A NEW TYPE OF NEIGHBOURHOODS USING SINE TOPOLOGY IN TRIGONOMETRIC TOPOLOGICAL SPACES

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ABSTRACT- In this article we introduce t_s-neighbourhoods using Sin-open sets in trigonometric topological spaces. In addition, we examine their basic properties. Furthermore, we introduce and study the fundamental properties of t_s*-Neighbourhoods in trigonometric topological spaces.

Key words- **t-open, t-closed, t_s-neighbourhood, t_s*-neighbourhood.**

I. INTRODUCTION

In this paper, we present t_s-neighbourhoods in Trigonometric topological spaces. These spaces are based on Sine and Cosine topologies. In a bitopological space we have considered two different topologies but in a trigonometric topological space the two topologies are derived from one topology. From this, we see that the trigonometric topological space is differs from the bitopological space.

II. PRELIMINARIES

Throughout this paper X denotes a topological space that has elements from $[0, \frac{\pi}{2}]$ and $T_u(X)$ denotes the Trigonometric topological space corresponds to the space X with trigonometric topology $\mathcal{T}$. Furthermore, $T_u(X)\backslash A^*$ denotes the complement of A^* in $T_u(X)$. The following definitions are very helpful in the subsequent sections.

Definition: 2.1 Let X be an arbitrary non-empty set that has elements from $[0, \frac{\pi}{2}]$. Let SinX be the set consisting of the Sine values of the corresponding elements of X. Define a function f_s:X→SinX by $f_s(x)$=Sin x. Then f_s is a bijective function. This implies, $f_s(\phi)=\phi$ and $f_s(X)$=Sin X. That is, Sin $\phi=\phi$.

Let τ_s be the set formed by the images (under f_s) of the corresponding elements of τ. Then τ_s form a topology on SinX. This topology is called Sine topology (briefly, Sin-topology) of X. The pair (SinX,τ_s) is called the Sine topological space corresponding to X. The elements of τ_s are called Sin-open sets. $\frac{c}{c}$.

Definition: 2.2 Let CosX be the set consisting of the Cosine values of the corresponding elements of X. Define a function f_c:X→CosX by $f_c(x)$=Cos x. Then f_c is bijective. Also, $f_c(\phi)=\phi$ and $f_c(X)$=CosX. This implies, Cos$\phi=\phi$.

Let τ_{cs} be the set formed by the images (under f_c) of the corresponding elements of τ. Then τ_{cs} form a topology on CosX. This topology is called Cosine topology (briefly, Cos-topology) of X. The pair (CosX,τ_{cs}) is called the Cosine topological space corresponding to X. The elements of τ_{cs} are called Cos-open sets.

Definition: 2.3 Let $T_u(X)$ be a trigonometric topological space. A subset $\mathcal{N}$ of $T_u(X)$ is said to be a t_s-neighbourhood (briefly, t_s-nbd) of $y \in T_u(X)$ if there exists an open set M such that $y \in$ SinM$\subseteq \mathcal{N}$.

Definition: Let $T_u(X)$ be a trigonometric topological space. A subset $\mathcal{N}$ of $T_u(X)$ is said to be a t_s-neighbourhood (briefly, t_s-nbd) of a subset $A\subseteq T_u(X)$ if there exists an open set M such that $A\subseteq \text{Sin}M\subseteq\mathcal{N}$.

