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# Optimizing Inventory Management through Mathematical Models for Deteriorating Items

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## Abstract

Effective inventory management is crucial for businesses, particularly when dealing with goods that deteriorate over time. This abstract presents a review of mathematical models aimed at optimizing inventory management for deteriorating goods. Traditional inventory models often fail to account for deterioration, leading to suboptimal inventory decisions and increased costs. To address this challenge, various mathematical approaches have been developed. Deterministic modeling techniques utilize equations to describe the relationship between inventory levels, demand, and deterioration rates. Stochastic modeling incorporates uncertainty in demand and deterioration rates using techniques such as stochastic differential equations and Markov processes. Optimization methodologies, including dynamic programming and heuristic algorithms, are employed to determine optimal inventory policies under deteriorating conditions. Recent advancements in machine learning enable predictive analytics, enhancing forecasting accuracy and decision-making in inventory management. These mathematical models and techniques offer businesses the opportunity to minimize holding costs while ensuring sufficient inventory levels to meet demand and mitigate stockouts.

## Introduction

Inventory management plays a pivotal role in the success of businesses across various industries, with effective management being essential for ensuring optimal operations and customer satisfaction. However, managing inventory becomes notably challenging when dealing with goods that deteriorate over time. The management of deteriorating inventory demands careful consideration of factors such as degradation rates, uncertain demand patterns, and optimal ordering policies to minimize costs while maintaining adequate stock levels. Traditional inventory management techniques often overlook the dynamic nature of deteriorating inventory, leading to inefficient resource allocation and increased costs. Consequently, there has been a growing emphasis on developing and implementing mathematical models tailored to address the unique challenges posed by deteriorating goods. This paper examines how mathematical models can optimize inventory management for deteriorating goods, drawing on deterministic and stochastic modeling techniques, optimization methodologies, and advancements in machine learning. By integrating these mathematical approaches, businesses can enhance their inventory management strategies to mitigate the impact of deterioration, minimize holding costs, and improve overall operational efficiency.

## Importance of the Study

The study on optimizing inventory management through mathematical models for deteriorating goods holds significant importance for businesses and industries facing challenges related to perishable or time-sensitive inventory. Effective management of deteriorating inventory is crucial for minimizing costs, reducing waste, and ensuring customer satisfaction. By

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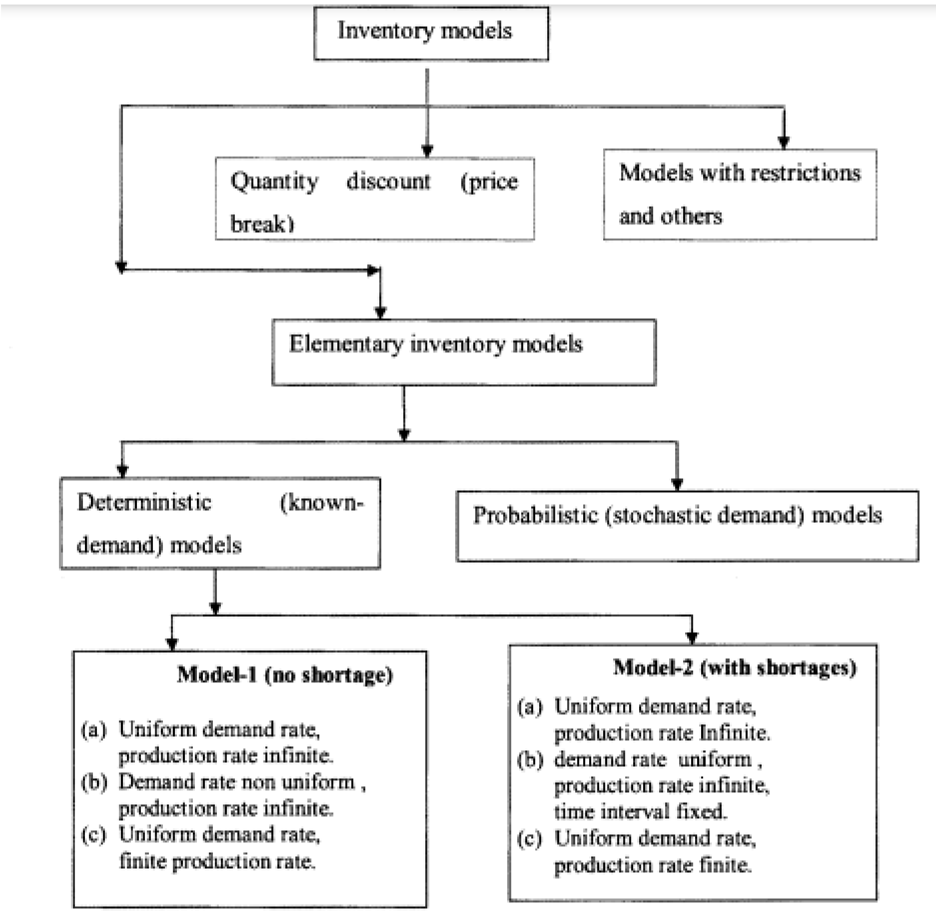
employing mathematical models tailored to address the unique dynamics of deteriorating goods, businesses can make informed decisions regarding inventory levels, ordering policies, and resource allocation. optimizing inventory management for deteriorating goods can lead to enhanced competitiveness in the marketplace, as businesses can better meet customer demands while maximizing profitability. Additionally, with increasing pressure to operate sustainably and reduce environmental impact, efficient management of deteriorating inventory can contribute to minimizing waste and promoting responsible resource utilization. the study's findings have implications for various industries, offering insights and strategies to improve inventory management practices and drive business success in an increasingly dynamic and competitive landscape.

