5	5	5.54	6.00	5	8.95	5.14	5	5.35	6.80
Total	150	150.00	150.00	150	150.00	150.00	150	150.00	150.00
Mean	1.19	-	-	1.25	-	-	1.19	-	-
Variance	1.95	-	-	1.92	-	-	2.46	-	-
Estimates of parameters	θ	0.84	0.85	θ	0.88	0.99	θ	0.85	0.77
	q	0.71	0.71	q	0.70	0.79	q	0.71	0.65
χ^2		1.89	1.97	χ^2	4.16	4.37	χ^2	χ^2	0.79
d.f.		3	3	d.f.	3	3	d.f.	d.f.	3

Table-5 focused that the insect pest related to green gram plants follows a Polya-Aeppli distribution because during 22 DAS of the first treatment (TM-37) the mean was smaller than variance of the distribution. The estimates of θ were 0.84 and 0.85 and the estimates of q were 0.71 and 0.71

respectively in MPZC and MM. The values of χ^2 were non-significant in both methods. Second treatment (TJM-160) the mean was smaller than variance of the distribution. The estimates of θ were 0.88 and 0.99 and the estimates of q were 0.70 and 0.79 respectively in MPZC and MM. The values of χ^2 were non-significant in both methods. And third treatment (TJM-196) the mean was smaller than variance of the distribution. The estimates of θ were 0.85 and 0.77 and the estimates of q were 0.71 and 0.65 respectively in MPZC and MM. The values of χ^2 were non-significant in both methods.

Table No.-6 Distribution of observed and expected number of green gram plants according to number of insect pest at 22 DAS of the treatment 4 (Shikha), treatment 5 (PDM-139), treatment 6 (TJM-140).

No. of major insect	Observed freq. treatment 4 (Shikha).	Poisson Distribution treatment 4 (Shikha).		Observed freq. treatment 5 (PDM-139)	Poisson Distribution treatment 5 (PDM-139)		Observed freq. treatment 6 (TJM-140)	Poisson Distribution treatment 6 (TJM-140)	
		MPZC	MM		MPZC	MM		MPZC	MM
0	62	62.00	57.03	65	65.00	60.55	62	62.00	56.58
1	37	38.43	43.02	37	40.64	44.50	36	38.71	43.04
2	21	23.15	25.69	24	22.96	24.81	22	16.91	25.83
3	18	12.76	13.31	17	10.99	11.82	17	12.74	13.44
4	7	6.57	6.35	4	5.18	4.85	8	6.50	6.48
5	5	7.09	4.06	2	2.16	2.04	5	13.14	4.63
Total	150	150.00	150.00	150	150.00	150.00	150	150.00	150.00
Mean	1.24	-	-	1.12	-	-	1.25	-	-
Variance	1.93	-	-	1.65	-	-	1.97	-	-
Estimates of parameters	θ	0.88	0.97	θ	0.84	0.91	θ	0.88	0.98
	q	0.71	0.78	q	0.75	0.81	q	0.71	0.78
χ^2		3.05	3.89	χ^2	4.18	4.10	χ^2	8.53	3.57
d.f.		3	3	d.f.	3	3	d.f.	3	3

Table-6 focused that the insect pest related to green gram plants follows a Polya-Aeppli distribution because during 22 DAS of the fourth treatment (Shikha) the mean was smaller than variance of the distribution. The estimates of θ were 0.88 and 0.97 and the estimates of q were 0.71 and 0.78 respectively in MPZC and MM. The values of χ^2 were non-significant in both methods. Fifth treatment (PDM-139) the mean was smaller than variance of the distribution. The estimates of θ were 0.84 and 0.91 and the estimates of q were 0.75 and 0.81 respectively in MPZC and MM. The values of χ^2 were non-significant in both methods. And sixth treatment (TJM-140) the mean was smaller than variance of the distribution. The estimates of θ were 0.88 and 0.98 and the estimates of q were 0.71 and 0.78

respectively in MPZC and MM. The values of χ^2 was non-significant in MPZC at 1% level of significance and in MM at 5% level of significance.

Table No.-7 Distribution of observed and expected number of green gram plants according to number of insect pest at 22 DAS of the treatment 7 (Virat), treatment 8 (TM-115), treatment 9 (TJM-141).