Example: $X=\{\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{2}\}$ with $\tau=\{\phi,\{\frac{\pi}{6}\},\{\frac{\pi}{4}\},\{\frac{\pi}{6},\frac{\pi}{4}\},X\}$. Then Sin $X=\{\frac{1}{2},\frac{1}{\sqrt{2}},1\}$ and $\tau_s=\{\phi,\{\frac{1}{2}\},\{\frac{1}{\sqrt{2}}\},\{\frac{1}{2},\frac{1}{\sqrt{2}}\},\text{Sin } X\}$. Also, $T_u(X)=\{0,\frac{1}{2},\frac{1}{\sqrt{2}},\frac{\sqrt{3}}{2},1\}$. Now, $\mathcal{T}=\{\phi,T_i(X),\{\frac{1}{\sqrt{2}},\frac{\sqrt{3}}{2}\},\{\frac{1}{2},\frac{1}{\sqrt{2}}\},\text{Sin } X,\text{Cos } X,\{0,\frac{1}{2},\frac{1}{\sqrt{2}},\frac{\sqrt{3}}{2}\},\{\frac{1}{2},\frac{1}{\sqrt{2}},\frac{\sqrt{3}}{2}\},\{\frac{1}{2},\frac{1}{\sqrt{2}},\frac{\sqrt{3}}{2},1\},T_u(X)\}$. Let $\mathcal{N}=\{\frac{1}{2},\frac{1}{\sqrt{2}},\frac{\sqrt{3}}{2}\}$. Then $\mathcal{N}$ is a t_s-nbd of $\frac{1}{2}$, since $\{\frac{\pi}{6}\}$ is an open set such that $\frac{1}{2}\in\text{Sin}\{\frac{\pi}{6}\}\subseteq\mathcal{N}$.

Remark: A t_s-nbd need not be Sin-open and t-open. For example, consider Example 3.3, the subset $\{\frac{1}{2},\frac{1}{\sqrt{2}},\frac{\sqrt{3}}{2}\}$ is a t_s-nbd of $\frac{1}{2}$ but it is not a Sin-open set. Also, the subset $\{1,0,\frac{1}{\sqrt{2}}\}$ is a t_s-nbd of $\frac{1}{\sqrt{2}}$ but it is not a t-open set.

Proposition: Let $T_u(X)$ be a trigonometric topological space and N be a subset of X. Then

1. $\mathcal{N}$ is a t_s-nbd of y if and only if there exists a Sin-open set $\mathcal{M}$ such that $y\in\mathcal{M}\subseteq\mathcal{N}$ &
2. $\mathcal{N}$ is Sin-open if and only if it is a t_s-nbd of each of its points.

Proof:

1. Assume that $\mathcal{N}$ is a t_s-nbd of y. Then there exists an open set H such that $y\in\text{Sin } H\subseteq\mathcal{N}$. Let $\mathcal{M}$=Sin H. Then $\mathcal{M}$ is a Sin-open set and $y\in\mathcal{M}\subseteq\mathcal{N}$. Conversely, assume that there exists a Sin-open set $\mathcal{M}$ such that $y\in\mathcal{M}\subseteq\mathcal{N}$. Since $\mathcal{M}$ is Sin-open, we have $\mathcal{M}$=Sin H, where H is open in X. Thus, there exists an open set H such that $y\in\text{Sin } H\subseteq\mathcal{N}$. Therefore, $\mathcal{N}$ is a t_s-nbd of y.
2. Assume that $\mathcal{N}$ is Sin-open. Then for each $y\in\mathcal{N}$, there exists a Sin-open set $\mathcal{N}$ such that $y\in\mathcal{N}\subseteq\mathcal{N}$. Therefore,   is a t_s-nbd of each of its points. Conversely, assume that $\mathcal{N}$ is a t_s-nbd of each of its points. Then for each point of $\mathcal{N}$, there exists a Sin-open set contained in $\mathcal{N}$. This implies, $\mathcal{N}$ is the union of these Sin-open sets. Therefore, $\mathcal{N}$ is Sin-open.

Remark: If $\mathcal{N}$ is a t_s-nbd of some of its points, then $\mathcal{N}$ need not be Sin-open. For example, consider Example 3.3, the subset $\mathcal{N}=\{1,0,\frac{1}{\sqrt{2}}\}$ is a t_s-nbd of $\frac{1}{\sqrt{2}}$ but not a Sin-open set.

Proposition: Let $T_u(X)$ be a trigonometric topological space. If $\mathcal{N}$ is a t-open set, then $\mathcal{N}$ is a t_s-nbd of each of the points of some Sin-open set $\mathcal{M}$.