**CLASSIFICATION OF INVENTORY MODELS:**

The classification of inventory models can be better understood using the following skeleton diagram (Fig-1).




As displayed in the above figure inventory models are arranged in three classifications, which are (1) Elementary models (2) Models with value breaks (3) Models with limitations. Again the rudimentary models are grouped into two classes, for example, (I) deterministic inventory model and (ii) probabilistic inventory model.

**Deterministic Inventory Model:**

At the point when a few or each of the boundaries which influence an inventory model like interest, lifetime of items put away, creation rate, lead time, crumbling rate, accessibility of room in store and so on and their utilitarian connections are thought to be known with sureness then the inventory model is called deterministic inventory model.

**The Probabilistic Inventory Model:**
If no less than one of the above talked about boundaries isn't known with conviction ahead of time, however it is viewed as an irregular variable with some known likelihood dispersion then the inventory model including such boundaries is called probabilistic inventory model. In our current exploration work we have concentrated on just the deterministic inventory model. Thusly we have given the arrangement of deterministic inventory model and brief depictions about some of them as follows.

**Classification of deterministic inventory model**
Model 1(a): Classical EOQ model (demand rate uniform, Replenishment rate infinite, no shortage):
It is extremely challenging to plan a solitary general model which considers all varieties in genuine frameworks. Indeed, regardless of whether a particularly model were created, it may not be systematically resolvable. Subsequently inventory models are generally created for some particular circumstances. The old style EOQ model is one of the least complex inventory models. It was first evolved by F. Harris in 1915. The monetary arranged amount (EOQ) is the amount of a thing to be requested to such an extent that it limits the complete yearly (or other time span as dictated by individual firms) cost of conveying inventory and cost of requesting under the accepted states of sureness and known interest.
For the most part the idea of EOQ applies to items which are renewed intermittently into inventory in parcels covering a few periods' need. Suppositions needed in this model are as per the following:

1. Demand is known and uniform (consistent).
2. Shortages are not allowed.
3. Replenishment of stock is momentary or renewal rate is limitless.
4. The framework relates to a solitary thing.
5. Inventory conveying cost and requesting cost per request stay consistent over the long run.
6. Cost of the thing stays consistent over the long run. There are no price breaks or amount markdown.
7. The thing is bought and renewed in parcels or bunches.