No. of major insect	Observed freq. treatment 7 (Virat).	Poisson Distribution treatment 7 (Virat).		Observed freq. treatment 8 (PDM-139)	Poisson Distribution treatment 8 (PDM-139)		Observed freq. treatment 9 (TJM-140)	Poisson Distribution treatment 9 (TJM-140)	
		MPZC	MM		MPZC	MM		MPZC	MM
0	64	64.00	57.74	63	63.00	55.88	65	65.00	60.36
1	34	37.83	42.45	34	38.37	43.59	34	34.10	40.65
2	21	22.82	25.37	22	23.19	26.16	22	24.47	24.26
3	19	12.66	13.25	19	12.84	13.49	17	12.66	12.95
4	7	6.69	6.48	8	6.69	6.09	6	6.68	6.27
5	5	6.00	4.71	4	5.91	4.79	4	3.56	2.52
Total	150	150.00	150.00	150	150.00	150.00	150	150.00	150.00
Mean	1.24	-	-	1.25	-	-	1.23	-	-
Variance	1.97	-	-	1.91	-	-	2.11	-	-
Estimates of parameters	θ	0.85	0.95	θ	0.87	0.99	θ	0.84	0.91
	q	0.69	0.77	q	0.70	0.79	q	0.68	0.74
χ^2		3.90	5.67	χ^2	3.54	6.05	χ^2	2.86	2.98
d.f.		3	3	d.f.	2	2	d.f.	3	3

Table-7 focused that the insect pest related to green gram plants follows a Polya-Aeppli distribution because during 22 DAS of the seventh treatment (Virat) the mean was smaller than variance of the distribution. The estimates of θ were 0.85 and 0.95 and the estimates of q were 0.69 and 0.77 respectively in MPZC and MM. The values of χ^2 were non-significant in both methods. Eighth treatment (TM-115) the mean was smaller than variance of the distribution. The estimates of θ were 0.87 and 0.99 and the estimates of q were 0.70 and 0.79 respectively in MPZC and MM. The values of χ^2 was non-significant in MPZC at 5% level of significance and in MM at 1% level of significance. And ninth treatment (TJM-141) the mean was smaller than variance of the distribution. The estimates of θ were 0.84 and 0.91 and the estimates of q were 0.68 and 0.74 respectively in MPZC and MM. The values of χ^2 were non-significant in both methods.

Table No.-8 Distribution of observed and expected number of green gram plants according to number of insect pest at 22 DAS of the treatment 10 (TJM-136), treatment 11 (TJM-111), treatment 12 (TJM-155), treatment 13 (TJM-137).

No. of major insect	Observed freq. treatment 10 (TJM-136).	Poisson Distribution treatment 10 (TJM-136).		Observed freq. treatment 11 (TJM-111)	Poisson Distribution Treatment 11 (TJM-111)		Observed freq. treatment 12 (TJM-155)	Poisson Distribution treatment 12 (TJM-155)		Observed freq. treatment 13 (TJM-137)	Poisson Distribution treatment 13 (TJM-137)	
		MPZC	MM		MPZC	MM		MPZC	MM		MPZC	MM
0	64	64.00	58.14	61	61.00	56.16	61	61.00	56.58	67	67.00	61.88
1	36	39.75	44.63	36	37.89	41.38	36	37.89	41.37	35	37.53	42.19
2	24	23.07	25.61	21	23.51	25.59	21	23.51	25.47	20	21.77	24.09
3	17	12.12	12.50	19	13.33	13.79	19	13.33	13.84	18	11.69	12.12
4	5	6.03	5.45	6	7.22	7.19	7	7.22	6.92	6	5.88	5.84
5	4	5.03	3.67	5	3.41	3.35	4	3.41	3.26	4	4.06	5.20
Total	150	150.00	150.00	150	150.00	150.00	150	150.00	150.00	150	150.00	150.00
Mean	1.17	-	-	1.25	-	-	1.23	-	-	1.15	-	-
Variance	1.73	-	-	1.91	-	-	2.11	-	-	1.83	-	-
Estimates of parameters	θ	0.85	0.95	θ	0.90	0.98	θ	0.90	0.98	θ	0.80	0.89
	q	0.73	0.81	q	0.69	0.75	q	0.69	0.75	q	0.70	0.77
χ^2		2.74	4.14	χ^2	2.98	4.29	χ^2	2.94	3.76	χ^2	4.46	5.17
d.f.		2	2	d.f.	3	3	d.f.	3	3	d.f.	2	2