Proof: Assume that $\mathcal{N}$ is a t-open set. Then $\mathcal{N}$ is the union of Sin-open, Cos-open and the set $T_i(X)$. Let this Sin-open set be $\mathcal{M}$. Then for each $y\in\mathcal{M}$, we have $y\in\mathcal{M}\subseteq\mathcal{N}$. This implies, $\mathcal{N}$ is a t_s-nbd of each point of $\mathcal{M}$. Hence the proof.

Remark: The converse of the above Result is not true. For example, consider Example 3.3, the subset $\mathcal{N}=\{\frac{1}{2}\}$ is a t_s-nbd of each of its points but it is not a t-open set.

Proposition: Let $T_u(X)$ be a trigonometric topological space. If A is a Sin-closed subset of Sin X and $y\in\text{Sin } X\backslash A$, then there exists a t_s-nbd $\mathcal{N}$ of y such that $\mathcal{N}\cap A=\phi$.

Proof: Assume that A is a Sin-closed set and $y\in\text{Sin}X\backslash A$. Then $\text{Sin}X\backslash A$ is a Sin-open set containing y. This implies, $\text{Sin}X\backslash A$ is a t_s-nbd of y. Let $\mathcal{N}=\text{Sin}X\backslash A\subseteq T_u(X)$. Then $\mathcal{N}$ is a t_s-nbd of y. Also, $\mathcal{N}\cap A=\phi$.

Definition: Let $T_u(X)$ be a trigonometric topological space and $y\in T_u(X)$. The set of all t_s-nbd of y is called the t_s-nbd system at y and is denoted by t_s-N(y).

Proposition: Let $T_u(X)$ be a trigonometric topological space. Then

1. $t_s\text{-N}(y)\neq\phi$ for all $y\in\text{Sin}X$,

2. if $\mathcal{N}\in t_s$-N(y), then $y\in\mathcal{N}$,
3. if $\mathcal{N}\in t_s$-N(y) and $\mathcal{N}\subseteq\mathcal{M}$, then $\mathcal{M}\in t_s$-N(y),
4. if $\mathcal{N}\in t_s$-N(y) and $\mathcal{M}\in t_s$-N(y), then $\mathcal{N}\cap\mathcal{M}$, $\mathcal{N}\cup\mathcal{M}\in t_s$-N(y).

Proof:

1. Since SinX is the Sin-open set, we have SinX is the t_s-nbd of each of its points.
2. Assume that $\mathcal{N}\in t_s$-N(y). Then by definition of t_s-nbd, $y\in\mathcal{N}$.
3. Assume that $\mathcal{N}\in t_s$-N(y) and $\mathcal{N}\subseteq\mathcal{M}$. Then there exists a Sin-open set H* such that $y\in H^*\subseteq\mathcal{N}$. This implies, $y\in H^*\subseteq\mathcal{M}$. Therefore, $\mathcal{M}$ is a t_s-nbd of y. Hence $\mathcal{M}\in t_s$-N(y).
4. Assume that $\mathcal{N},\mathcal{M}\in t_s$-N(y). Then there exist Sin-open sets H_1^* and H_2^* such that $y\in H_1^*\subseteq\mathcal{N}$ and $y\in H_2^*\subseteq\mathcal{M}$. This implies, $y\in H_1^*\cup H_2^*\subseteq\mathcal{N}\cup\mathcal{M}$ and $y\in H_1^*\cap H_2^*\subseteq\mathcal{N}\cap\mathcal{M}$. Since H_1^* and H_2^* are Sin-open, we have $H_1^*\cup H_2^*$ and $H_1^*\cap H_2^*$ are Sin-open. Therefore, $\mathcal{N}\cup\mathcal{M}$ and $\mathcal{N}\cap\mathcal{M}$ are t_s-nbd of y. Hence $\mathcal{N}\cup\mathcal{M}$, $\mathcal{N}\cap\mathcal{M}\in t_s$-N(y).

Proposition: Let $T_u(X)$ be a trigonometric topological space and $y\in T_u(X)$. If $\mathcal{N}\in t_s$-N(y), then there exists $\mathcal{M}\in t_s$-N(y) such that $\mathcal{M}\subseteq\mathcal{N}$ and $\mathcal{M}\in t_s$-N(x) for all $x\in\mathcal{M}$.