Notations used in this model are as follows:
R: Uniform demand rate.
C; cost per unit of the item.
$C_0$: Minimum average cost per unit time.
C, cost of holding one unit in the inventory for a unit of time.
$C_2$: Shortage cost which is infinite i.e. no shortages.
$C_3$ Ordering cost per order.
1: Inventory level.
t: Time.
q0; Optimum quantity to be ordered
The derivation of EOQ formula gives the following results:
Optimum quantity to be ordered during each order is given by, $q_0 = \sqrt{\frac{2C_3R}{C_1}}$

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This is known as the economic order quantity or optimal lot size formula due to R.H. Wilson. It is also called Square root formula or Harris lot size formula.

Average total cost per unit time, $C(t) = \frac{C_1 Rt}{2} + \frac{C_3}{t} = \frac{C_1 q}{2} + \frac{C_3 R}{q}$

Where, q = R t, is the quantity to be ordered in each order. The resulting minimum average cost per unit time,

$$C_0(q) = \sqrt{2C_1C_3F}$$

The total minimum cost per unit time, including the cost of the item

$$\sqrt{2C_1C_3R} + CR$$

Expansion in recurrence of orders will build requesting cost where as diminishing in recurrence of request will expand the volume of each request bringing about increment of holding cost consequently expansion in requesting cost goes with decline in holding cost. The EOQ model decides the amount of request size which limits the amount of these two expenses.

**Mathematical Model:**

The inventory level is zero at the beginning stage for example at/= 0. The creation and supply start all the while and the creation stops at t = Tx at which the most extreme inventory i(M) is reached. The inventory developed at a rate p-d in the span [o,7]] and there is no disintegration during this timeframe. After the time Tx , the delivered units start disintegration and supply is proceeded at a limited rate. As request rate thought to be consistent there is no fall sought after and the inventory diminishes to zero level. This finishes the creation process duration after which the following creation cycle starts and creation begins. Our concern is to observe the ideal creation cycle.

The inventory level of the item at time t over the period [o, T] can be addressed by the differential conditions

$$\frac{dI_1(t)}{dt} = p - d$$

Where, p-d> 0 since shortages are not allowed.

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -d, T_1 \le t \le T$$

Here θ = at, i.e. linear deterioration, where o < a < 1.

With boundary condition $I_1(0) = I_2(T_2) = 0$

Using the boundary condition Ix(0) = 0 solution of equation (1) is

$I_1(t) = (p-d)t$ , $0 \le t \le T_1$

Equation (39) is a linear differential equation.

Integrating Factor of equation (39) is

$$= e^{\int \alpha t dt} = e^{\frac{\alpha t^2}{2}}$$

Using the above integrating factor, solution of equation (39) is

$$I_2(t) e^{\frac{\alpha t^2}{2}} = \int -de^{\frac{\alpha t^2}{2}} dt + c$$

Since 0 < a < 1, neglecting the terms involving second and higher powers of a in the expansion of exponential function we get,

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$$\Rightarrow I_2(t)e^{\frac{\alpha t^2}{2}} = -d\int \left(1+\frac{\alpha t^2}{2}\right)dt + c$$

$$\Rightarrow I_2(t)e^{\frac{\alpha t^2}{2}} = -d\left(t+\frac{\alpha t^3}{6}\right) + c$$

Now using the initial condition $I_2(T_2) = 0$the above we can find the required solution of equation (39) as

$$I_2(t) = d\left(T_2+\frac{\alpha T_2^3}{6} - t - \frac{\alpha t^3}{6}\right)e^{-\frac{\alpha t^2}{2}}$$

**T1≤t≤T**

The production cost per unit time is given by

PC =pk $T_1$/T

The set up cost per unit time is given by

**SC= A/T**

The holding cost per unit time is given by

$$HC = \frac{1}{T}\left[\int_0^{T_1} h(t)I_1(t)dt + \int_0^{T_2} h(t)I_2(t)dt\right]$$

$$= \frac{1}{T}\left[\int_0^{T_1}(a+bt)(p-d)tdt + d\int_0^{T_2}(a+bt)\left[T_2+\frac{\alpha T_2^3}{6} - t - \frac{\alpha t^3}{6}\right)e^{\frac{-\alpha t^2}{2}}dt\right]$$