Table-8 indicated that the insect pest related to green gram plants follows a Polya-Aeppli distribution because during 22 DAS of the tenth treatment (TJM-136) the mean was smaller than variance of the distribution. The estimates of θ were 0.85 and 0.95 and the estimates of q were 0.75 and 0.81 respectively in MPZC and MM. The values of χ^2 were non-significant in both methods. Eleventh treatment (TJM-111) the mean was smaller than variance of the distribution. The estimates of θ were 0.90 and 0.98 and the estimates of q were 0.69 and 0.75 respectively in MPZC and MM. The values of

χ^2 were non-significant in both methods. And thirteenth treatment (TJM-137) the mean was smaller than variance of the distribution. The estimates of θ were 0.80 and 0.89 and the estimates of q were 0.70 and 0.77 respectively in MPZC and MM. The values of χ^2 were non-significant in both methods.

Table No.- 9 Distribution of observed and expected number of green gram plants according to number of insect pest at 29 DAS of the treatment 1 (TM-37), treatment 2 (TJM-160), treatment 3 (TJM-196).

No. of major insect	Observed freq. treatment 1 (TM-37).	Poisson Distribution treatment 1 (TM-37).		Observed freq. treatment 2 (TJM-160)	Poisson Distribution treatment 2 (TJM-160)		Observed freq. treatment 3 (TJM-196)	Poisson Distribution treatment 3 (TJM-196)	
		MPZC	MM		MPZC	MM		MPZC	MM
0	60	60.00	59.11	60	60.00	54.41	60	60.00	55.91
1	37	36.29	36.88	35	34.08	38.07	33	35.73	38.63
2	21	23.31	23.67	20	22.64	25.13	22	23.15	24.93
3	15	13.88	14.00	19	14.12	15.03	18	13.94	14.55
4	7	7.76	7.83	8	9.20	8.42	9	7.98	7.94
5	5	4.20	4.20	5	4.85	4.49	4	3.65	4.13
6	3	2.16	2.21	3	2.72	2.30	3	2.34	2.06
7	2	2.40	2.10	2	2.39	2.15	1	3.21	1.85
Total	150	150.00	150.00	150	150.00	150.00	150	150.00	150.00
Mean	1.39	-	-	1.47	-	-	1.41	-	-
Variance	2.73	-	-	2.81	-	-	2.57	-	-
Estimates of parameters	θ	0.92	0.93	θ	0.92	1.01	θ	0.92	0.99
	q	0.66	0.67	q	0.62	0.69	q	0.65	0.70
χ^2		0.60	0.74	χ^2	2.38	3.07	χ^2	1.74	2.42
d.f.		4	4	d.f.	4	4	d.f.	3	3

Table-9 focused that the insect pest related to green gram plants follows a Polya-Aeppli distribution because during 29 DAS of the first treatment (TM-37) the mean was smaller than variance of the distribution. The estimates of θ were 0.92 and 0.93 and the estimates of q were 0.66 and 0.67 respectively in MPZC and MM. The values of χ^2 were non-significant in both methods. Second treatment (TJM-160) the mean was smaller than variance of the distribution. The estimates of θ were 0.92 and 1.01 and the estimates of q were 0.62 and 0.69 respectively in MPZC and MM. The values of χ^2 were non-significant in both methods. And third treatment (TJM-196) the mean was smaller than variance of the distribution. The estimates of θ were 0.92 and 0.99 and the estimates of q were 0.65 and 0.70 respectively in MPZC and MM. The values of χ^2 were non-significant in both methods.

Table No.-10 Distribution of observed and expected number of green gram plants according to number of insect pest at 29 DAS of the treatment 4 (Shikha), treatment 5 (PDM-139), treatment 6 (TJM-140).