Proof: Let $\mathcal{N}\in t_s$-N(y). Then there exists a Sin-open set $\mathcal{M}$ such that $y\in\mathcal{M}\subseteq\mathcal{N}$. Since $\mathcal{M}$ is Sin-open, we have $\mathcal{M}$ is a t_s-nbd of each of its points. Therefore, $\mathcal{M}\in t_s$-N(x) for all $x\in\mathcal{M}$. In particular, $\mathcal{M}\in t_s$-N(y).

Proposition: Let $T_u(X)$ be a trigonometric topological space and $y\in T_u(X)$. If $y\notin$SinX, then t_s-N(y)=ϕ.

Proof: Assume that $y\notin$SinX. Then there is no Sin-open set $\mathcal{M}$ such that $y\in\mathcal{M}\subseteq\mathcal{N}$. This implies, $\mathcal{N}$ is not a t_s-nbd of y. Therefore, t_s-N(y)=ϕ for every $y\notin$SinX.

t_s*-NEIGHBOURHOODS

In this section we introduce a new type of neighbourhoods namely t_s*-neighbourhood. Also, we furnish some of their basic properties.

Definition: Let $T_u(X)$ be a trigonometric topological space. A subset N of X is said to be a t_s*-neighbourhood (briefly, t_s*-nbd) of $x\in X$ if there exists a trigonometric open set M such that Sin $x\in M\subseteq$Sin N.

Definition: Let $T_u(X)$ be a trigonometric topological space. A subset N of X is said to be t_s*-nbd of a subset A of X if there exists a trigonometric open set M such that $\text{SinA}\subseteq M\subseteq\text{SinN}$.

Example: Let $X=\{\frac{\pi}{6},\frac{\pi}{4},\frac{\pi}{2}\}$ with $\tau=\{\phi,\{\frac{\pi}{6}\},\{\frac{\pi}{2}\},\{\frac{\pi}{6},\frac{\pi}{2}\},X\}$. Then Sin $X=\{\frac{1}{2},\frac{1}{\sqrt{2}},1\}$ and $\tau_s=\{\phi,\{\frac{1}{2}\},\{1\},\{\frac{1}{2},1\},\text{Sin X}\}$. Also, $T_u(X)=\{0,\frac{1}{2},\frac{1}{\sqrt{2}},\frac{\sqrt{3}}{2},1\}$. Now, $\mathcal{T}=\{\phi,T_i(X),\{\frac{1}{\sqrt{2}},\frac{\sqrt{3}}{2}\},\{0,\frac{1}{\sqrt{2}}\},\{\frac{1}{2},\frac{1}{\sqrt{2}}\},\{1,\frac{1}{\sqrt{2}}\},\text{Sin X},\text{Cos X},\{0,\frac{1}{2},\frac{1}{\sqrt{2}}\},\{1,\frac{1}{\sqrt{2}},\frac{\sqrt{3}}{2}\},\{1,0,\frac{1}{\sqrt{2}}\},\{0,\frac{1}{2},\frac{1}{\sqrt{2}},\frac{\sqrt{3}}{2}\},\{1,0,\frac{1}{\sqrt{2}},\frac{\sqrt{3}}{2}\},\{\frac{1}{2},\frac{1}{\sqrt{2}},\frac{\sqrt{3}}{2}\},\{\frac{1}{2},\frac{1}{\sqrt{2}},\frac{\sqrt{3}}{2},1\},\{0,\frac{1}{2},\frac{1}{\sqrt{2}},1\},T_u(X)\}$ is a trigonometric topology corresponding to X. Let $N=\{\frac{\pi}{6},\frac{\pi}{4}\}$. Then N is a t_s*-nbd of $\frac{\pi}{6}$, since $\mathcal{M}=\{\frac{1}{2},\frac{1}{\sqrt{2}}\}$ is a trigonometric open set such that $\text{Sin}(\frac{\pi}{6})\in\mathcal{M}\subseteq N$.

V. CONCLUSION

In this paper we have introduced and studied the basic properties of t_s-Neighbourhoods and t_s*-Neighbourhoods in trigonometric topological spaces.

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