Since 0 < a < 1, neglecting the terms involving second and higher powers of a in the expansion of exponential function we get,

$$HC = \frac{1}{T}\left[\int_0^{T_1}(a+bt)(p-d)tdt + d\int_0^{T_2}(a+bt)\left(T_2+\frac{\alpha T_2^3}{6} - t - \frac{\alpha t^3}{6}\right)\left(1-\frac{\alpha t^2}{2}\right)dt\right]$$

$$\Rightarrow HC = \frac{1}{T}\left[(p-d)\int_0^{T_1}(at+bt^2)dt\right.$$

$$\left. +d\int_0^{T_2}\left(a-\frac{a\alpha t^2}{2}+bt-\frac{b\alpha t^3}{2}\right)\left(T_2+\frac{\alpha T_2^3}{6}-t-\frac{\alpha t^3}{6}\right)dt\right]$$

$$\Rightarrow HC = \frac{1}{T}\left[\frac{a(p-d)T_1^2}{2}+\frac{b(p-d)T_1^3}{3}\right]$$

$$+d\frac{1}{T}\left[aT_2^2-\frac{aT_2^2}{2}+\frac{bT_2^3}{2}-\frac{bT_2^3}{3}+\frac{a\alpha T_2^4}{6}-\frac{a\alpha T_2^4}{24}-\frac{a\alpha T_2^4}{6}+\frac{a\alpha T_2^4}{8}\right.$$

$$\left.+\frac{b\alpha T_2^5}{12}-\frac{b\alpha T_2^5}{30}-\frac{b\alpha T_2^5}{8}+\frac{b\alpha T_2^5}{10}-\frac{a\alpha^2 T_2^6}{36}+\frac{a\alpha^2 T_2^6}{72}-\frac{b\alpha^2 T_2^7}{48}+\frac{b\alpha^2 T_2^7}{84}\right]$$

$$\Rightarrow HC = \frac{a(p-d)T_1^2}{2T}+\frac{b(p-d)T_1^3}{3T}+\frac{adT_2^2}{2T}+\frac{bdT_2^3}{6T}+\frac{da\alpha T_2^4}{12T}$$

$$+\frac{db\alpha T_2^5}{40T}-\frac{da\alpha^2 T_2^6}{72T}-\frac{db\alpha^2 T_2^7}{112T}$$

Deterioration cost:

The number of units that deteriorate in a cycle is the difference between the maximum inventory and the number of units used to meet the demand. Hence the deterioration cost per unit time is given as

$$DC = \frac{k}{T}\left[I_2(0) - \int_0^{T_2} ddt\right]$$
$$= \frac{k}{T}\left[d\left(T_2 + \frac{\alpha T_2^3}{6}\right) - dT_2\right]$$
$$= \frac{k\alpha dT_2^3}{6T}$$

Price discount:

Price discount per unit time is offered as a fraction of production cost for the units in the period [o,r2]

$$\Rightarrow PD = \frac{kr}{T}\int_0^{T_2} ddt = \frac{krdT_2}{T}$$

Therefore the average total cost per unit time is given by

$$TVC(T) = PC + SC + HC + DC + PD$$
$$= \frac{pkT_1}{T} + \frac{A}{T} + \frac{a(p-d)T_1^2}{2T} + \frac{b(p-d)T_1^3}{3T} + \frac{adT_2^2}{2T} + \frac{bdT_2^3}{6T} + \frac{da\alpha T_2^4}{12T} + \frac{db\alpha T_2^5}{40T} - \frac{da\alpha^2 T_2^6}{72T} - \frac{db\alpha^2 T_2^7}{112T} + \frac{k\alpha dT_2^3}{6T} + \frac{krdT_2}{T}$$

Let us express Tx and T2 in terms off . At the moment when production run is terminated during a cycle i.e.