No. of major insect	Observed freq. treatment 4 (Shikha).	Poisson Distribution treatment 4 (Shikha).		Observed freq. treatment 5 (PDM-139)	Poisson Distribution treatment 5 (PDM-139)		Observed freq. treatment 6 (TJM-140)	Poisson Distribution treatment 6 (TJM-140)	
		MPZC	MM		MPZC	MM		MPZC	MM
0	60	60.00	57.57	60	60.00	58.48	60	60.00	54.74
1	36	37.94	39.69	34	31.89	33.05	32	36.29	39.72
2	24	23.75	24.80	20	21.87	22.56	25	23.31	25.53
3	15	13.61	13.92	14	14.34	14.52	17	13.88	14.67
4	8	7.35	7.28	7	9.05	8.96	8	7.82	7.82
5	4	3.96	3.60	6	5.46	5.36	4	4.25	3.95
6	2	1.86	1.71	4	3.29	3.12	3	2.22	1.92
7	1	1.53	1.43	3	1.88	1.77	1	2.23	1.65
Total	150	150.00	150.00	150	150.00	150.00	150	150.00	150.00
Mean	1.33	-	-	1.57	-	-	1.40	-	-
Variance	2.34	-	-	3.68	-	-	2.51	-	-
Estimates of parameters	θ	0.92	0.96	θ	0.92	0.94	θ	0.92	1.01
	q	0.69	0.72	q	0.58	0.60	q	0.66	0.72
χ^2		0.32	0.64	χ^2	2.33	1.41	χ^2	1.39	2.42
d.f.		3	3	d.f.	4	4	d.f.	3	3

Table-10 indicated that the insect pest related to green gram plants follows a Polya-Aeppli distribution because during 29 DAS of the fourth treatment (Shikha) the mean was smaller than variance of the distribution. The estimates of θ were 0.92 and 0.96 and the estimates of q were 0.50 and 0.72 respectively in MPZC and MM. The values of χ^2 were non-significant in both methods. Fifth treatment (PDM-139) the mean was smaller than variance of the distribution. The estimates of θ were 0.92 and 0.94 and the estimates of q were 0.58 and 0.60 respectively in MPZC and MM. The values of χ^2 were non-significant in both methods. And sixth treatment (TJM-140) the mean was smaller than variance of the distribution. The estimates of θ were 0.92 and 1.01 and the estimates of q were 0.66 and 0.72 respectively in MPZC and MM. The values of χ^2 were non-significant in both methods.

Table No.-11 Distribution of observed and expected number of green gram plants according to number of insect pest at 29 DAS of the treatment 7 (Virat), treatment 8 (TM-115), treatment 9 (TJM-141).

No. of major insect	Observed freq. treatment 7 (Virat).	Poisson Distribution treatment 7 (Virat).		Observed freq. treatment 8 (PDM-139)	Poisson Distribution treatment 8 (PDM-139)		Observed freq. treatment 9 (TJM-140)	Poisson Distribution treatment 9 (TJM-140)	
		MPZC	MM		MPZC	MM		MPZC	MM
0	60	60.00	56.40	60	60.00	58.37	60	60.00	57.57
1	34	34.64	36.96	33	35.19	36.36	33	32.99	34.73
2	21	22.82	24.32	23	22.98	23.69	21	22.26	23.33
3	15	14.06	14.67	16	14.01	14.25	13	14.12	14.61
4	9	8.27	8.34	8	8.13	8.12	10	8.63	8.72
5	6	4.70	4.55	5	6.11	4.44	6	5.12	5.01
6	3	2.60	2.39	4	2.48	2.36	4	2.88	2.81
7	2	2.91	2.37	1	1.10	2.41	3	1.62	1.53
Total	150	150.00	150.00	150	150.00	150.00	150	150.00	150.00
Mean	1.24	-	-	1.43	-	-	1.23	-	-
Variance	1.97	-	-	2.72	-	-	2.11	-	-
Estimates of parameters	θ	0.92	0.98	θ	0.92	0.94	θ	0.92	0.96
	q	0.63	0.67	q	0.64	0.66	q	0.60	0.63
χ^2		0.68	1.45	χ^2	1.19	0.68	χ^2	0.64	0.98
d.f.		4	4	d.f.	4	4	d.f.	4	4

Table -11 revealed that the insect pest related to green gram plants follows a Polya-Aeppli distribution because during 29 DAS of the seventh treatment (virat) the mean was smaller than variance of the distribution. The estimates of θ were 0.92 and 0.98 and the estimates of q were 0.63 and 0.67 respectively in MPZC and MM. The values of χ^2 were non-significant in both methods. Eighth treatment (TM-115) the mean was smaller than variance of the distribution. The estimates of θ were 0.92 and 0.94 and the estimates of q were 0.64 and 0.66 respectively in MPZC and MM. The values of χ^2 were non-significant in both methods. And ninth treatment (TJM-141) the mean was smaller than variance of the distribution. The estimates of θ were 0.92 and 0.96 and the estimates of q were 0.60 and 0.63 respectively in MPZC and MM. The values of χ^2 were non-significant in both methods.