$$I_1(T_1) = I_2(0)$$
$$(p-d)T_1 = d\left(T_2 + \frac{\alpha T_2^3}{6}\right)$$

Neglecting the term containing a from the right hand side we get

$(p - d)T_1 = dT_2$

$(p - d)T - T_2 = dT_2$

T2 = (p-d) T/p

T1 = dT /p

Using these values of t1 and T2 in equation (8) and (11) we get HC and TVC respectively as

$$HC = \frac{a(p-d)d^2T}{2p^2} + \frac{b(p-d)d^3T^2}{3p^3} + \frac{ad(p-d)^2T}{2p^2} + \frac{bd(p-d)^3T^2}{6p^3} + \frac{da\alpha(p-d)^4T^3}{12p^4} + \frac{\alpha bd(p-d)^5T^4}{40p^5} - \frac{\alpha^2 ad(p-d)^6T^5}{72p^6} - \frac{\alpha^2 bd(p-d)^7T^6}{112p^7}$$

**And**

$$TVC(T) = kd + \frac{A}{T} + \frac{a(p-d)d^2T}{2p^2} + \frac{b(p-d)d^3T^2}{3p^3} + \frac{ad(p-d)^2T}{2p^2} + \frac{bd(p-d)^3T^2}{6p^3} + \frac{da\alpha(p-d)^4T^3}{12p^4} + \frac{\alpha bd(p-d)^5T^4}{40p^5} - \frac{\alpha^2 ad(p-d)^6T^5}{72p^6} - \frac{\alpha^2 bd(p-d)^7T^6}{3p^3} + \frac{k\alpha d(p-d)^3T^2}{2p^2} + \frac{krd(p-d)}{6p^3}$$

To obtain the optimal cycle time we have to minimize the TVC. To minimize the TVC, first of all we have to differentiate with respect to T and equate it to zero

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$$\frac{d}{dT}(TVC(T)) = 0$$

$$\Rightarrow -\frac{A}{T^2} + \frac{a(p-d)d^2}{2p^2} + \frac{2b(p-d)d^3T}{3p^3} + \frac{ad(p-d)^2}{2p^2} + \frac{bd(p-d)^3T}{3p^3} + \frac{da\alpha(p-d)^4T^2}{4p^4} + \frac{\alpha bd(p-d)^5T^3}{10p^5} - \frac{5\alpha^2 ad(p-d)^6T^4}{72p^6} - \frac{3\alpha^2 bd(p-d)^7T^5}{56p^7} + \frac{k\alpha d(p-d)^3T}{3p^3} = 0$$

$$\frac{d^2}{dT^2}(TVC(T)) = \frac{2A}{T^3} + \frac{2b(p-d)d^3}{3p^3} + \frac{bd(p-d)^3}{3p^3} + \frac{da\alpha(p-d)^4T}{2p^4} + \frac{3\alpha bd(p-d)^5T^2}{10p^5} - \frac{5\alpha^2 ad(p-d)^6T^3}{18p^6} - \frac{15\alpha^2 bd(p-d)^7T^4}{56p^7} + \frac{k\alpha d(p-d)^3}{3p^3}$$

The values of T found from equation minimizes the total average variable cost ifthe second order derivative is positive for that T

**Conclusion**

The optimization of inventory management through mathematical models tailored for deteriorating goods is paramount for businesses aiming to enhance operational efficiency, reduce costs, and meet customer demands effectively. This study has underscored the importance of addressing the unique challenges posed by deteriorating inventory, including degradation rates, uncertain demand patterns, and optimal resource allocation. By leveraging deterministic and stochastic modeling techniques, businesses can gain insights into inventory dynamics and make informed decisions to mitigate the impact of deterioration on inventory levels and costs. Optimization methodologies further enable businesses to determine optimal ordering policies and inventory levels that balance costs and service levels efficiently. the integration of machine learning and data-driven approaches empowers businesses with predictive analytics, enabling proactive decision-making and accurate forecasting of demand and deterioration patterns. the findings of this study offer practical strategies and insights for businesses across various industries to improve their inventory management practices for deteriorating goods. By optimizing inventory management, businesses can minimize waste, reduce costs, and enhance customer satisfaction, ultimately gaining a competitive edge in the marketplace. Moving forward, continued research and innovation in mathematical modeling will further advance inventory management capabilities, enabling businesses to adapt to evolving market dynamics and achieve sustained success.

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