Table No.-12 Distribution of observed and expected number of green gram plants according to number of insect pest at 29 DAS of the treatment 10 (TJM-136), treatment 11 (TJM-111), treatment 12 (TJM-155), treatment 13 (TJM-137).

No. of major insect	Observed freq. treatment 10 (TJM-136).	Poisson Distribution treatment 10 (TJM-136).		Observed freq. treatment 11 (TJM-111)	Poisson Distribution Treatment 11 (TJM-111)		Observed freq. treatment 12 (TJM-155)	Poisson Distribution treatment 12 (TJM-155)		Observed freq. treatment 13 (TJM-137)	Poisson Distribution treatment 13 (TJM-137)	
		MPZC	MM		MPZC	MM		MPZC	MM		MPZC	MM
0	60	60.00	56.70	60	60.00	56.96	60	60.00	56.96	60	60.00	57.09
1	32	32.99	35.30	35	34.08	35.85	33	34.08	35.85	36	35.73	38.06
2	20	22.26	23.70	20	22.64	23.84	20	22.64	23.84	20	23.15	24.48
3	19	14.12	14.76	12	14.12	14.66	18	14.12	14.66	18	13.94	14.34
4	7	8.63	8.73	10	8.42	8.55	8	8.42	8.55	7	7.98	7.88
5	5	5.12	4.97	7	4.85	4.79	5	4.85	4.79	4	4.40	4.13
6	4	2.88	2.75	4	2.72	2.63	3	2.72	2.63	3	2.34	2.13
7	2	1.62	1.47	2	3.17	2.72	2	1.49	1.37	2	2.42	1.89
Total	150	150.00	150.00	150	150.00	150.00	150	150.00	150.00	150	150.00	150.00
Mean	1.52	-	-	1.49	-	-	1.23	-	-	1.15	-	-
Variance	3.20	-	-	3.11	-	-	2.11	-	-	1.83	-	-
Estimates of parameters	θ	0.92	0.98	θ	0.92	0.97	θ	0.92	0.97	θ	0.92	0.97
	q	0.60	0.64	q	0.62	0.65	q	0.62	0.65	q	0.65	0.69
χ^2		2.26	2.87	χ^2	1.90	2.47	χ^2	1.57	1.79	χ^2	1.74	2.20
d.f.		4	4	d.f.	4	4	d.f.	4	4	d.f.	3	3

Table-12 focused that the insect pest related to green gram plants follows a Polya-Aeppli distribution because during 29 DAS of the tenth treatment (TJM-136) the mean was smaller than variance of the distribution. The estimates of θ were 0.92 and 0.98 and the estimates of q were 0.60 and 0.64 respectively in MPZC and MM. The values of χ^2 were non-significant in both methods. Eleventh

treatment (TJM-111) the mean was smaller than variance of the distribution. The estimates of θ were 0.92 and 0.97 and the estimates of q were 0.62 and 0.65 respectively in MPZC and MM. The values of χ^2 were non-significant in both methods. Twelfth treatment (TJM-155) the mean was smaller than variance of the distribution. The estimates of θ were 0.92 and 0.97 and the estimates of q were 0.62 and 0.65 respectively in MPZC and MM. The values of χ^2 were non-significant in both methods. And thirteenth treatment (TJM-137) the mean was smaller than variance of the distribution. The estimates of θ were 0.92 and 0.97 and the estimates of q were 0.65 and 0.69 respectively in MPZC and MM. The values of χ^2 were non-significant in both methods.

Conclusion- In the suitable probability distribution namely, Poisson and Polya-Aeppli the two methods of estimation of their parameter like MPZC and MM were taken into consideration. The chi square test of goodness of fit was used for fitting of the distributions of the occurrence pattern of insect pest on green gram. Poisson distribution is found to be most adequate distribution for describing the spatial spread of insect pest viz., whitefly, leafhopper and aphids for each treatment during the 15 DAS. The Polya-Aeppli distribution which is a mixture of Poisson and Geometric distribution was also found to be adequate for describing the inherent variability of the insect pest population on green gram during 22 and 29 DAS. The method of proportion of zero and method of moments were found to be suitable for estimating the parameters in the concerned distributions.

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